Harmonic Series:
\[ \sum_{k=1}^{\infty} \frac{1}{k} = \text{Diverges} \]

Divergence Test:
If \( \lim_{k \to \infty} a_k \neq 0 \), then the series diverges. If not, it’s inconclusive.

Integral Test:
\[ \sum_{k=1}^{\infty} a_k \text{ and } \int_{1}^{\infty} f(x)dx \text{ both converge or both diverge (for continuous, positive, decreasing function)} \]
*When doing an integral test with natural logs, make it as simplified as possible. Then, it might help to bring the limit inside the function. For instance, \( \lim_{x \to \infty} \ln \left( \frac{3x+1}{3x+4} \right) \rightarrow \ln(\lim_{x \to \infty} \frac{3x+1}{3x+4}) \rightarrow \ln(1) = 0 \)

*Can also be done with trig functions, etc.*

*Note: \( \int a^u du = \frac{a^u}{\ln a} + C \)

*Note: For integrals, if it’s \( \frac{1}{3^k} \), make it the integral of \( \left( \frac{1}{3} \right)^k \)

P-Test:
\[ \sum_{k=1}^{\infty} \frac{1}{k^p} \text{ converges when } p > 1 \text{ and diverges when } p \leq 1 \]

Estimations for Converging Series:
\[ R_n \leq \int_{n}^{\infty} f(x)dx \]
*Note: The first value that will produce the correct value is rounded up (next integer)*

\[ S_n + \int_{n+1}^{\infty} f(x)dx \leq \sum_{k=1}^{\infty} a_k \leq S_n + \int_{n}^{\infty} f(x)dx \]

*When doing inequalities, multiplying or dividing by a negative switches the sign, and so does doing the inverse of both sides*