



2 **Construction, categorization, and consensus: student**  
3 **generated computational artifacts as a context**  
4 **for disciplinary reflection**

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8 **Abstract** There are increasing calls to prepare K-12 students to use computational tools and  
9 principles when exploring scientific or mathematical phenomena. The purpose of this paper is  
10 to explore whether and how constructionist computer-supported collaborative environments  
11 can explicitly engage students in this practice. The Categorizer is a Javascript-based inter-  
12 active gallery that allows members of a learning community to contribute computational  
13 artifacts they have constructed to a shared collection. Learners can then analyze the collection  
14 of artifacts, and sort them into user-defined categories. In a formative case study of the  
15 Categorizer for a fractal activity in three middle grade (ages 11–14) classrooms, there was  
16 evidence that participating students began to evaluate fractals based on structural and  
17 mathematical properties, and afterward could create algorithms that would generate fractals  
18 with particular area reduction rates. Further analysis revealed that students' construction and  
19 categorization experiences could be better integrated by explicitly scaffolding discussion and  
20 negotiation of the categorization schemes they develop. This led to the development of a new  
21 module that enables teachers and students to explore points of agreement and disagreement  
22 across student categorization schemes. I conclude with a description of limitations of the  
23 study and environment, implications for the broader community, and future work.

24 **Keywords** Computational thinking · Constructionism · Collaborative  
25 environments · Middle school · Disciplinary practices · Mathematics education  
26

27 **Introduction**

28 Many collaborative technological environments are beginning to incorporate the con-  
29 struction of computational artifacts—such as simulations, games, or algorithms—as a way

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30 to participate in a learning community. As a collection, these artifacts can illustrate  
31 important patterns and themes in a domain of study. In this paper I draw from existing  
32 theories of learning and pedagogy to argue that such collections of student-generated  
33 artifacts hold untapped potential to help learners connect what they learn from constructing  
34 individual examples of a topic in math and science to the ways in which they organize and  
35 investigate that topic more generally. Specifically, I explore whether encouraging learners  
36 to identify patterns within their own collective work is one way to help them to attend to  
37 deep structural properties of objects within a domain, rather than only surface features—an  
38 important and difficult skill.

39 The Categorizer is a Javascript-based interactive gallery designed to encourage students  
40 to reflect on the themes and patterns evident within collections of their own and other's  
41 computational artifacts. It allows members of a learning community to (1) build and  
42 contribute artifacts to a shared space, (2) organize those artifacts into meaningful cate-  
43 gories learners define themselves, and (3) review similarities and differences across dif-  
44 ferent categorization systems. The Categorizer is designed to engage learners in making  
45 sense of connections between *construction* by defining computational rules used to produce  
46 an artifact, and *disciplinary reflection* by allowing them to explore and organize those same  
47 artifacts according to the themes and properties they find most relevant for the domain they  
48 represent.

49 I report on a case study of the Categorizer in three middle grade (ages 11–14) class-  
50 rooms during a lesson about the mathematical structure of fractals, an increasingly popular  
51 way to explore fractions and functional reasoning in the middle grades (NCTM Illumi-  
52 nations 2003; Romburg and Kaput 1999). Findings suggest participating students identified  
53 connections between the computational rules they used to construct the fractals and ways  
54 of organizing those fractals mathematically, as evidenced by their use of the environment  
55 itself and on pre–post questionnaires. Further analysis highlights classroom interactions  
56 that especially encouraged students to explore these connections. These findings led to  
57 refinement of the tool itself, and suggested activities and challenges to consider for future  
58 implementations of shared collaborative galleries. I describe limitations of the study and  
59 environment, implications for the broader community, and future work.

## 60 Motivation and background

61 Educators and policymakers agree that contemporary education should engage students in  
62 the practices, skills, and core ideas that underlie a discipline—for example, by engaging in  
63 argumentation and supporting claims with evidence in science (NRC 2007, 2012), or  
64 finding patterns and moving across representations in mathematics (CCSSI 2010; NCTM  
65 2000). Many technology-mediated collaborative learning environments provide tools and  
66 infrastructures for learners to engage in such practices by contributing to a shared col-  
67 lection of knowledge, or working toward a common goal. Examples include CSILE/  
68 Knowledge Forum (Scardamalia and Bereiter 1994; Scardamalia 2004), the Collaboratory  
69 Notebook (Edelson et al. 1996), the Math Forum (Renninger and Shumar 2002), WISE  
70 (Linn et al. 2003), SAIL (Slotta and Aleahmad 2009), and Science Online (Forte and  
71 Bruckman 2007). A major goal of such environments is to enable students to aggregate,  
72 engage with, and make sense of their collective contributions to a shared knowledge base.

73 At the same time, creating and using technology-mediated artifacts and tools is in itself  
74 a central aspect of what it means to learn, participate, and create knowledge in a discipline  
75 (diSessa 2000; NRC 2010, 2012; Papert 1980, 1996; Wing 2006). Simulations, statistical



76 models and data, interactive visualizations, and technology-mediated experimentation all  
77 serve important roles in STEM practice (Chandrasekharan 2009; Kress and van Leeuwen  
78 2001; Sabelli 2006). Correspondingly, many learning environments enable students to  
79 construct and use computational artifacts to explore ideas in math and science (diSessa and  
80 Abelson 1986; Jackson et al. 2000; Kahn 1996; Konold and Miller 2005; Papert 1980;  
81 Reppenning et al. 2000; Resnick et al. 2009; Wilensky 1999).

82 Recently, many learning environments have started to integrate both collaborative and  
83 constructive approaches: so that *constructing artifacts* is one of the very ways that learners  
84 can contribute to *collaborative inquiry*. The WebLabs and Playground environments  
85 enable students to construct and share programmed games and mathematical models (Noss  
86 and Hoyles 2006) and the Science Created by You (SCY) environment requires students to  
87 build executable simulations as part of collaboratively pursuing a problem scenario solu-  
88 tion (de Jong et al. 2012). Code Breaker has students collaborate over a network to  
89 construct and test cipher algorithms to decrypt a coded message (White 2009), and SAIL  
90 Smart Space allows students to assign tags to and sort student solutions for analysis  
91 (Tissenbaum et al. 2012). Networked SimCalc aggregates the results of students' algebraic  
92 investigations using a multi-representational collective display. Studies suggest that these  
93 sorts of integrated environments can facilitate community discourse (Ares et al. 2009), help  
94 students connect personal experiences to disciplinary learning (Hegedus and Moreno-  
95 Armella 2009) and afford powerful new learning activity structures (Brady et al. 2013).

96 The current project builds on this work by putting both construction and aggregation/  
97 classification of computational artifacts into students' hands to be explored and negotiated.  
98 Like constructing knowledge, determining how that knowledge is organized is an  
99 important component of reasoning in the STEM disciplines. The purpose is to explore the  
100 pedagogical potential of such an approach, and determine how designers and educators can  
101 realize that potential.

## 102 Theoretical framework

103 This project adopts the perspective that there is something special about creating and  
104 classifying computational artifacts as a way to participate in collaborative mathematical  
105 and scientific inquiry: computational ideas provide new and powerful ways of thinking  
106 about math and science phenomena (diSessa 2000; Papert 1980).

