Advanced Fuel Assembly For

Sodium Cooled Fast Reactors

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Abstract

Sodium Cooled Fast reactors with metallic fuel are currently an important area of research. In improving the nuclear and thermal-hydraulic performance of the reactor, it is important to consider the geometry of the assembly. This research proposes the use of the tube- in- duct or vented fuel design rather than the conventional pin cell. It explores the geometry of the TID cell including constraints, volume fractions, pressure drop and other relevant thermal hydraulic parameters. It also compares the results with a reference pin cell design. The reference pin cell has a fuel volume fraction of 42%, a temperature limit of 200 and a pressure drop of 0.77 MPa. NOVEX 1.1, the primary design in this project, gives a fuel volume fraction of 60.5%, meeting the total temperature limit of 200 while having a pressure drop of 0.45MPa. Thermal analysis has been done using COSMOS WORKS which is a simulation tool in SOLID WORKS. The aim was to develop an ideal vented cell with full description of the parameters.

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Introduction

Since the start of nuclear energy, the potential of the liquid fast metal breeder reactor has been realized. It was Enrico Fermi who said "The country which first develops the breeder reactor will have greater competitive advantage in atomic energy.[1]" The breeder reactor is a nuclear fission reactor designed to convert more fertile material into fissile material than it consumes. As a result it produces a substantial excess of fissile material that can be used to fuel either new breeders or ordinary nuclear power reactors. Although the first experimental breeder reactor started up several decades ago, the development of breeder reactors is far behind that of thermal reactors.

Under the Gen-IV project, significant research at MIT has been carried out on a vented-tube-in duct (TID) assembly for Gas-cooled fast reactors. Since there is a worldwide and US interest in sodium-cooled fast reactors (SFR), it is of importance to evaluate the extent to which the use of TID assemblies can also benefit the performance of the sodium-cooled fast reactors.

One advantage is a high fuel volume fraction which leads to high internal conversion ratio. Secondly because of the TID configuration and absence of fission gas plena, there will be a reduced pressure drop, hence pumping power. Also since venting prevents clad strain by high internal fission gas pressure, a longer fuel lifetime is possible.

However there is a difference between the Sodium-cooled Fast reactor and Gas-cooled fast reactor. The Sodium- cooled Fast reactor has a lower coolant pressure which leads to a

1 Tang, Y.S. *Thermal Analysis of Liquid Metal fast Breeder Reactors*. Illinois: The American Nuclear Society, 1978. Print.

proportional increase in fission product gas specific volume which makes holdup in vent-path plena much shorter, hence increasing the escape rate of shorter half life radio nuclides.

The purpose of this research is to quantify features of the SFR TID. As a result the geometry of the TID assembly will be explored. In the second phase calculations will be carried out on the thermal-hydraulic aspects of sodium bonded IFR type metal fuel to determine the pressure drop, fuel temperature, acceptable dimensions and power densities. Finally the collected data will be compared to current pin-type fuel assemblies and results will be used to come up with a candidate assembly design.

2. The concept of the unit cell

A Reactor fuel is generally in the form of pellets. The pellets are then encased in metal tubes to form fuel rods, which are arranged into a fuel assembly ready for introduction into a reactor. In metal fueled sodium cooled fast reactors, a solid cylindrical metal fuel slug is encapsulated within the cladding and liquid sodium added as a thermal bond between the fuel and cladding. A free volume (fission gas plenum) is provided in the upper part of the fuel pin to reduce the pressure due to fission gas release from the fuel.

Fuel elements are arranged in a regular manner. These regular manners can be squares, hexagons or rings. We call it a square lattice if the elements are arranged in squares and a hexagonal lattice if the elements are placed at the corners of a hexagon. The only regular polygons that can tile over a given plane are equilateral triangles, squares or hexagons. Mathematically, they will reflect good symmetry, simplifying reactor lattice calculations. In any type of reactor lattice it is possible to find a single repetitive fragment composed of either fuel or coolant. A fictitious boundary can be introduced that divides nearest elements. An element, for example a fuel cylinder, surrounded by cladding and adjacent coolant portion forms a unit cell. Figure 1.1 shows an example of a unit cell where the element is a fuel pin and it is surrounded by a coolant.

For a hexagonal assembly with side S and the rods arranged in regular manner, it is possible to deduce certain relationships about the total number of rods. Figure 1.2 shows a hexagonal TID assembly with fuel external to coolant tubes. Clearly as we go from the center, the number of coolant regions (channels) increases. In the first ring we have 7 rods; in the

second ring we have 19, in the third ring 37 and so on. A mathematical formula for the total number of rods is as follows.

$$N = 3n^2 + 3n + 1$$

Where n is the number of the rings of rods. Note that the center rod is the "zeroth" ring. For example if n =2, N= 19

