(1) Give an example of a well-defined function from $\mathbb{Z}_{10} \to \mathbb{Z}_3$ that is neither injective nor surjective. (Or if none exists, say why not.)

> The constant map defined by $f(x) = [2]_3$ for all $x \in \mathbb{Z}_{10}$ is certainly well-defined because all inputs are mapped to a valid output. It is not injective because ten different elements map to $[2]_3$. It is not surjective because nothing maps to $[1]_3$.

(2) Let $S = \{1, 2, 3, 4, 5\}$ and let $F$ be the set of all functions from $S \to S$. Define an equivalence relation by saying that $f \sim g$ if and only if $f(5) = g(5)$.

Consider the function $h : S \to S$ defined by $h(x) = \begin{cases} 3, & x \text{ even} \\ 2, & x \text{ odd.} \end{cases}$

Name a function different from $h$ that is in $[h]$.

> Define $p : S \to S$ by $p(x) = 2$ for all $x \in S$. Then $p(5) = h(5) = 2$, so $p \sim h$, so $p \in [h]$. On the other hand, $p(4) = 2 \neq 3 = h(4)$, so $p \neq h$.

EC1: How many such functions exist in #1?

> No map from a 10-element set to a 3-element set can be injective. How many are not surjective? Consider the different possibilities for the number of elements in the image of $f$. (Because $f$ is not surjective iff its image has 1 or 2 elements, rather than all 3.) There are 3 different ways to have $|Im(f)| = 1$, namely the three different constant maps. In order to have $|Im(f)| = 2$, there are 3 different ways to pick what point to miss, then $2^{10}$ ways to map $\mathbb{Z}_{10}$ to the other two elements, though two of these are constant maps so they were already counted. So there are $3(2^{10} - 2)$ ways to have two points in the image. That means the number of functions between these sets that are neither injective nor surjective is $3+3(2^{10}-2) = 3(2^{10} - 1) = 3(1023) = 3069$.

EC2: How many elements does each equivalence class have in #2?

> To be in $[h]$, a function just needs to send 5 to 2. The other four elements can be sent to any of the five elements of $S$. So there are $5^4 = 625$ ways to do that.