107 In describing the role of computation in creating new knowledge through mathematical  
108 experimentation, Bailey and Borwein (2011) note that "Never have we had such a cor-  
109 nucopia of ways to generate intuition. The challenge is to learn how to harness them, how  
110 to develop and how to transmit the necessary theory and practice" (p. 1419). This high-  
111 lights the interrelationship of two aspects of integrating computation and disciplinary  
112 practice: (1) understanding how to use computational tools to explore a topic in math or  
113 science, and (2) identifying how doing so can inform one's exploration of disciplinary  
114 phenomena more broadly.

115 Constructionism: creating public computational artifacts

116 There is a long tradition of research exploring how programming and computational  
117 construction can help students explore STEM phenomena (diSessa 2000; Kafai and Re-  
118 snick 1996; NRC 2010, 2012; Wilensky and Resnick 1999; Wilensky and Reisman 2006).  
119 One goal of such approaches is to connect mathematical and scientific ideas to students'



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120 experiences or expectations of how things work by having them make things “work”  
121 themselves. For example, the LOGO programming language allows students to generate  
122 complex geometric figures by instructing a *turtle* to combine actions like moving and  
123 turning in complex ways (Papert 1980). Constructing computational artifacts also  
124 encourages students to combine multiple ideas into a cohesive process, organize their  
125 understandings in new ways, and ‘debug’ understandings if their instructions produce  
126 something unexpected. The approach has especially been linked to students’ learning of  
127 underlying structure, causal mechanism, and the epistemological aspects of a domain of  
128 study (Blikstein and Wilensky 2009; Harel and Papert 1991; Sherin 2001).

129 Another important component of Constructionism is that the artifacts students create  
130 should be public, so that students feel ownership over their constructions, learn from one  
131 another, and receive critique. This is particularly important when thinking about the role  
132 that collaborative learning environments can play in supporting computational construction  
133 for learning. For the purposes of this study, I will use the term *computational artifacts* to  
134 refer specifically to digital objects that students have created using a programming lan-  
135 guage or computational construction kit and have contributed to a public collaborative  
136 environment. Given the importance of the relationship between building and sharing in  
137 Constructionism, both a representation of the programmed rules or building blocks used to  
138 generate the artifact (the representation of “how things work”) as well as a representation  
139 of the outcome when those rules are executed, are included as part of the artifacts that are  
140 shared among users.

#### 141 Computational thinking: strategies for complex problems

142 Computational thinking (NRC 2010; Wing 2006) is often described as a set of ideas,  
143 strategies, and habits of mind that are useful for solving problems across curricular  
144 domains. It is one aspect of what diSessa (2000) calls computational literacy: the use of  
145 computational tools, ideas, and representations in the same way text and language are used  
146 in traditional literacy. For example, ideas such as automation, optimization, and recursion  
147 are useful for thinking about how to approach complex problems in any domain. This has  
148 led educators to explore integrating computational principles and ideas into STEM courses  
149 (Clark and Ernst 2008; Hambruch et al. 2009) and beyond (Dierbach et al. 2011). A  
150 considerable amount of this work has focused on K-12 education (Barr et al. 2011; Bers  
151 2010; Repenning et al. 2010), given the increased attention to problem solving and  
152 knowledge construction at these levels. While many suggest that programming and  
153 computational thinking approaches can increase students’ analytical thinking skills (Kur-  
154 land et al. 1986) and learning of other STEM content, research is still needed (Grover and  
155 Pea 2013; NRC 2010). For the purposes of this study, I am interested in whether students  
156 connect the *computational ideas* they leverage to construct individual artifacts—that is, the  
157 programmed rules and building blocks that are used—to inform what *patterns and themes*  
158 they identify in the learning community’s collection of work.

159 The current project builds on the theories of constructionism and computational  
160 thinking to posit the following theoretical conjecture: one important part of making sense  
161 of a domain is exploring its themes, core ideas, and patterns as a disciplinary community  
162 (collaborative disciplinary inquiry). If those themes, core ideas, and patterns are illustrated  
163 by categorizing objects that students themselves create as they explore that domain  
164 (computational artifacts), students are likely to leverage the shared computational  
165 knowledge, experiences, and strategies they used when constructing those objects (com-  
166 putational thinking) in order to do so. To explore this conjecture, the case study I present in



167 this paper focuses on one particular relationship between computational thinking and  
168 disciplinary inquiry skills, in one particular domain of study: that between computational  
169 *algorithms/rules*, and the skill of *pattern finding and classification* in the study of fractals  
170 and fractal structure.

171 Thinking about rules and finding patterns

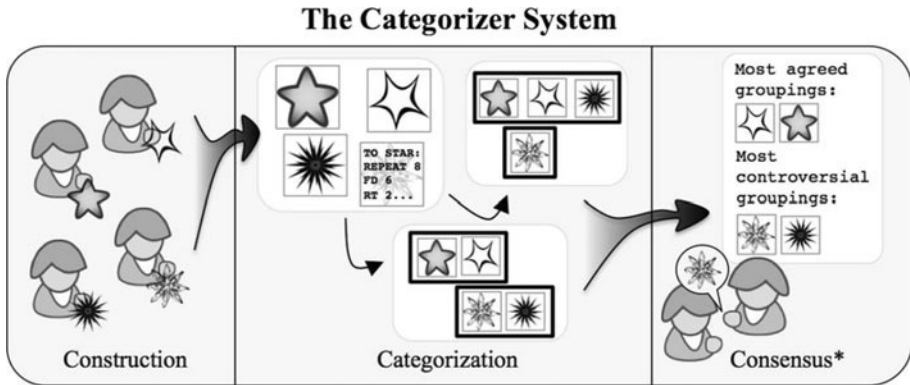
172 *Algorithms* are a core idea in computer science and mathematics. Although many defini-  
173 tions of “algorithm” exist, they are generally characterized as a set of rules for how to take  
174 some input or starting state and produce a corresponding output or end state (NRC 2010)—  
175 for example, algorithms are used to multiply multi-digit numbers, or to tell a computer how  
176 to sort a list of numbers. Another important aspect of being able to understand algorithms  
177 is being able to predict what the output of a given algorithm will be for a given input (NRC  
178 2012; Wing 2006). In The Categorizer, users must construct their objects by defining some  
179 such set of rules that the computer follows to generate the object; and are exposed to a  
180 collection of other objects that have been generated by other combinations of those same  
181 rules.

182 *Pattern finding and classification* involves observing and noting similarities and dif-  
183 ferences across related specimens that reflect a mathematical or scientific question or  
184 phenomenon. Making sense of patterns has been identified as a core crosscutting skill in  
185 the recent K-12 Science Framework (NRC 2012), and students’ ability to identify and  
186 make sense of structure and express regularity has been valued by the mathematical  
187 community for years (CCSMI 2010; NCTM 2000). An important part of classification  
188 involves evaluating and grouping objects at different levels of analysis or representation,  
189 including at the level of microscopic elements, underlying structures, or relational/  
190 behavioral processes. The Categorizer is designed to help students to explore whether and  
191 how the rules they use to construct objects might produce certain observable patterns in the  
192 finished objects, and to experience the often dramatic differences in how objects look at the  
193 rules versus output level.