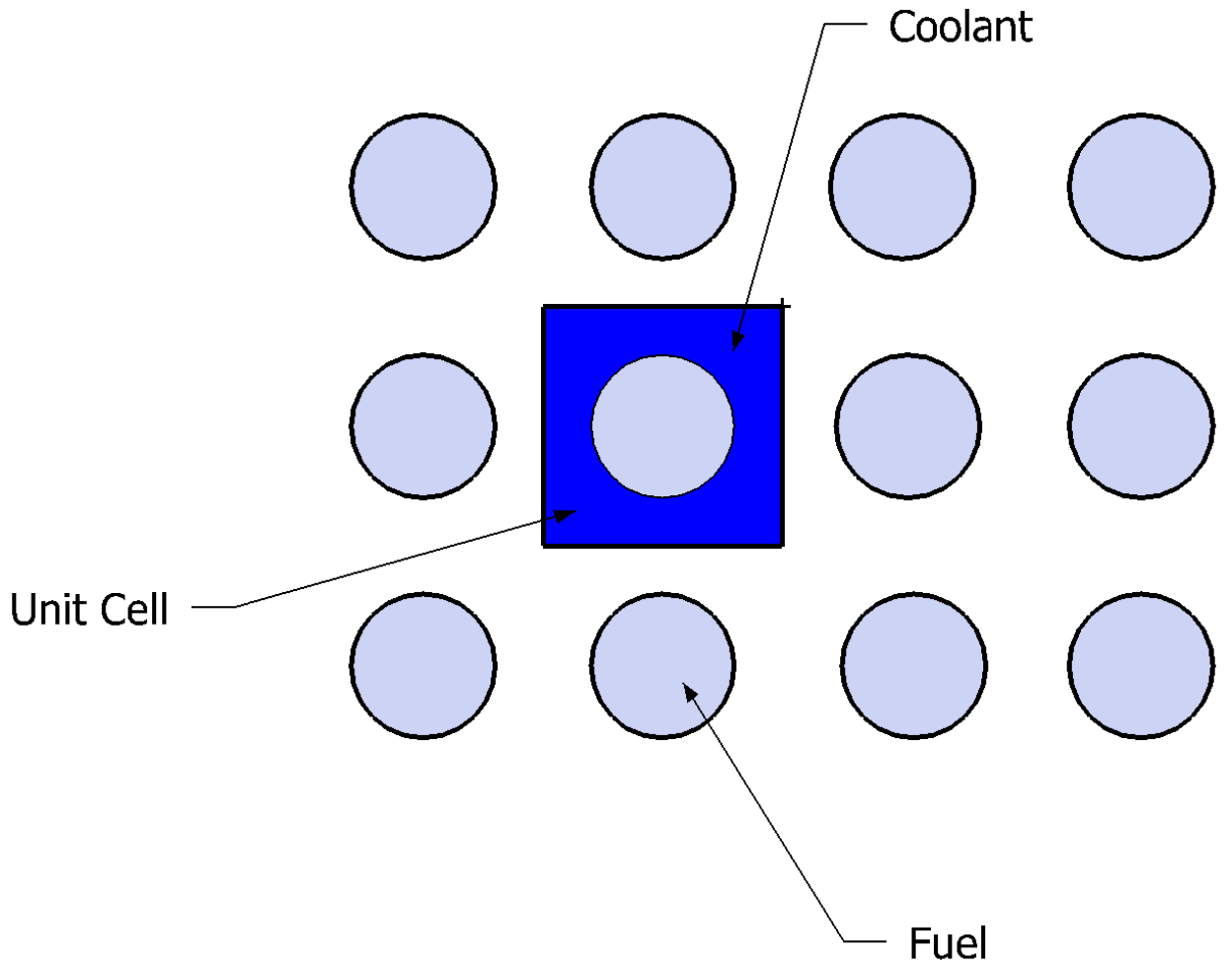


Figure 1.1 A square Unit Cell For a Pin cell Core

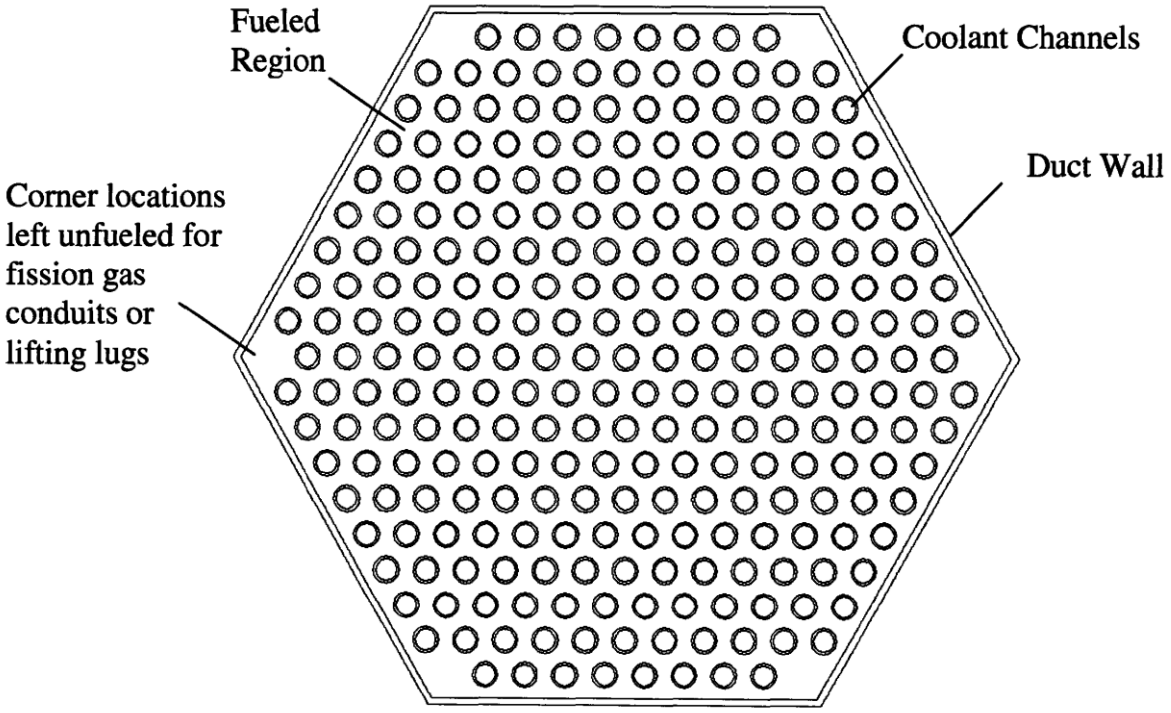


Figure 1.2 A Hexagonal TID Assembly

When using the concept of a unit cell usually, it is often convenient to reduce hexagons and squares into an equivalent circle. That is we create a cylindrical cell whose volume is equal to that of the given shape. Thus the outer boundary of the cell is transformed from a square or hexagon into an equivalent circle. Figure 1.3 illustrates this concept.

The formulas for equivalent diameter for square and hexagonal cells or assemblies are as follows. The directions are provided in Appendix A.

For a Square Lattice, the outer diameter is

$$d_{Cell} = \frac{2}{\sqrt{\rho}} P$$

Where P = distance between centers of adjacent squares.

For Hexagonal Lattice (of interest to this report)

$$d_{Cell} = \sqrt{\frac{2\sqrt{3}}{\rho}} p$$

Where p = the distance across the flats of the hexagon.

This formula will be made use of in subsequent analysis.

