QUIZ 1 Math 145, Abstract Algebra, Duchin

The grading of this quiz will focus on clear argumentation.

(a) Clearly prove that $b \mid a, \quad b \mid (a+c) \implies c \in b\mathbb{Z}$.

Proof. The definition of $b\mathbb{Z}$ is the set of all integer multiples of b, so $x \in b\mathbb{Z} \iff b \mid x \iff x = bk$ for some $k \in \mathbb{Z}$. So I will assume $b \mid a$ and $b \mid (a+c)$ and I must prove that $b \mid c$. From these assumptions, we know that $a = bm$ and $a + c = bn$ for some $m, n \in \mathbb{Z}$. But then

$$c = (a+c) - a = bn - bm = b(n-m),$$

and $n - m$ is in $\mathbb{Z}$ because it is the difference of two integers, so we have verified $b \mid c$. □

(b) If $a = 120$ and $b = 4$, does $b \mid a$? Does $2b \mid a$?

Proof. Again, the meaning of $r \mid s$ is that $s = rk$ for some $k \in \mathbb{Z}$. But $120 = 4 \cdot 30$ and $120 = 8 \cdot 15 = (2 \cdot 4) \cdot 15$, so the statements are verified. □

(c) Show that $b \mid a \not\Longrightarrow 2b \mid a$. Clearly explain the logic of your answer.

Proof. I must show that $b \mid a \not\Longrightarrow 2b \mid a$; in other words, I must show that $b \mid a \implies 2b \mid a$ is FALSE. The meaning of an implication $P \implies Q$ is that every time P is upheld, Q is also upheld. To show that this is false I need only supply one case in which P is true but Q is false. So I just need to give one example of numbers a and b such that $b \mid a$ but $2b \nmid a$. For example, if $a = 2$ and $b = 10$, I have $2 \mid 10$ while $4 \nmid 10$. □