194 The intersection of *rules* and *pattern finding* is powerful for a number of reasons. There  
195 is a large body of literature documenting the importance—and difficulty—that learners  
196 face in differentiating between surface and deep structure in science and mathematics (e.g.,  
197 Chi et al. 1981). More generally, understanding how learners organize a collection of  
198 examples can reveal what they understand to be the core ideas and perspectives in a  
199 domain. By allowing users to generate their own categorization schemes and make explicit  
200 the purpose of each category, the tool emphasizes that there are multiple ways to organize a  
201 domain of study and encourages students to make their reasoning explicit. The Categorizer  
202 seeks to encourage learners to build connections at deeper levels by using “deep level”  
203 rules and algorithms to construct their own computational objects, then providing them  
204 access to a shared gallery that allows students to explore both the rules and algorithms of  
205 other constructions, as well as their final forms, when deciding how to organize and make  
206 sense of the collection as a whole.

## 207 Design of the categorizer

208 The Categorizer is a flexible, web-based Javascript framework that integrates three  
209 interfaces representing three related activities: a Construction Interface, Categorization  
210 Gallery, and Theme Processor (Fig. 1). Its design is based on the theory and conjecture



**Fig. 1** A schematic of the categorizer system, which aggregates student constructions into a shared gallery, and student categorizations of those constructions into classroom-level themes. The theme processor module to support consensus-building (marked by an *asterisk*) was added as a result of findings from the current study

211 articulated above, though the Theme Processor was introduced during a second round of  
212 development based on findings from the preliminary implementation, as described in the  
213 next section.

214 To work within the Categorizer environment, students create one or more artifacts using  
215 the *Construction Interface*, which can be any computational toolkit that allows students to  
216 export a visual representation of (1) a set of rules and (2) the resulting artifact to a URL as  
217 image files. In this paper, I describe a study that allowed users to upload the rules (set of  
218 transformation functions) and resulting recursive patterns for Iterated Function System  
219 (IFS) fractals. However, any topic areas characterized by a complex relationship between  
220 underlying rules/processes and surface structure would work, such as functions and their  
221 resulting graphs (Leinhardt et al. 1990), iteratively generated geometric figures and their  
222 generating code (Papert 1980), or emergent visuospatial patterns derived from similar  
223 individual interaction rules (Goldstone and Wilensky 2008).

224 Each contribution from a community member is then uploaded for display to a shared  
225 *Categorization Gallery* with all other artifacts constructed by a given learning community.  
226 Users visiting the gallery can double click on any object to see its underlying rules. When  
227 ready, a user can create one or more category windows to sort the artifacts into meaningful  
228 groupings that they deem important, or that have been requested by a facilitator or teacher.  
229 The user would enter a name and description for each category window they create. Once  
230 all of the objects have been sorted into categories, the user can save the categorization  
231 scheme.

232 The *Theme Processor* was added to the Categorizer as a result of the formative case  
233 study described in this paper, and allows a facilitator or teacher to view an aggregated  
234 summary of how a particular community of learners is choosing to organize their col-  
235 lection. This summary uses simple matrix decomposition methods to analyze the collection  
236 of categorization schemes that users produce. This information can then be used to inform  
237 the facilitator of which sets of gallery objects students often group together, and which  
238 items are not grouped similarly by different students and hence may reflect borderline or  
239 controversial cases for the classroom population as a whole.

240 These three different modules reflect three underlying broad and interrelated theories of  
241 learning guiding the overall development and implementation of the Categorizer:





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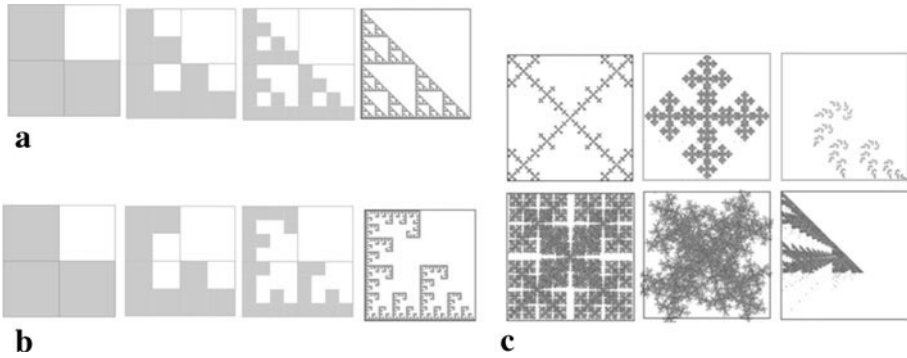
242 constructivism/constructionism, collaborative knowledge-building, and disciplinary  
243 engagement. Drawing from constructivist theories of learning and constructionist theories  
244 of pedagogy, students create and obtain feedback about their own fractals using the  
245 Construction Interface and when interacting with others in the Categorization Gallery,  
246 always have access to the rules used to create those fractals. To support collaborative  
247 knowledge-building students' constructions are all accessible to one another, and the  
248 Theme Processor helps the class explore other ways of classifying and thematically ana-  
249 lyzing those contributions. Finally, as students contribute diverse objects, the nature of  
250 their potential classifications may shift or be redefined. Since the categories students use to  
251 sort and describe one another's objects are created by students themselves and made visible  
252 through the Theme Processor, students' *own* meaning-making processes around the topic  
253 of interest, as well as the class' consensus building around those themes, are emphasized,  
254 rather than organizations introduced by an outside authority.

### 255 Case study: creating and analyzing fractal structure

256 To explore whether the Categorizer does support students' exploration of the connections  
257 between underlying computational rules and ways of classifying objects that represent a  
258 particular phenomenon, I conducted one implementation of the Categorizer tool in the  
259 context of a lesson on iterated function system fractals (Demko et al. 1985) in three middle  
260 school mathematics classrooms. This provided a context to test the first version of the tool,  
261 explore the learning theories that underlie its design, and inform subsequent development  
262 and refinement. Case studies such as this are one way to conduct research on educational  
263 interventions in situ, especially during early phases of development. They are well-suited  
264 for research that aims to maintain sensitivity to contextual influences, and can reveal  
265 unexpected dimensions that affect how technology designs are used in real settings such as  
266 classrooms (Khan 2008). They also allow researchers to collect in-depth data that can  
267 speak to students' social interactions and understandings of complex subject matter, and to  
268 challenge or extend theory (Yin 2008).

269 *Iterated Function System Fractals*, henceforth referred to as IFS fractals or just fractals,  
270 are self-similar geometric figures defined by a set of geometric affine transformation  
271 functions (the "function system") to be applied to a set of points, in this case to a unit  
272 square. The fractal is generated by recursively copying all points in the unit square into  
273 each transformation, such that a copy of the unit shape is repeated inside each defined  
274 transformation. These repeated transformations reflect the *algorithm* that is the focus of the  
275 exploration, the squares that define the transformations are its rules and the resultant fractal  
276 the output. For example, Fig. 2a, b feature two IFS fractals: Fig. 2a is a version of the  
277 familiar Seirpinski gasket that is defined by three transformations defining scale reductions  
278 and translations of the unit square to a non-overlapping triangle arrangement. Figure 2b is  
279 the fractal generated when the topmost transformation also includes a 90° rotation. Fig-  
280 ure 2c features a number of fractals created by students during the study to illustrate the  
281 diversity of forms that can be generated.