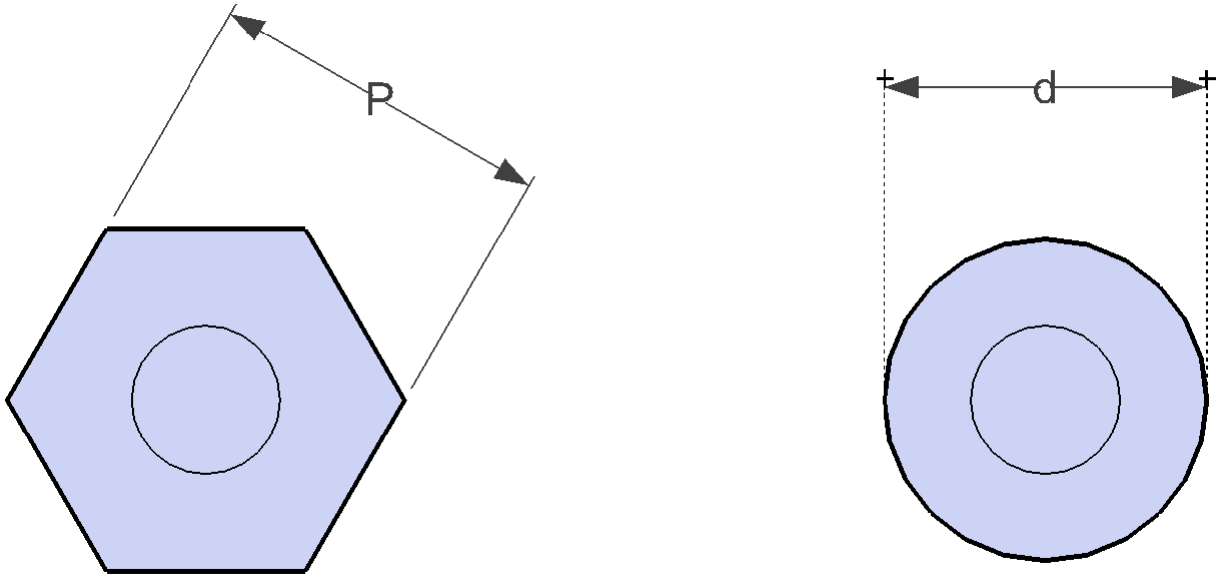


Figure 1.3 The Concept of an Equivalent Diameter

In reactor core neutronic and thermal-hydraulic design, a unit cell is the smallest repeating feature which can be replicated in tiling fashion, to model, to a good approximation, fuel assemblies, which can in turn be arranged to describe a complete reactor core.

Most commonly (e.g. for LWR, SFR fuel pin arrays) the (rod centered) unit cell consists of a cylindrical fuel pin surrounded by its metal cladding and associated coolant flow channels. In the "Inverted" (e.g. tube-in-duct: TID) case the (coolant centered) unit cell is the metal tube containing a coolant flow channel surrounded by an associated fuel region. In thermal-hydraulic analysis, use of coolant- centered unit cells is fairly common.

By the further imposition of symmetry and reflection, the unit cell can frequently be subdivided into smaller (e.g. pie slice) sectors, which can often simplify analytic and numerical modeling. Note that all such cells are defined strictly speaking, by zero gradient (neutron flux or coolant flow) boundaries.

3. The Reference Core Design

3.1 Introduction

Since the objective of this report is to be able to replace a conventional pin cell design with a TID design, it is very important to understand the pin cell design, its parameters and the particular reference case used. This section introduces the pin cell design so that a proper comparison can be made in the next section.

The reference core design was taken from Appendix 5A of the MIT-NFC-PR-101 report. This appendix describes analysis of a conceptual core design of a 2400 MWt sodium-cooled fast reactor with unity conversion ratio. The reactor is a pool-type modular design and utilizes sodium coolant and metallic fuel. It employs U-TRU-Zr metallic fuel with the fuel pins and assemblies in hexagonal arrangement. Figure 1.4 shows the core layout while Table 1.1 and 1.2 describe important parameters and material properties respectively.

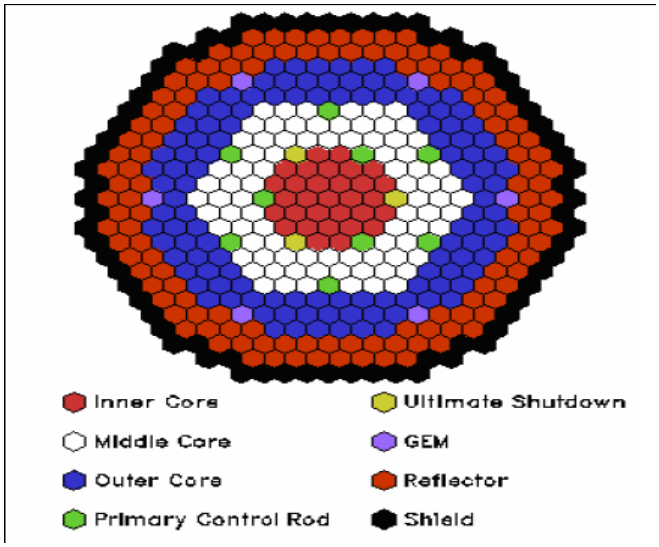


Figure 1.4 Reference Design Core Layout

Core thermal power (MWt)		2400
Maximum radial power peaking coefficient		1.23
Assembly Geometry	Assembly pitch, hexagon flat-to-flat (cm)	16.14
	Assembly can thickness (cm)	0.40
	Inter assembly gap (cm)	0.43
	Total number of fuel assemblies	360
	Number of CRD assemblies	19
	Number of fuel pins per assembly	271
Fuel pin geometry	Pin outer diameter (cm)	0.808
	Cladding thickness (cm)	0.0559
	Gap thickness (cm)	0.05
	Fuel heated length (cm)	102.0
	Fuel pin pitch (cm)	0.89
	Pitch-to-diameter ratio	1.1
	Plenum height (cm)	120.0
	Cladding material	HT-9
	Gap bond	Na
Core average	Coolant mass flow rate (kg/s)	12561
	Inlet temperature (°C)	352.0
	Outlet temperature (°C)	502.0
	Re number	67727
	Pr number	5.1E-3
	Nu number (Westinghouse correlation)	5.62
	Heat transfer coefficient (kW/m2-K)	139.2
	Core pressure drop* (MPa)	0.20
Average subchannel	Coolant average temperature (°C)	427.0
	Cladding temperature limit (°C)	600.0
	Cladding inner surface temperature (°C)	461.7
	Fuel centerline temperature (°C)	630.0
	Coolant velocity (m/s)	7.86
Hot subchannel	Peak cladding temperature (°C)	568.0
	Fuel centerline temperature (°C)	788.0
Power	Linear power, average (kW/m)	24.12
	Power density (kW/l)	289.7
	Specific power (kW/kgHM)	64.80

* The pressure drop is only for the active fuel height.

