MATH 145-02: WORKSHEET

Problem A True or false: in any symmetric group, the conjugate of a k-cycle by a transposition is always a k-cycle. If true, prove it. If false, give a counterexample.

Problem B Let $G = \langle a, b \mid aba^{-1} = b^5 \rangle$. Show that $H = \langle b \rangle$ is normal. To do this, explain what an arbitrary element of H looks like and show what happens when you conjugate it by the generators of G.

Problem C Consider the presentation $D_{10} = \langle r, f \mid r^{10} = e, f^2 = e, frf = r^{-1} \rangle$. Find the index of $\langle r \rangle$ in D_{10}. Is it normal?

Problem D In D_4, let $H = \langle r^2 \rangle$. The elements of D_4/H are cosets of the form aH.

(a) Show that $rH = r^3H$.

(b) Write down one representative of each distinct coset, making sure you get the right number of cosets overall.

(c) Using these, write down a multiplication table for the quotient group.

(d) This is isomorphic to one of the groups we often discuss, so you know it by another name. Which one?