282 As a content area, fractals represent an especially productive context for studying the  
283 integration of computational object creation and collaborative inquiry for a number of  
284 reasons. By their nature, the space of IFS fractals is rich with a diversity of themes that  
285 may emerge as students select different sets of transformation rules. These sets of rules  
286 exhibit systematic, mathematically important relationships. For example, IFS fractals often  
287 feature geometric, branch-like, or shell/fern-like structures. They are often 'crisp' or



**Fig. 2** Examples of IFS fractals and their underlying rules. **a** Sierpinski triangle, **b** Sierpinski w/rotation, **c** Example IFS fractals

288 well-defined when transformations do not intersect, but cloudy or fuzzy when they do.  
 289 Certain arrangements of transformations produce figures that do not illustrate fractal  
 290 structure because they do not appear to repeat within themselves. And, the area occupied  
 291 by points that undergo recursive transformations reduces measurably during each iteration  
 292 of the function system. Fractals have been applied to the study of a number of topics, from  
 293 the study of cancer (Baish and Jain 2000) to the development of computer graphics  
 294 (Demko et al. 1985). Using fractals to explore fractions and functional reasoning is  
 295 increasingly popular in middle grades mathematics because they are engaging, cognitively  
 296 complex, and technology-mediated (NCTM 2001, 2003; Romberg and Kaput 1999).

297 Constructing public fractals can also provide learners with new lenses into the structure  
 298 of the content area and new motivations to explore that structure. Complex fractals are  
 299 often generated from a small set of simple rules requiring deep understanding. There is a  
 300 high potential for rich diversity of student-produced fractal types to be categorized, and  
 301 designing fractals is seen by many as a personally expressive/aesthetic pursuit.

### 302 Research question

303 Given (a) the increasing centrality of computational objects in STEM practice and in  
 304 computer-supported collaborative inquiry environments, (b) the claim that such approaches  
 305 are useful in particular because they might provide a new way to help students integrate  
 306 computational thinking into disciplinary inquiry, and (c) a particular focus on connections  
 307 between *algorithms/rules* and *classification*, the research question driving this study is:

308 To what extent do learners who use the Categorizer build connections between the  
 309 *computational rules/algorithms* used to construct individual fractal objects, and the  
 310 *organizing themes* they identify within collections of such objects?

### 312 Methods

#### 313 *Participants*

314 This study was conducted with three middle grades mathematics classes at a small, sub-  
 315 urban K-8 school in the Midwestern United States. Six 6th grade, six 7th grade, and eight





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316 8th grade students (total  $n = 20$ ) students consented to participate in the study. The stu-  
317 dents' teacher assisted in planning the classroom activities, and was present during the  
318 implementation. All students had prior exposure to the fractal construction tool through a  
319 previous workshop. During that workshop, the classroom teacher and facilitators decided  
320 during some class sessions to print out students' fractals and compare them with one  
321 another at the end of class. This decision led to the conjecture that more tightly coupling  
322 construction and classification activities using a computer-mediated environment might  
323 encourage students to draw connections between deep structure and pattern finding. The  
324 present study was the first time students interacted with the first version of the Categorizer.

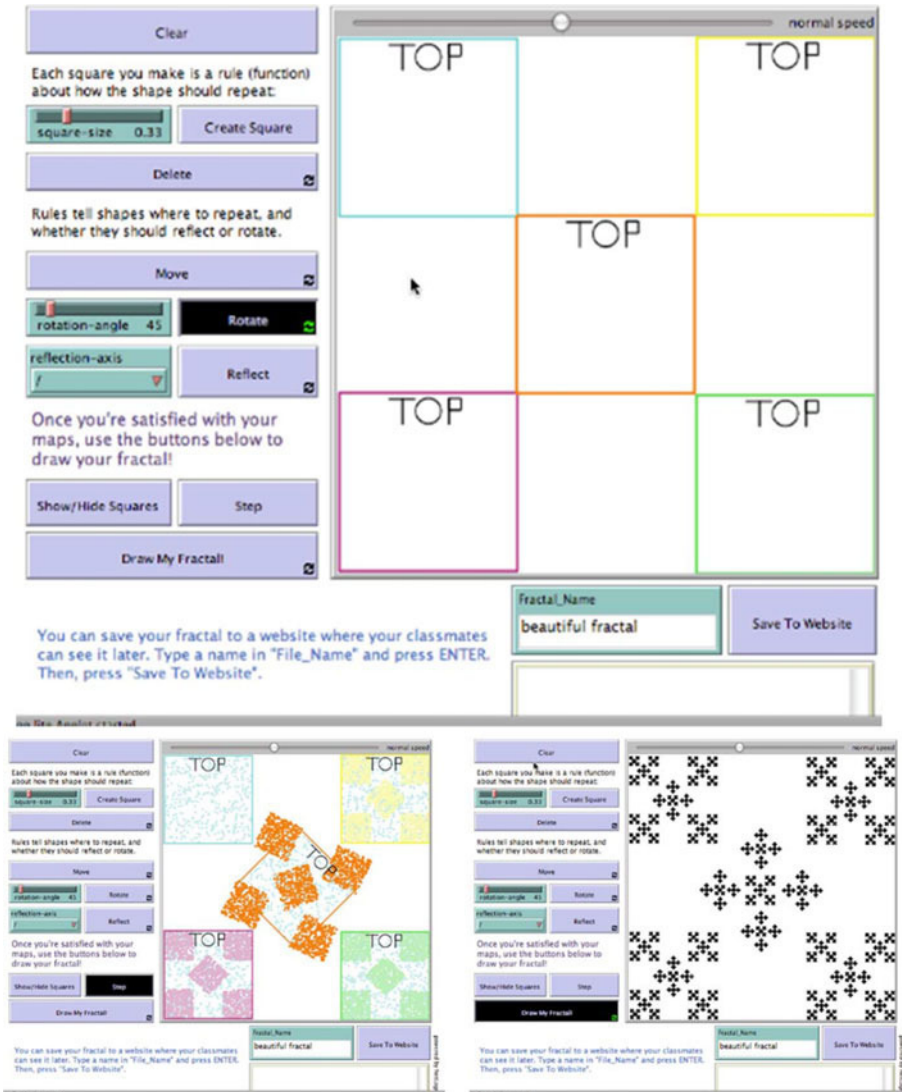
### 325 *Class activities*

326 Each class session was 1 h long. First, each session started with a short paper-and-pencil  
327 warm up activity designed to remind students of their prior lesson on fractals and of the  
328 construction interface, as well as to collect baseline data on students' understandings of  
329 fractal structure, described in more detail below. Next, students were allowed to freely  
330 explore the fractal construction interface (Fig. 3), including the new "save to gallery"  
331 feature. We asked students to try and make sense of the connections between certain rules  
332 they chose to construct fractals and the resulting features within those fractals. During our  
333 previous lesson, students had completed challenges to create fractals with specific qualities  
334 (such as "spiral fractals" or "spongy fractals") and were hence familiar with such a  
335 request.