Table 1.1 Summary of the main core parameters

Parameter	Value
Sodium	
Coolant average temperature, °C	427.0
Density, g/cc	0.846
Thermal conductivity, W/m-K	66.70
Specific heat, J/kg-K	1272.00
Viscosity, kg/m-s	2.65E-04
Steel HT-9	
Thermal conductivity, W/m-K	20.35
Metallic fuel (U-TRU-Zr)	
Assumed thermal conductivity, W/m-K	12.0

Table 1.2 Select properties of materials at average coolant temperature

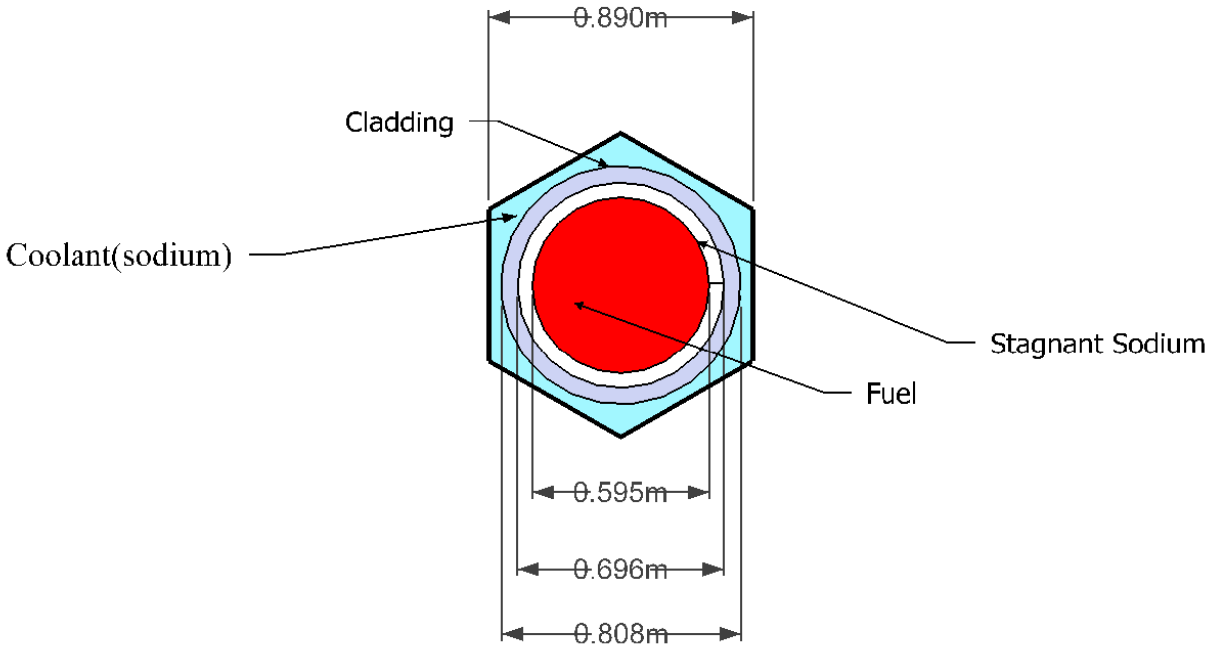


Figure 1.5 A unit cell of reference pin cell

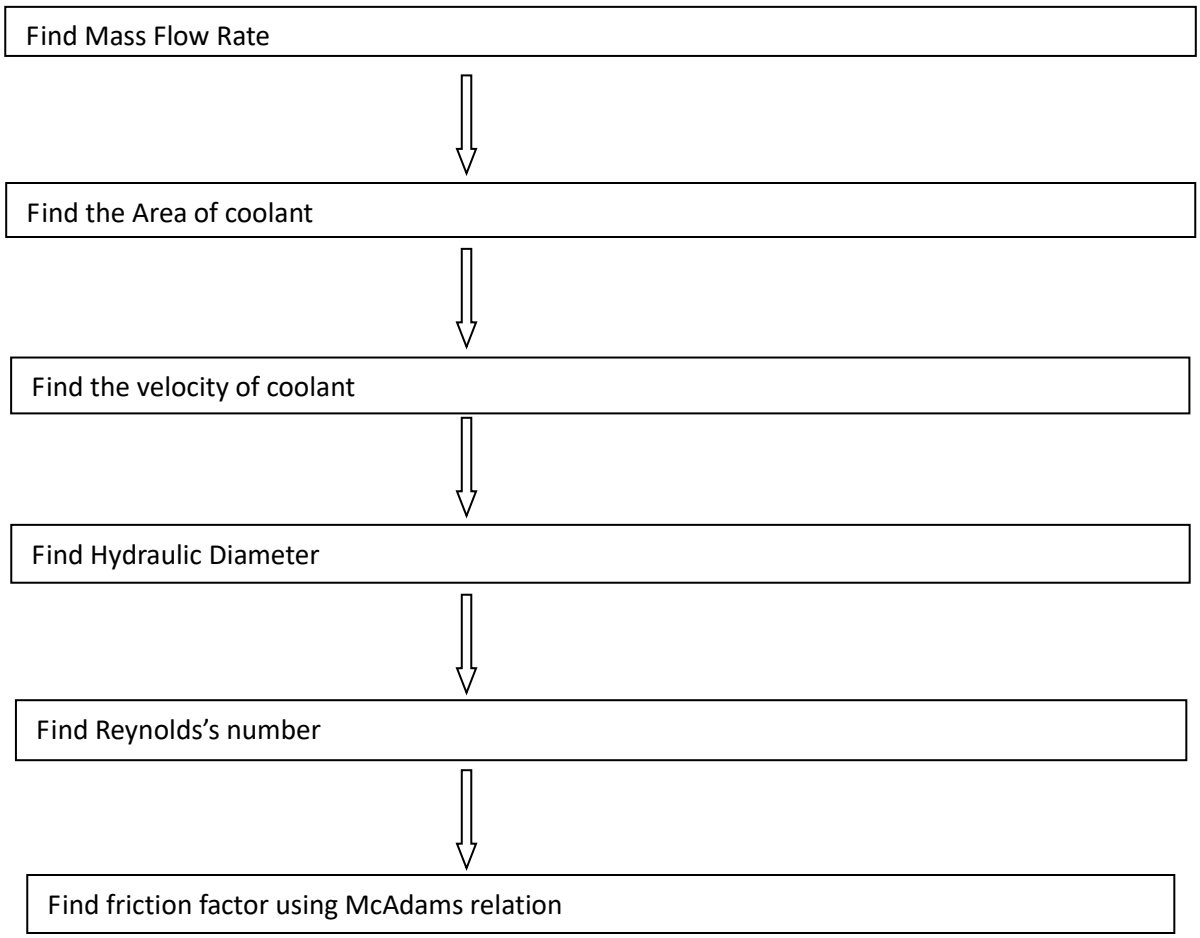
3.2 Pressure Drop Calculation for reference Core

The pressure drop in Table 1.1, which is 0.2 Mpa, is only for the active fuel. Since the coolant flows along other parts, it was necessary to calculate a new pressure drop.

$$\Delta P = \left(f \frac{l}{d_e} + K \right) \frac{\rho v^2}{2}$$

Where f is the friction factor, L is the channel length, $\boldsymbol{d_e}$ is the equivalent (or hydraulic) diameter p is the fluid density, and v is the fluid velocity. K is the entrance and exit loss and a maximum value of 1.5 is used in this calculation.

The flow chart below shows the steps used in calculating the pressure drop.



