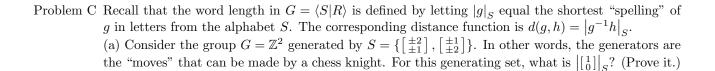
MATH 145-02: WORKSHEET

This worksheet, which is your last problem set of the semester, covers the two post-midterm 2 topics: (A-B) all groups are quotients of free groups, and (C-D) free groups are "paradoxical."

Problem A Let G be the quotient $F_2/\langle b \rangle$. (a) What is a simplified form of $ab^8a^5b^{10}$? (b) What is a normal form for the elements of G? (In other words, what is a full set of representatives for the cosets?) (c) What familiar group is G isomorphic to?

Problem B Let G be the quotient $F_2 / \langle a^4, b^4, aba^{-1}b^{-1} \rangle$. In other words, this quotient is formed by the equivalence relations $a^4 \equiv e, \ b^4 \equiv e, \ ab \equiv ba$. Now answer (a),(b),(c) just like in the last question!



(b) Consider the group $G=\mathbb{Z}^2$ generated by $S=\{[\frac{1}{0}],[\frac{0}{1}]\}$. How many group elements g have $|g|_S\leq n$? (Work your way up to this with n=2,3,4.) Sketch a picture that shows the words of length $\leq n$ in the Cayley graph.

Problem D Show that left-multiplication by any group element is a rigid motion by showing that d(g,h) = d(jg,jh) for any $j \in G$. (This is needed to show that the group F_2 rigidly covers itself two times over!)