336 After about 10 min, students were introduced to the Categorization interface of the tool  
337 (Fig. 4). This introduction included an explicit mention that they could double click a  
338 given image to see the rules used to generate it. Students were then given a series of  
339 prompts on the board to complete over the class session. There were not assigned times  
340 that each task would begin and end, although students were given a warning about 15 min  
341 before the end of the activity. Instead the goal was to better understand how students  
342 themselves would interact with the environment as a constructive and collaborative tool.  
343 The first prompt was to create groupings that reflected any patterns they found interesting,  
344 to familiarize themselves with the interface, and to collect their first impressions of what  
345 themes make sense to capture from the fractal collection. Next, they were to try and create  
346 categories that reflected the mathematical properties about fractals discussed during the last  
347 class session: things such as density, "crispness" or "fuzziness;" structure (twisting,  
348 branching, sponging); area reduction; and so on. The hope was that since students had  
349 spent time during both class sessions exploring the connections between rules and struc-  
350 ture, they would explore and cite fractal rules as part of this classification process. At the  
351 end of the session, students completed a short follow up questionnaire.

### 352 *Data collection*

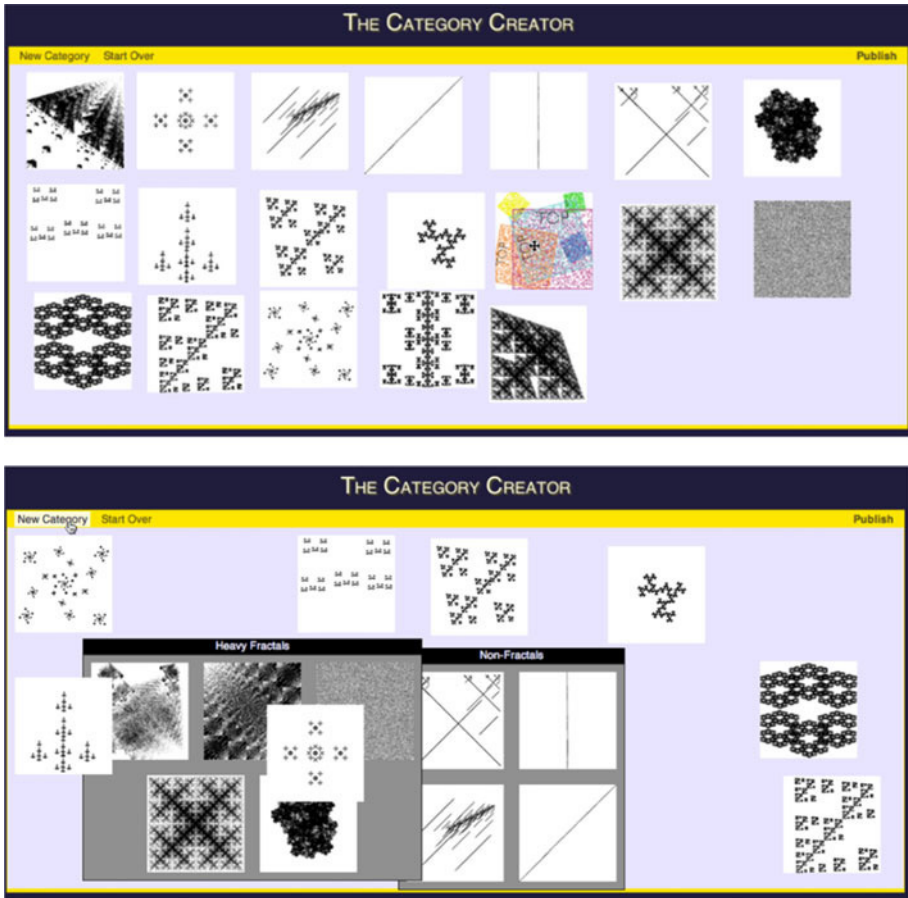
353 Data collected during the study included the written pre and post questionnaires, syn-  
354 chronized video and screen capture of one consenting focal student per class using  
355 Camtasia (Techsmith 2004), and Categorizer usage log files. At the beginning and end of  
356 each class, students completed a pre questionnaire that asked them to predict the next  
357 "step" or iteration for a set of fractal rules ("Item 1"; understanding of algorithm), to  
358 predict from a set of four possibilities the fractal figure that would result from those rules  
359 ("Item 2", understanding of algorithm), and to generate two sets of rules that would  
360 produce two different fractals exhibiting a particular area reduction factor ("Item 3,"



**Fig. 3** Fractal construction interface (*top*) and resulting computational objects, illustrating stepwise iterations (*bottom left*) and final 'infinite' product (*bottom right*)

361 connection between algorithm and theme). Since IFS fractals are a relatively new,  
 362 exploratory topic area in the middle grades, items were loosely adapted from a higher-level  
 363 textbook (Alligood et al. 2000) specifically for this study. Items for students in grade 6  
 364 were simpler than those in grades 7 and 8 in that items for 7th and 8th graders included not  
 365 only translation, but also rotation rules.

366 Camtasia video captured students' discussions with one another and class facilitators, as  
 367 well as all on-screen activity including the fractals that focal students generated and their  
 368 ongoing interactions with the Categorizer interface. Finally, the Categorizer log files captured  
 369 students' categorization schemes created over the course of each class session including time



**Fig. 4** Categorization gallery featuring fractal objects and their underlying rules, accessed by double-clicking a fractal (*top*). Fractals can be placed into one or more user-generated category windows (*bottom*)

370 stamps, the text entered by students as titles and descriptions of categories and categorization  
371 scheme, and adjacency tables that indicated which objects students grouped together.

### 372 *Analysis*

373 Categorizer log file data was analyzed using a bottom-up grounded theory approach  
374 (Glaser and Strauss 1967) to characterize themes students generated while classifying  
375 artifacts within the environment, and identify to what extent those themes included aspects  
376 of the deep structure rules they were using to generate those artifacts. Pre and post  
377 questionnaires were scored for whether students produced correct or incorrect responses  
378 for each item, which would indicate that they had started to understand the algorithms that  
379 generate fractals (Items 1 & 2), and build connections between fractal rules and thematic  
380 mathematical properties (Item 3) after working with the environment. Finally, video of  
381 focal students was coded to identify what activities within the software tool (e.g., con-  
382 struction, categorization, exploration) and within the classroom environment (e.g., dis-  
383 cussion) they engaged in over the course of the class session. Detailed descriptions and



384 examples of coded data for each of these coding schemes are provided in the results  
385 section.

386 To establish reliability, an independent rater also analyzed a representative subset of at  
387 least 20 % of each data corpus. Agreement for log file coding was 87 %, questionnaire  
388 scoring was 100 %, and video coding was 86 %. Most disagreements in video coding were  
389 a result of student multitasking—in particular, *discussing* while also engaging in some  
390 other activity with the software. When these discussion disagreements were resolved  
391 among coders, reliability of video coding rose to 95 % agreement.