To find the mass flow rate,

$\dot{m} = \frac{q'}{c\Delta T}$ where q' is linear power, c is the specific heat of sodium and ΔT is temperature difference across the core

Using q' =24.12KW/m, c =1272.00J/kg-k and ΔT=150°c

$\dot{m}$ =0.126 kg/s

To find area of coolant

$$A_{ct} = \frac{\sqrt{3}}{2} p^2 - \frac{\pi d_{pin}^{\ 2}}{4}$$

Where P is the fuel pin pitch and d_{pin} is the diameter of the fuel pin. Given P=0.89cm and d_{pin} =0.808 cm, the area of coolant = $0.1732 \times 10^{-4} m^2$

To find the coolant velocity we use $v = \frac{\dot{m}}{\rho A_{ct}}$ where $\dot{m}$ is the mass flow rate, ρ is the density of sodium (846g/cc), A_{ct} is the area of coolant. Thus the velocity of the coolant is 8.6 m/s.

The hydraulic diameter of a hexagonal array of fuel pins is related to its lattice parameters by;[2]

[2] Pope, Michael. "Thermal Hydraulic Design of a 2400Mw, Direct Supercritical CO2-cooled Fast Reactor." (2006): 1-244

$$D_{e,pin} = D_{pin}\left(\frac{2\sqrt{3}}{\pi}\left(\frac{P_{pin}}{D_{pin}}\right)^2 - 1\right)$$

Thus $D_{e,pin}$ =2.73 mm

Reynolds number is defined as

$R_e = \frac{D_{e,pin} v \rho}{u}$; u is the viscocity of sodium and it has a value of 2.65×10^{-4}

R_e = 74952.4

Using McAdams relation for friction factor,

$$f = \frac{0.184}{R_e^{0.2}} \text{ ; } f \text{ =0.0195}$$

The length is the sum of all the heights in the reactor layout (excluding Lower Core Support Grid Plate) = (0.4+0.4+1.02+1.20+0.2) m =3.22m

Given all these values;

$$\Delta P = \left(f\frac{l}{d_e} + K\right)\frac{\rho v^2}{2}$$

ΔP = 0.77 Mpa

3.3 Thermal Analysis for Reference Core

Thermal Conductivity: The thermal conductivity of a metallic fuel depends on the temperature, composition, porosity and fraction of the sodium infiltrated into the fuel. It is assumed that sodium infiltrates 40 % of the open (interconnected) porosity. Since a detailed calculation of thermal conductivity after smearing is beyond the scope of this report, a simple correlation is used. The porosity was assumed to be 20%; with 5% closed and 15% open.

The thermal conductivity of a fresh metallic fuel is given by[3]:

$K_0 = a + bT + cT^2$ (T is in Kelvin, K_0 in W/m/k)

Where

$$a = 17.5\left(\left(\frac{1-2.23W_{Zr}}{1+1.61W_{Zr}}\right) - 2.62W_{Pu}\right)$$

$$b = 1.54\times10^{-2}\times\frac{1+0.061W_{Zr}}{1+1.61W_{Zr}}$$

$$c = 9.38\times10^{-6}(1-2.7W_{Pu})$$

W_{Zr} : Zirconium weight fraction in the fuel

W_{Pu} : Plutonium weight fraction in the fuel

With Temperature of 550^0c, 15% Zirconium and 17% plutonium (assuming all TRU equals plutonium)

a = 1.59, b= 0.012517849 and c = 5.075×10^{-6}

Therefore K_0 = 15.33

[3] Karahan, Aydin. Modeling of Thermo-Mechanical and Irradiation Behavior of Metallic and Oxide Fuels for Sodium Fast Reactors. Thesis. MIT, 2009

Thermal conductivity after smearing is given by:

$$K_f = P_c \times (1-P)^{1.5} \times K_o$$

Where K_f = Thermal Conductivity

P =Fuel Porosity Fraction

K_o = As Fabricated thermal conductivity of fuel

P_c (The porosity correction factor with sodium infiltration) is expressed as:

$$P_c = 1 - \frac{3P_{Na}}{1-P} \times \frac{1-\frac{K_{Na}}{K_o}}{1.163+1.837\frac{K_{Na}}{K_o}}$$

Where K_{Na} = Sodium Thermal Conductivity; P_{Na} = Fraction of the Fuel which is filled with Sodium

At average coolant temperature of 427°c: K_{Na} =66.7W/m-k, K_o = 12w/m-k, P =0.2

$P_{Na} = P_{Na} = 15\% \times 40\% = 0.06$

P_c = 1.08

Therefore K_f = 11.85

.

Thermal Analysis before Smearing:

Figure 1.5 shows how the pin cell looks like before smearing.

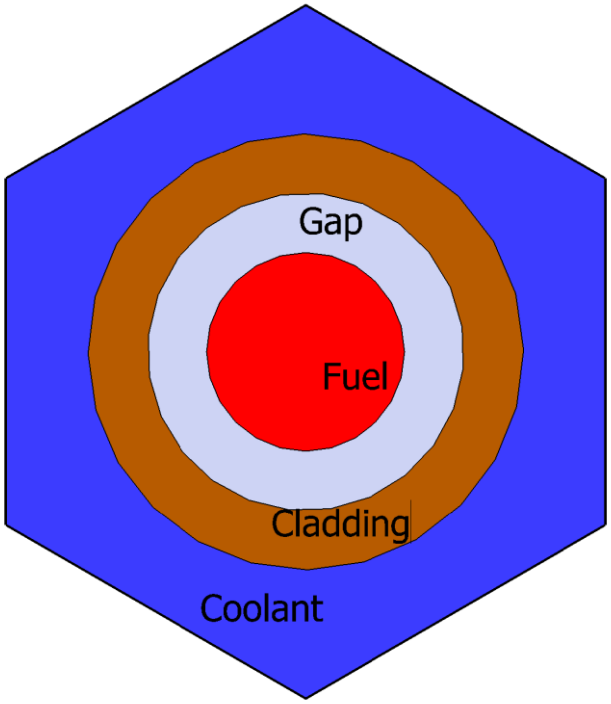


Figure 1.6 Reference pin cell Before Smearing

Temperature Drop across Fuel: The temperature drop across the fuel meat is given by:

$$\Delta T_{fuel} = \frac{q'}{4\pi k_f}$$ With q' =24.12KW/m and K_o =15.33W/m-k

Thus ΔT_{fuel} =125.21°c

Temperature Drop across bond (Stagnant Sodium): is given by

$\Delta T_{gap} = \frac{q'}{2\pi r_f}\left(\frac{t_g}{k_g}\right)$ Where t_g is thickness of gap and k_g is gap conductivity

ΔT_{gap} =9.65°c

Temperature Drop across Cladding:

$\Delta T_{cladding} = \frac{q'}{2\pi\left(r_f + t_g\right)}\left(\frac{t_c}{k_c}\right)$; Where t_g is thickness of gap, t_c is thickness of cladding and

k_c is the clad conductivity. This gives us a value of 30.29°c.

