MATH 145-02: WORKSHEET

Problem A For $SL_2(\mathbb{Z})$, consider $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Find the orders of S, T, and ST in the group.

Problem B Recall that the dihedral group D_n is the group of rigid motions of the regular n-gon. In any dihedral group, we will write f for the flip that preserves the vertex labeled 1, and r for the rotation $(1234\ldots n)$. By discussing where the map sends the labels, show that $frf = r^{-1}$ in any D_n. Using that, simplify $rfr^2fr^2fr^{-1}$.

Problem C Draw Cayley graphs for the following groups and generating sets:

(a) $F_2 = \langle a, b \rangle$, (b) $\mathbb{Z}^2 = \langle \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right] \rangle$, (c) $\mathbb{Z}^2 = \langle \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right], \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right], \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right] \rangle$, (d) $D_5 = \langle r, f \rangle$, (e) $D_\infty = \langle r, f \rangle$, (f) $S_3 = \langle (12), (23), (13) \rangle$