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Geometric Series:

If $|r| < 1$, then $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$

If $|r| \geq 1$, then the series diverges

*Note: If the geometric series does not start at $k=0$, it can still be solved for. The NEW a value must be computed (the first value of the series). Simply write them out every time.

Eg: $\sum_{k=4}^{\infty} \left( \frac{1}{5} \right)^k$ $|r| < 1$, converges $\frac{a}{1-r} = \frac{\left( \frac{1}{5} \right)^4}{1-\frac{1}{5}} = \frac{1}{500}$

*Note: NEW a value for not starting at k=0 needs to be checked (should always do so)

Eg: $\sum_{k=1}^{\infty} 3 \left( \frac{1}{2} \right)^k \rightarrow a \text{ value} = 1$ [NOT 3]

*Note: $\sum_{k=1}^{\infty} \frac{2^k}{3^{k+2}} = \frac{1}{3} \sum_{k=1}^{\infty} \left( \frac{2}{3} \right)^k$ [for addition]

$S_n = \frac{a(1-r^n)}{(1-r)}$ (finite series)

*Note: If it is raised to a power different than just “r,” it can still be solved (just make it “r”) [for multiplication]

Eg: $\sum_{k=0}^{20} \left( \frac{2}{5} \right)^{2k} = 1 * \frac{1 - \left( \frac{4}{25} \right)^{21}}{1 - \frac{4}{25}}$

Eg: $\sum e^{-2k} \rightarrow \sum \left( \frac{1}{e^2} \right)^k$

Telescoping series:

Expand out the sums to find a grouping pattern that can simplify in order to take the limit of $S_n$

*Note: Some series can be turned into subtraction via partial fractions or by log rules

Eg: $\sum \frac{1}{(k+1)(k+2)} = \sum \left( \frac{1}{k+1} - \frac{1}{k+2} \right)$