Temperature Drop across Film:

The temperature difference between cladding surface and bulk fluid is given by

$\Delta T_{coolant} = \frac{q'}{2\pi h\left(r_f + t_g + t_c\right)}$; Where h is heat transfer coefficient.

Thus the temperature drop across coolant is 6.83°c

Thus the total temperature drop is the sum of all 125.21°c+9.65°c+30.29°c+6.83°c= 171.98°c

Thermal Analysis After Smearing:

Figure 1.6 shows how the pin cell looks like after smearing.

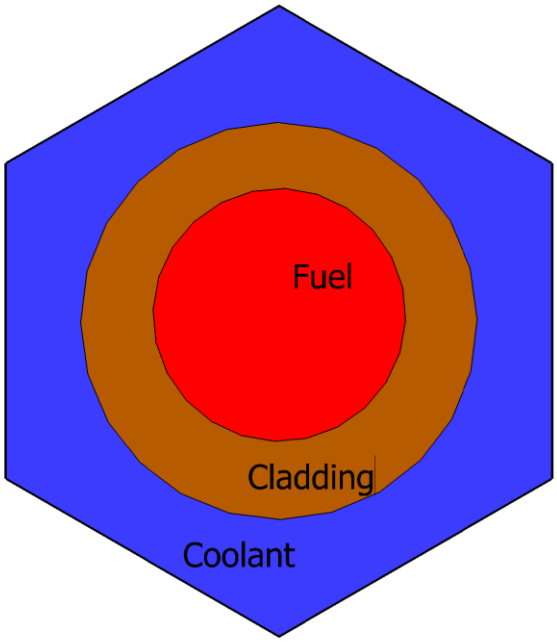


Figure 1.7 Reference pin cell After Smearing

Temperature Drop across Fuel: The temperature drop across the fuel meat is given by:

$$\Delta T_{fuel} = \frac{q'}{4\pi k_f}$$ With q' =24.12KW/m and k_f =11.85W/m-k

Thus ΔT_{fuel} = 161.98°c

Temperature Drop across Cladding:

$$\Delta T_{cladding} = \frac{q'}{2\pi\left(r_{final}\right)}\left(\frac{t_c}{k_c}\right)$$ Where r_f is the fuel radius after swelling.

Thus we get a cladding temperature difference of 30.29°c.

Temperature Drop across Film:

The temperature difference between cladding surface and bulk fluid is given by

$$\Delta T_{coolant} = \frac{q'}{2\pi h\left(r_{final} + t_c\right)}$$

This is the same as before smearing case which is 6.83°c.

Thus the total temperature drop is the sum of all 161.98+30.29+6.83 =199.1°c

3.4 Core Volume and Number of Fuel Assemblies

In both NOVEX 1.1 and NOVEX 1.2, the core volume has been fixed. Here is how the core volume of the reference pin cell was calculated.

$$V_{core} = A_{core} \times h_{fissile}$$
$$A_{core} = A_{assembly} \times N_{assemblies}$$
$$A_{core} = \frac{\sqrt{3}}{2} p^2 \times N_{assemblies}$$
$$A_{core} = \frac{\sqrt{3}}{2} (16.14cm)^2 \times 360$$
$$A_{core} = 8.12m^2$$

Then the core volume can be calculated as;

$$V_{core} = 8.12m^2 \times 1.02m$$
$$V_{core} = 8.28m^3$$

To confirm these values, we can check using the given value for power density in Table 1.1.

Power density is defined as $Q''' = \frac{\dot{Q}}{V_{core}}$

Using 289.7 Kw/l for power density and 2400Mwt for core power we get 8.28m^3 which makes our calculation consistent.

Number of slugs for a TID core: Here is how the number of slugs for a TID cell was calculated. It will be used frequently in the next two designs.

The average linear power is expressed as:

$$q' = \frac{\dot{Q}}{N \times h_{fissile}}$$

Where $\dot{Q}$ is the core thermal power, N is the number of fuel slugs and $h_{fissile}$ is the height of the fissile material. N can be written as:

$$N = 2 \times \frac{A_{core}}{\frac{\sqrt{3}}{2} p_{cell}^{\ 2}} \times c$$

Where p_{cell} the pitch of a unit cell and c is the correction factor. The correction factor for the reference pin cell is used.

Correction factor for pin cell: The inner assembly is assumed to be made of unit cells. Thus the correction factor can be expressed as:

$$c = \left(\frac{W_i}{P_a} \right)^2$$

Given P_a =16.14cm and W_i =14.91, c = 0.853

Note that we overestimate the number of fuel slugs since we assume where in reality there is gap for control rods and edge spacing.

Since A_{core} = 8.12 m^2 , and c = 0.853

$$N = \frac{16}{p_{cell}^{\ 2}}$$

$$\text{Thus } q' = \frac{2400 \times 10^6 W_t}{h_{fissile} \times \frac{16}{p_{cell}^{\ 2}}}$$

$$\boxed{q'_{per\ slug} = \frac{1.5 \times 10^8}{h} p_{cell}^{\ 2}}$$

This equation is very crucial since it determines how varying the pitch will affect the linear power and as a result the pressure drop. It will be used in the NOVEX 1.1 design.

4. Introduction to TID design

TID fuel assemblies consist of a hexagonal duct with coolant tubes inside. Fuel would be placed around coolant tubes and inside the hexagonal duct.

The TID design in this report is vented to reduce wall stresses during the steady state and LOCA conditions. Venting of Sodium fast reactor fuel is of interest because it eliminates the differential pressure across cladding caused by fission product gas accumulation, which has the potential of significantly increasing fuel burn up[4]. It also eliminates the long fission gas plenum on fast reactor fuel pins, which can be comparable to the active fuel zone. This significantly reduces core pressure drop and pumping power, and helps promote natural circulation. Venting was in fact employed on the Dounreay fast reactor. GE also considered venting for SFR in the 1960s.

[4] Driscoll, Michael J. *Radiological Aspects of Venting SFR fuel*. Technical Note. 2009.