## 392 Results

393 Overall, results indicate that while students generated a number of categorization systems  
394 including many that reflected *potential* connections between rules and organizational  
395 themes, only a few students explicitly linked these connections in their category  
396 descriptions. But although this suggests some students may not have formed such con-  
397 nections, significantly more students did create multiple sets of rules to generate fractals  
398 belonging to a requested mathematical category after the intervention than before. In this  
399 section, these findings are described in more detail, and further analysis of one focal  
400 student's interaction with the Categorizer is provided to shed light on why students may  
401 not have made more explicit connections between rules and categories in the environment.  
402 The following section describes resulting modifications to the environment and activity  
403 structures to address these findings.

### 404 Log file data

405 Over the course of the entire implementation, a total of 68 categorization schemes were  
406 uploaded. Of these, 35 % of the schemes either explicitly referred to rules, or identified  
407 features of fractals that were directly related to features of their underlying rules. The  
408 majority of categorization schemes, however, did not mention rules explicitly, and 65 %  
409 could be related to rules only tangentially or not at all. Table 1 includes full descriptions of  
410 the categories identified, explicit criteria used to identify each category during analysis,  
411 examples of each, and overall results of the analysis. There was no evidence of meaningful  
412 differences between grade levels in categorization schemes created. For example, there  
413 was not consistent change in the proportion of rules-based to non-rules-based schemes  
414 students employed by grade level, and one instance of rule-based categories emerged from  
415 a Grade 6 student and the other from a Grade 8 student.

### 416 Pre-post questionnaire data

417 While the categories that students generated while interacting with the Categorizer indicate  
418 that only some students explicitly referenced rules when generating categories for their  
419 class' fractal collection, there is more evidence that students began to link rules to deep  
420 structure in the pre and post questionnaire data. This is especially evident when analyzing  
421 student responses to Item 3 of the pre-post questionnaire.<sup>1</sup>

1FL01 <sup>1</sup> Items 1 and 2 on the post questionnaire were designed to be more difficult than those on the pre  
1FL02 questionnaire. Pre-post differences on both items were not significant (Item 1,  $W = 12, p = n.s.$ ; Item 2,  
1FL03  $W = 28, p = n.s.$ ).



Construction, categorization, and consensus

**Table 1** Student categorization schemes, by presence of connection between theme and underlying fractal rules

	Description	Example	N (/68)	%
More evidence of connection	At least one category explicitly cites <i>rules</i> in its description	“Simple first steps” ( <i>features fractals made from only a few nonoverlapping transformations</i> )	2	3
	At least once category refers to a feature <i>uniquely determined by a rule</i> (examples include density, rotations, reduction in area)	“These are fractals that look fuzzy” ( <i>features fractals that result from nonoverlapping transformations</i> )	22	32
	At least one category refers to <i>fractal structure</i> such as shape or self-similarity	“They all have triangles” ( <i>features fractals with triangular structure</i> )	19	28
Less evidence of connection	At least one category identifies fractals as <i>recognizable or aesthetically pleasing</i>	“Each fractal looks like there are little people inside of them” ( <i>features fractals for which a link to systematic rules is difficult to determine</i> )	12	18
	Categories do not satisfy any of the above descriptions	“Mine/not mine”	13	19

Examples drawn from student log file data



**Table 2** Number of students with correct responses on pre and post questionnaires by item ( $N = 20$ ; Wilcoxon paired signed rank test)

	Pre	Post	W	$N_{diff}$	$p$
Item 3 (at least one rule)	5	14	0	9	$p < .005$
Item 3 (two or more rules)	4	11	5	9	$p < .025$

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Item 3 dealt explicitly with connecting mathematical themes to construction rules by asking students to create two different sets of rules that would produce fractals that would look different, but that each exhibit the same rate of area reduction (area reduction rates varied between  $\frac{3}{4}$  and  $\frac{5}{9}$ , all well within target grade levels; NCTM 2000). Table 2 reports the number of correct responses on Item 3 of pre and post versions of the questionnaires administered during the intervention. Student responses are marked correct if there is evidence students included boxes intentionally sized to approximate fractional units of the area reduction rate, positioned in nonoverlapping configurations. For example, a reduction factor of  $\frac{3}{4}$  would be marked correct if it included three boxes, each approximately one-fourth of the total area of the square, positioned so no area of the squares are overlapping. Both responses were marked correct if at least one box was visibly translated, rotated, or reflected in the second picture. A two-tailed Wilcoxon paired signed rank test indicates that significantly more students produced more sets of rules that would generate fractals belonging to a category that exhibits a certain rate of area reduction for Item 3 on the post questionnaire, including a significant number of students who moved from generating no rule sets to two or more.

One explanation for this is that over the course of the session, students began to attend to the ways in which fractal rules mimic the area model of fraction because this was one of many prompts used during the activity. However, like Items 1 and 2, this topic had been covered in our previous session, yet there was marked improvement on this item and not the others. Furthermore, of the students that generated rules for Item 3, some included features specific to the construction interface, or extra transformations of the rule beyond only defining the needed area (see Fig. 5). This suggests that at least some students were actively connecting their constructive experience *within* the environment with normative ways of classifying *multiple different* fractal structures.

But why, if so few students explicitly mentioned rules in their organization of fractal categories, were they so adept at articulating rules to create multiple different fractals exhibiting a particular mathematical theme? And, how could this apparent connection be leveraged and built upon using the Categorizer environment?

451 In-depth analysis

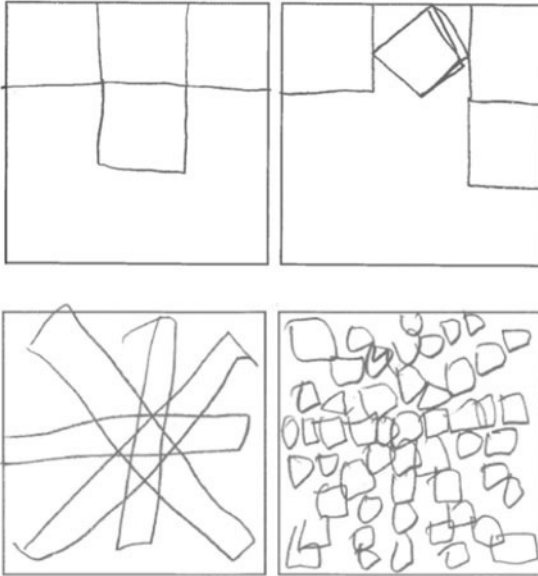
452 One way to shed light on this apparent tension is through in-depth analysis of what actually  
 453 happens when students interact with the Categorizer environment. For the purposes of this  
 454 paper, analysis focuses on one of the four focal students, 6th grader Carol.<sup>2</sup> I focus on  
 455 Carol because she never explicitly, or ostensibly implicitly, connected fractal rules to the  
 456 organizations she created within the Categorizer: her only saved scheme was coded in  
 457 Table 1 as “Aesthetic”. To describe Carol’s experience using the tool, I present her  
 458 activity within the environment, as well as important events completed within it, using a

2FL01 <sup>2</sup> Pseudonym.

