(1) Write out the elements of $\mathbb{Z}$ from -10 to 10 arranged in a line. Using four different symbols, mark the four cosets of $4\mathbb{Z}$. What are the names of those four cosets? What is the structure of the quotient group $\mathbb{Z}/4\mathbb{Z}$?

(2) Is $\langle ab,\ ba\rangle$ a proper subgroup of $F_2 = \langle a, b\rangle$? What is its index?

(3) Consider the Heisenberg group presented as

$$H(\mathbb{Z}) = \langle a, b, c \mid aba^{-1}b^{-1} = c,\ ac = ca,\ bc = cb\rangle.$$

(a) Show that $a^i b^j c^k$ is a normal form. (That is, every different group element can be written uniquely in that form, and that form covers all possible group elements.)

(b) For the subgroup $H = \langle c\rangle$ in $H(\mathbb{Z})$, does $aH = bH$? Does $babH = ab^2H$?

(c) What familiar group is $H(\mathbb{Z})/\langle c\rangle$ isomorphic to?

(d) With the same generating set, draw all edges that are present between the nine elements $e, a, b, ab, ba, aba^{-1}, bab^{-1}, c, c^{-1}$ in the Cayley graph.