5. NOVEX 1.1

5.0 Introduction, Methodology and Motivation

NOVEX 1.1 was the initial design in this project. The design objectives were

1. High fuel Volume Fractions
2. Tolerable Pressure Drop
3. Operating within the fuel Temperature Limit
4. Reasonable Number of Fuel Slugs

To meet these design objectives the following methodology was employed. Here are some of the main assumptions made in the design of NOVEX 1.1.

a) Radius of the Core

b) Core Power

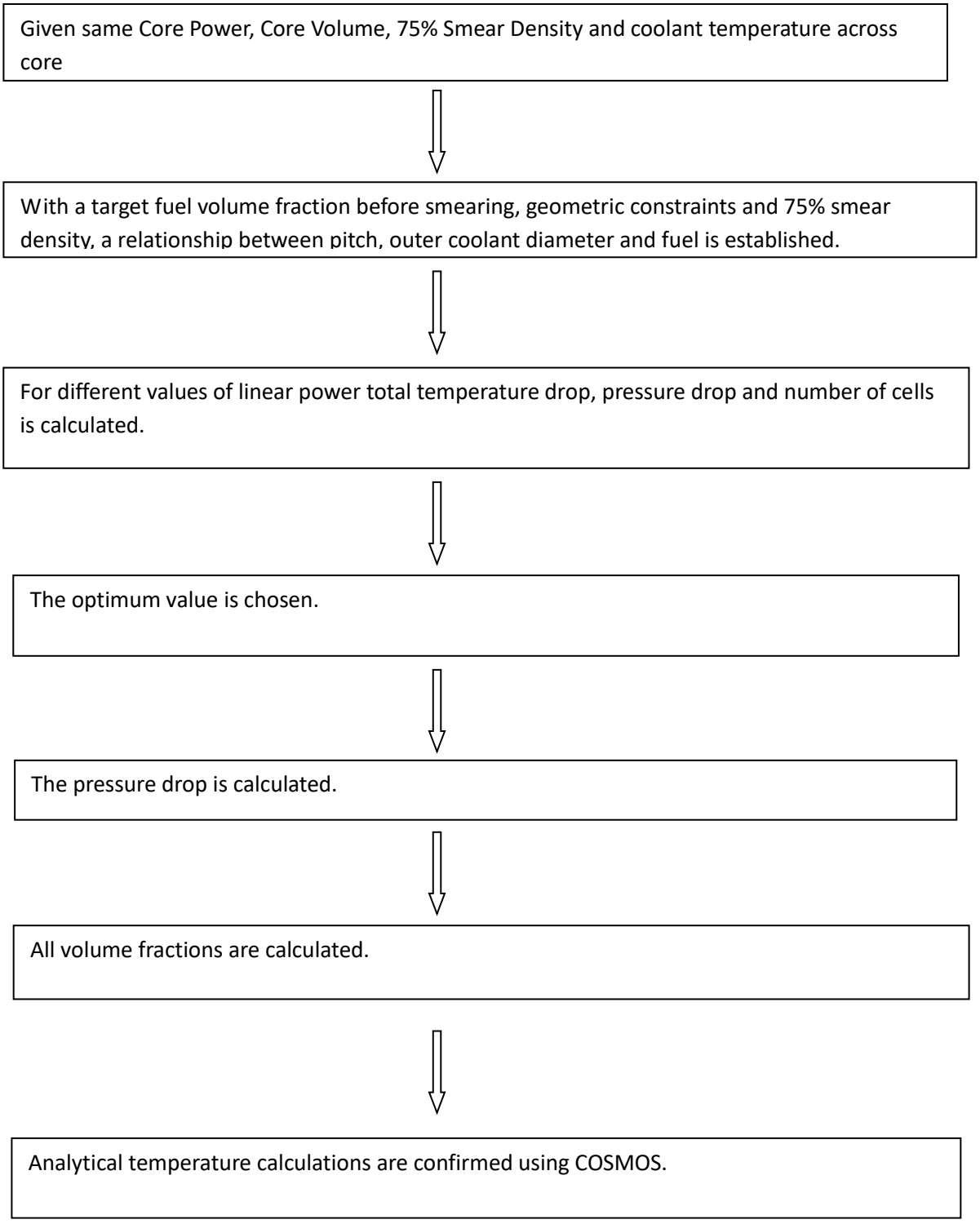
c) 75% Smear Density after swelling

d) Coolant Temperature Across Core

The TID flow tube thickness was also kept the same as the pin cell clad

The flow chart in the next page shows the procedures in NOVEX 1.1 design.

FLOW CHART FOR TID CELL DESIGN



Step 1: Volume Fraction Calculations

Volume fraction tells us how much of a given material exists with a given volume. In calculating the volume fraction of a given core, it is very important to distinguish the two ways of doing it. One is volume fraction within a unit cell. The second one is volume fraction averaged over an assembly. The latter one takes into account the duct spacing and coolant flowing in between assemblies. In this report volume fraction refers to a unit cell unless otherwise specified. A TID unit cell is shown below and with that in mind it is possible to calculate the volume fractions.

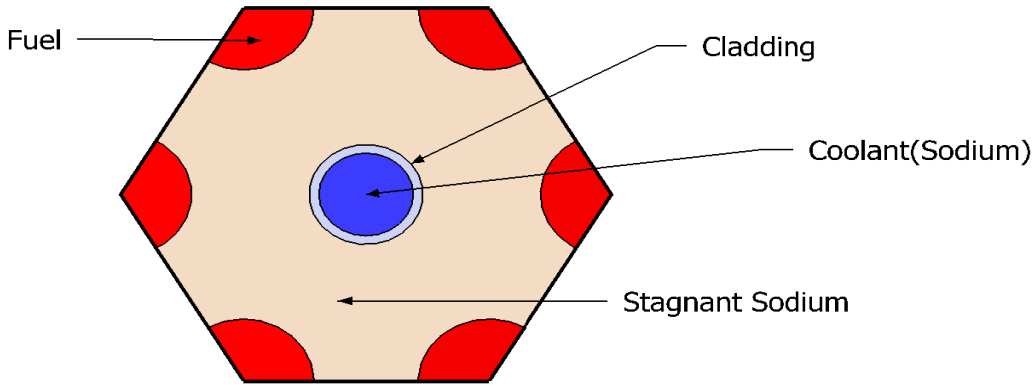


Figure 1.8 A TID Unit Cell

For such a TID unit cell of pitch P, diameter of fuel d_f and coolant tube diameter d_c, the following equations hold true.