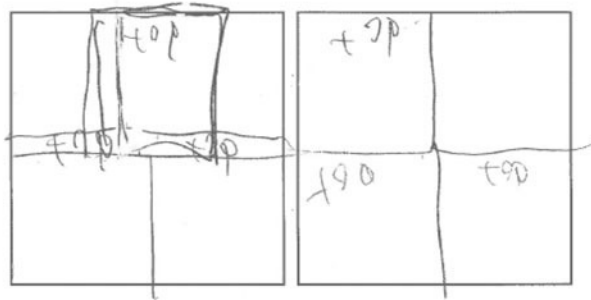




Below, draw the rules for two fractals that will look different, but that will both shrink to  $\frac{4}{9}$  of their existing area in each step:



Below, draw the rules for two fractals that will look different, but that will both shrink to  $\frac{3}{4}$  of their existing area in each step:

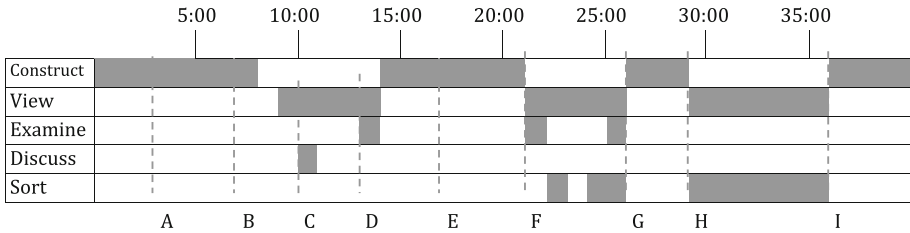


**Fig. 5** Examples of student post questionnaire responses measuring students' linking of mathematical and computational properties. The first examples show valid (*top*) and invalid (*middle*) rules for fractals that illustrate an area reduction factor of  $\frac{4}{9}$  (7th & 8th grade question). The second example (*bottom*) shows valid rules for an area reduction factor of  $\frac{3}{4}$  (6th grade question)

459 time series diagram (Fig. 6; similar to, but simpler than, those described in Hmelo-Silvere  
 460 et al. 2011).

461 Figure 6 represents Carol's navigation within and across different parts of the Categ-  
 462 orizer on a minute-to-minute basis over the course of her class session. The timeline was  
 463 constructed using screen capture and synchronized student video to identify times when  
 464 Carol was:

- 465 • *constructing* fractals (viewing and interacting with the Categorizer construction  
 466 screen),



**Fig. 6** Timeline of Categorizer use by focal student, Carol

- 467 • *viewing* those of her classmates (viewing the categorization screen, including moving
- 468 fractals within the interface without placing them into categories)
- 469 • *examining* the rules of particular fractals in the gallery (double-clicking fractals in the
- 470 categorization interface to view their rules)
- 471 • *discussing* what she was doing with her peers (speaking with other students during the
- 472 activity as captured on synchronized video)
- 473 • *sorting* gallery items (creating categories and placing fractals into them)

474 Three things are immediately obvious: First, Carol did not apparently spend much time  
 475 examining her peers' fractals, or discussing them with others. Second, she did not begin to  
 476 sort the fractals into categories until almost the end of class, even though students were  
 477 asked to do so sooner. Third, Carol spent most of her time moving between constructing  
 478 her own fractals and viewing and examining particular fractals within the shared gallery,  
 479 rather than analyzing the fractals as a group.

480 An analysis of exactly what objects Carol constructed, referenced, and examined during  
 481 this time sheds further light on this pattern. The vertical lines labeled A-I in Fig. 6 cor-  
 482 respond to the events listed in Table 3. It appears that Carol was primarily moving between  
 483 the Categorizer Gallery and the Construction Interface so that she could identify particular  
 484 patterns she especially liked and reproduce them as her own—for example, she twice  
 485 returned from examining a particular fractal in the Gallery to reconstruct that fractal herself  
 486 (or something close to it, see events D, E and G, H). This pattern also seems consistent with  
 487 the way that Carol did categorize the objects when she engaged in sorting activity (events  
 488 G and I)—she seemed primarily concerned with which fractals she liked, and which she  
 489 herself constructed.

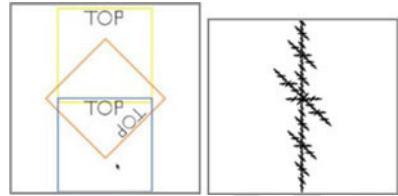
490 This suggests some important strengths and areas for improvement in Carol's experi-  
 491 ence. First, it is clear that Carol not only sensed ownership for her constructions, but was  
 492 trying to systematically learn more about how she could build objects she found inter-  
 493 esting—by uncovering and reproducing their underlying rules. Second, Carol was  
 494 exploring a particular 'theme' that connected rules and output—one that reflected her own  
 495 interest in objects she created or wished to create. This theme was reflected in her sorted  
 496 categories as well as in the way she identified and attempted to reproduce particular  
 497 patterns. Third, Carol was interested in identifying, learning more about, and discussing  
 498 her peers' constructions.

499 It seems, then, that Carol found construction and her own sense of ownership over  
 500 artifacts as interesting and rewarding enough that it interrupted (during event G) and  
 501 played an important role (during event I) in how she classified the larger group of objects.  
 502 This general pattern was evident in other case studies of focal students, and was noticed  
 503 more generally by facilitators at the implementation. What was missing from such an

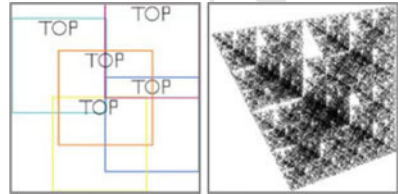


**Table 3** Review of events marked on the timeline featured in Fig. 6

A Carol contributes fractal 1 (right)

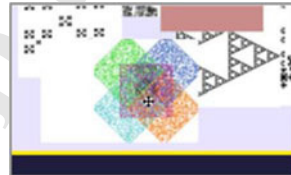


B Carol contributes fractal 2 (right)

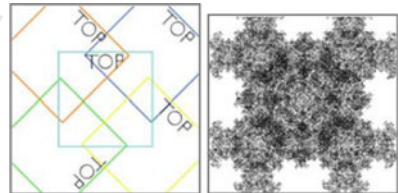


C Carol calls out to two classmates that she has located their fractals in the categorization gallery

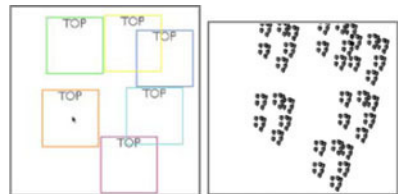
D Carol examines a fractal in the gallery (right)



E Carol contributes fractal 3, which appears to be a copy of the rules of the fractal she examined (right)



F Carol contributes fractal 4 (right)



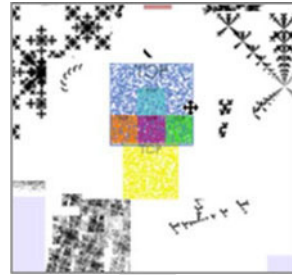
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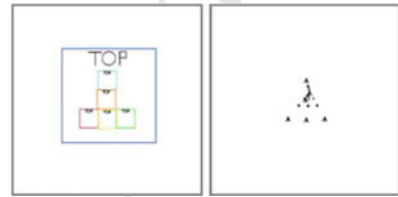


**Table 3** continued

G Carol examines fractal (right) in the midst of categorizing by ownership into categories entitled “mine” and “others”, and returns to the construction interface



H Carol contributes a new fractal, whose rules mimic but do not replicate features of the rules of the fractal she examined (right)



I Carol sorts fractals into aesthetic categories (entitled “iwish” and “other”)

504 investigation wasn't Carol's sense of connection or an emphasis on the relationship  
505 between rules, outcomes, and themes—but rather a motivation to push themes beyond  
506 aesthetic interest toward structural or mathematical foci. In the next section, I describe  
507 modifications to the tool and to supporting activities that might provide such motivation.

