**Use the form \((x-r)\):** If it is \((3-x)\), change it to \(-(x-3)\)

**Simple Partial Fractions:** \((x-r)\) factors

1. Factor denominator
2. Solve for the roots of the denominator
3. Plug in those roots into the numerator and denominator for both to create partial fractions

\[
\int \frac{5x - 10}{x^2 - 3x - 4} \, dx = \int \frac{5x - 10}{(x - 4)(x + 1)} \, dx = \int \frac{A}{x - 4} + \frac{B}{x + 1} \, dx
\]

\[
A = \frac{5(4) - 10}{4 + 1} = \frac{10}{5} = 2
\]

\[
B = \frac{5(-1) - 10}{-1 - 4} = \frac{-15}{-5} = 3
\]

\[
\int \frac{2}{x - 4} + \frac{3}{x + 1} \, dx = 2 \ln|x - 4| + 3 \ln|x + 1| + C
\]

Alternate method:

1. Factor denominator
2. Decompose the fraction
3. Multiply both sides of the equation by the original function’s denominator
4. Group coefficients of each power of \(x\)
5. Set each grouped order equal to the coefficients of those powers of the original numerator
6. Solve for the numerator constants

\[
\frac{5x - 10}{x^2 - 3x - 4} \, dx = \int \frac{5x - 10}{(x - 4)(x + 1)} \, dx = \int \frac{A}{x - 4} + \frac{B}{x + 1} \, dx
\]

\[
5x - 10 = A(x + 1) + B(x - 4)
\]

\[
5x - 10 = Ax + A + Bx - 4B
\]

\[
= (A + B)x + A - 4B
\]

\[
A + B = 5 \quad \text{and} \quad A - 4B = -10
\]

\[
A = 2, B = 3
\]

\[
\int \frac{2}{x - 4} + \frac{3}{x + 1} \, dx = 2 \ln|x - 4| + 3 \ln|x + 1| + C
\]

Extra factoring tip:

\[x^4 - 10x^2 + 9\]

To break this down, you know there are two second order differences of squares

To make it easier to see, set \(u = x^2\):

\[u^2 - 10u + 9 \rightarrow (u - 9)(u - 1) \rightarrow (x^2 - 9)(x^2 - 1) \rightarrow (x - 3)(x + 3)(x - 1)(x + 1)\]
Repeated Linear Factors Partial Fractions: Higher powered factors

1) Factor the denominator however possible
2) Check the highest order power for each factored term
   a. There must be a partial fraction for each power of \((x-r)\)
3) Solve for the known quantities as in the alternate method provided before

\[
\int \frac{2x + 4}{x^3 - 2x^2} \, dx = \int \frac{2x + 4}{x^2(x - 2)} \, dx = \int \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x - 2} \, dx
\]

\[
2x + 4 = A(x - 2) + Bx(x - 2) + Cx^2
\]

\[
= Ax - 2A + Bx^2 - 2Bx + Cx^2
\]

\[
= x^2(B + C) + x(A - 2B) - 2A
\]

\[
B + C = 0 \quad \text{and} \quad A - 2B = 2 \quad \text{and} \quad -2A = 4
\]

\[
A = -2, \ B = -2, \ C = 2
\]

\[
\int \frac{-2}{x^2} + \frac{-2}{x} + \frac{2}{x - 2} \, dx = \frac{2}{x} - 2 \ln |x| + 2 \ln |x - 2| + C
\]

Irreducible Quadratic Factors: Factors in the denominator that are quadratic but not reducible (eg: \(x^2+1\))

1) Factor the denominator however possible
2) If left with an irreducible quadratic in the denominator, check the highest order
   a. The numerator for that partial fraction should contain a variable of the order: degree of denominator-1 and all those until \(x^0\)
3) Set up partial fractions and solve via the alternate method as provided before

\[
\int \frac{x^2 + x - 2}{3x^3 - x^2 + 3x - 1} \, dx = \int \frac{x^2 + x - 2}{x^2(3x - 1) + 3x - 1} \, dx = \int \frac{x^2 + x - 2}{(3x - 1)(x^2 + 1)} \, dx = \int \frac{A}{3x - 1} + \frac{Bx + C}{x^2 + 1} \, dx
\]

\[
x^2 + x - 2 = A(x^2 + 1) + (Bx + C)(3x - 1)
\]

\[
= Ax^2 + A + 3Bx^2 - Bx + 3Cx - C
\]

\[
= x^2(A + 3B) + x(-B + 3C) + A - C
\]

\[
A + 3B = 1 \quad \text{and} \quad -B + 3C = 1 \quad \text{and} \quad A - C = -2
\]

\[
A = -\frac{7}{5}, \ B = \frac{4}{5}, \ C = \frac{3}{5}
\]

\[
\int \frac{-\frac{7}{5}}{3x - 1} + \frac{\frac{4}{5}x + \frac{3}{5}}{x^2 + 1} \, dx = -\frac{7}{5} \int \frac{1}{3x - 1} \, dx + \int \frac{\frac{4}{5}x}{x^2 + 1} \, dx + \int \frac{\frac{3}{5}}{x^2 + 1} \, dx ...
\]

\[
= -\frac{7 \ln |3x - 1|}{15} + \frac{4}{5} \int \frac{x}{x^2 + 1} \, dx + \frac{3}{5} \int \frac{1}{x^2 + 1} \, dx = -\frac{7 \ln |3x - 1|}{5} + \frac{2 \ln |x^2 + 1|}{5} + \frac{3 \arctan(x)}{5} + C
\]

Random: \(\frac{1}{3x^3 - x^2 + 3x - 1} = \frac{1}{(3x - 1)(x^2 + 1)}\) \(\Rightarrow\) If \(a/b = c/d\), then make it \((Ax^3+Bx^2)+(Cx+D)\). Then factor out \(x^2\)