508 Carol's story is not unique. Of the four focal students for which I have data like Carol's,  
509 all of the students spent the most time constructing artifacts (between 60 and 71 % of time  
510 spent, versus Carol's 55 %). But unlike Carol, the rest of the students spent less time  
511 viewing their peers' fractals without sorting them (between 8 and 10 % of time spent,  
512 versus Carol's 40 %). Interestingly, the only focal student to have spent more than 2 % of  
513 his time examining the underlying rules of fractals was also the only to have classified  
514 fractals by themes that bore explicit mathematical meaning (related to density and self-  
515 similarity).

516 These patterns of use suggest that even though students were encouraged to explore  
517 these themes during the session, they did not spend much time doing so. It makes sense that  
518 they might not find those themes intrinsically interesting right away. What students *did*  
519 engage with was the construction activity and their ability to view and share artifacts and  
520 identify those of their peers. There are indications even in student log file data that this may  
521 have been a widespread pattern: Students created on average more than 2 fractals for each  
522 categorization scheme saved (143 fractals/68 categorization schemes  $\approx 2.1$ ).

## 523 Discussion

524 The Categorizer is a specific ongoing project, but reflects a broader goal shared by the  
525 educational technology community: to engage students in STEM knowledge construction



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526 practices by enabling them to express and test their ideas using a computational medium.  
527 Therefore, there are two levels of contribution of this work. One lies in the design and  
528 refinement of the tool itself, described in this section, and the other lies in the design  
529 principles gleaned from its use and study that might be more generally informative to the  
530 educational technology community, described in the next section.

531 In terms of the design and refinement of the Categorizer tool in particular, these findings  
532 suggest that more connection between student construction—something the students are  
533 motivated and engaged in doing—and the identification of more mathematically and sci-  
534 entifically relevant themes might be in order. In particular, it seems that one way to engage  
535 students in exploring more scientifically or mathematically relevant themes is to tie that  
536 investigation explicitly to more opportunities for students to construct and investigate  
537 individual fractals.

538 This finding directly led to one refinement to the Categorizer tool itself, and two specific  
539 activity structures that will be integrated into future implementations of the tool. First, the  
540 *Theme Processor*, described in the Design section toward the beginning of this paper, was  
541 added to the environment as a result of this study. This will highlight points of agreement  
542 and disagreement at the category level. The aim is to provide students a more explicit sense  
543 of ownership and opportunity for discussion around categorization themes, much in the  
544 same way they were already engaging in discussion around the individual objects they  
545 constructed. Second, future implementations of the tool will involve activities designed to  
546 motivate a connection between fractal rule sets and *categorization themes* (versus only  
547 specific *fractal objects*). Two examples of activities that can help foster such integration  
548 include (1) a “Build for My Category” challenge—where students must create novel  
549 fractals that can be classified as members of existing themes determined by their peers or  
550 the classroom as a whole; and (2) a “Recreate My Categories” challenge, where students  
551 challenge their peers to uncover and articulate the connective threads that pull together  
552 different categories in a given student’s scheme.

553 While the reported study was only a preliminary case-based exploration of the original  
554 Categorizer environment, a larger design-based research project (DBR; Brown 1992;  
555 Collins 1992) can shed even further light on whether and how students leverage knowledge  
556 related to computational thinking to make sense of mathematical and scientific phenomena,  
557 and how they can be supported. The DBR approach involves designing and researching  
558 theory-based educational interventions in real educational contexts in a way that is iterative  
559 and reciprocal. The goal is to develop interventions, learning theories, and design  
560 frameworks that are scalable and sensitive to the realities of educational practice. Scholars  
561 have highlighted the potential DBR especially holds for the development and study of  
562 technology-enhanced learning environments (Wang and Hannafin 2005), especially those  
563 that involve new educational content, practices, or approaches (Cobb et al. 2003). This  
564 study would have benefitted from returning to the same classroom to investigate how the  
565 same students might interact and learn differently given the new design features. However,  
566 this was difficult given the realities of the academic school year. This highlights the  
567 importance and tension of working with school and site partners on iterative design pro-  
568 jects that require repeated, and at times unexpected, rounds of formative study and testing.

## 569 Conclusions

570 The goal of this study was to explore whether and how a computer-based environment—  
571 based on constructionist and collaborative learning principles—could support the insight



572 that computational construction activities might contribute to learners' more general  
573 STEM inquiry practices. Findings suggest that (1) such an environment can help students  
574 begin to explore the connections between rules and the features of their resulting outputs,  
575 and that (2) while some students also begin to connect themes they identify to features of  
576 rule sets, (3) this exploration might be even better supported by giving students more  
577 of use to assume ownership of *classification schemes themselves*, in addition to the objects  
578 within.

579 This study was preliminary, and only took place over the course of 1 day during a short  
580 mathematics unit. Despite this, students generated sophisticated fractal structures, began to  
581 explore important mathematical properties of those structures, and offered more ways to  
582 construct fractals with particular mathematical properties after interacting with the system.  
583 These early results should be extended by investigating longer sustained classroom  
584 engagement with such an environment, and by investigating how students and teachers  
585 may use the environment in other domains.

586 Although exploratory, this study does suggest that The Categorizer represents one way  
587 of leveraging students' sense of creativity and ownership to help them explore relation-  
588 ships between computational exploration and other forms of scientific and/or mathematical  
589 inquiry. It also highlights the difficulty and importance of merging these practices in ways  
590 that remain authentic to and contribute to mathematical and scientific inquiry. A larger  
591 design-based research project that investigates the relationship between all three designed  
592 modules, students' interaction with the system and one another, and their learning of  
593 mathematical relationships among fractal objects can further highlight how different  
594 aspects of students' reasoning are supported by these design elements. On a theoretical  
595 level, the findings suggest a new area to explore regarding how computer-based learning  
596 environments that attempt to integrate construction and inquiry allow students to explicitly  
597 establish ownership and investment in products of *both* of these processes in a connected  
598 way: the artifacts they generate, as well as the ways in which they might generate meaning  
599 and insight into what those artifacts represent.

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605  
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