Area of the cell: $A_c = \frac{\sqrt{3}}{2} p^2$

Area of Fuel: $A_f = \frac{\pi}{2} d_f{}^2$

Area of coolant tube: $A_t = \frac{\pi}{4} d_c{}^2$

Total area available for Swollen Fuel: $A_s = \frac{\sqrt{3}}{2} p^2 - \frac{\pi}{4} d_c{}^2$

Area of Cladding: $A_{cd} = \frac{\pi}{4}\left(d_c{}^2 - d_i{}^2\right)$

Smear density is defined as $s = \frac{A_f}{A_s} = \frac{\frac{\pi}{2} d_f{}^2}{\frac{\sqrt{3}}{2} p^2 - \frac{\pi}{4} d_c{}^2}$

Simplifying this further we can find that $s\left(\frac{\sqrt{3}}{\pi} p^2 - \frac{1}{2} d_c{}^2\right) = d_f{}^2$ Eq.1

Fuel Volume fraction: $V_f = \frac{\pi}{\sqrt{3}}\left(\frac{d_f}{p}\right)^2$

Coolant Volume Fraction: $V_c = \frac{\pi}{2\sqrt{3}}\left(\frac{d_i}{p}\right)^2$

Cladding Volume Fraction: $V_{cd} = \frac{\frac{\pi}{4}\left(d_c{}^2 - d_i{}^2\right)}{\frac{\sqrt{3}}{2} p^2}$

Stagnant sodium (bond) volume fraction: $V_b = 1 - \frac{\pi}{2\sqrt{3}}\left(\frac{d_c}{p}\right)^2$

There are two important geometric constraints that need to be taken into account.

1. To prevent fuel slugs not overlap (i.e. touching)

$$d_{f_{\max}} \leq \frac{p}{\sqrt{3}}$$

2. To insure slug is trapped between flow tubes (i.e. fuel slug and coolant tube outside diameter must not overlap)

$$d_c + d_f \leq \frac{2}{\sqrt{3}} P$$

Eq.1 Is another important constraint since we assume smear density to be 75 %(the same as the reference design).

From our flow chart, the next step is to solve for pitch, coolant diameter and fuel diameter with our target fuel volume fraction. In NOVEX 1.1 we are looking for the maximum a fuel volume fraction that can be attained (that also satisfies the constraints.)

From constraint 1 it can easily be seen that the maximum fuel volume fraction is expressed as

$$V_f \leq \frac{\pi}{3\sqrt{3}}$$

Thus the maximum fuel volume fraction we can attain is 60.5%. Since our design objective was to have the maximum fuel volume fraction, this is taken as our fuel volume fraction. Using equation 1, the fuel volume fraction can also be written as

$$V_f = s\left(1-\frac{\pi}{2\sqrt{3}}\left(\frac{d_c}{p}\right)^2\right)$$

Setting smear density of 0.75 and fuel volume fraction of 60.5%, we obtain

$$\frac{d_c}{p} = \sqrt{\frac{2\sqrt{3}}{\pi}-\frac{8}{9}}$$
$$\frac{d_c}{p} = 0.46$$
$$\frac{d_f}{p} = \frac{1}{\sqrt{3}}$$

Given the pitch, it is possible to calculate fuel diameter slug and outside diameter of coolant tube. However picking the value of pitch will affect the pressure drop and number of cells. To ensure design objectives it is necessary to choose a pitch that result in a tolerable pressure drop and meets thermal constraints. With high fuel volume fraction, it is a challenge to meet design objectives. The next section shows how the value of pitch has been picked.

Step 2: **Thermal Analysis for a TID**

For a TID cell after smearing (figure 1.7) the temperature drop across the various regions is evaluated.

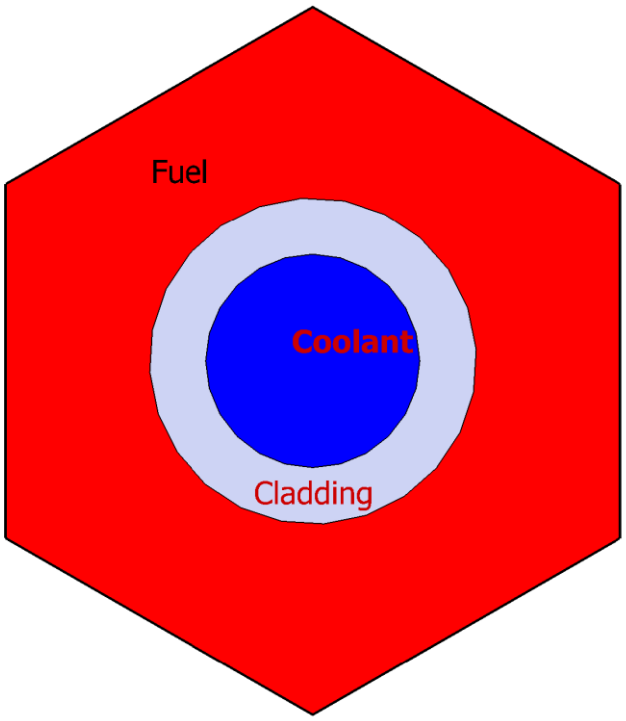


Figure 1.9 TID unit cell after smearing

Temperature drop across film: is given by

$$\Delta T_{film} = T_{wall} - T_{bulk} = \frac{q''_{wall}}{h}$$

Where ΔT_{film} is the temperature difference between the clad surface and the bulk fluid, T_{wall} is the clad surface temperature, T_{bulk} is the bulk fluid temperature, q''_{wall} is the heat flux at the

clad surface in contact with coolant, and h is the heat transfer coefficient. This can be simplified as

$$\Delta T_{film} = \frac{q'}{\pi d_h h}$$

The heat transfer coefficient can be evaluated known from the following equation

$$Nu = \frac{hd_h}{k}$$

Where Nu is the Nusselt number and k is the coolant conductivity

Rearranging to get h

$$h = \frac{Nuk}{d_h}$$

To find the Nusselt number, the Lyon-Martinelli equation was used:

$$Nu = 7 + 0.025P_e^{0.8}$$

Where P_e is Peclet number and defined as:

$$P_e = R_e \times P_r$$

Where R_e is Reynolds number and P_r is Prandtl number. At average coolant temperature of 427°c, P_r =0.0048

$R_e = \frac{d_h v\rho}{u}$ Where u is viscosity of sodium at 427°c and has a value of 2.65×10^{-4} kg/m-s

Simplifying this: $R_e = \frac{4q'}{c\Delta T \pi d_h u}$ Where c is specific heat capacity and T is temperature rise above the core

Using 1272J/kg-k for specific heat capacity, ΔT 150°c, 2.65×10^{-4} kg/m-s and

u 2.65×10^{-4} kg/m-s

We get

$$R_e = 0.025181748\frac{q'}{d_h}$$

Thus P_e =0.000120872 $\frac{q'}{d_h}$

Using $Nu = 7 + 0.025P_e^{0.8}$,

$$Nu = 7 + 0.000018357\left(\frac{q'}{d_h}\right)^{0.8}$$

Therefore the temperature drop across film can be expressed as:

$\Delta T_{film} = \frac{q'}{\pi d_h h}$: Expressing h as $h = \frac{Nuk}{d_h}$ we obtain

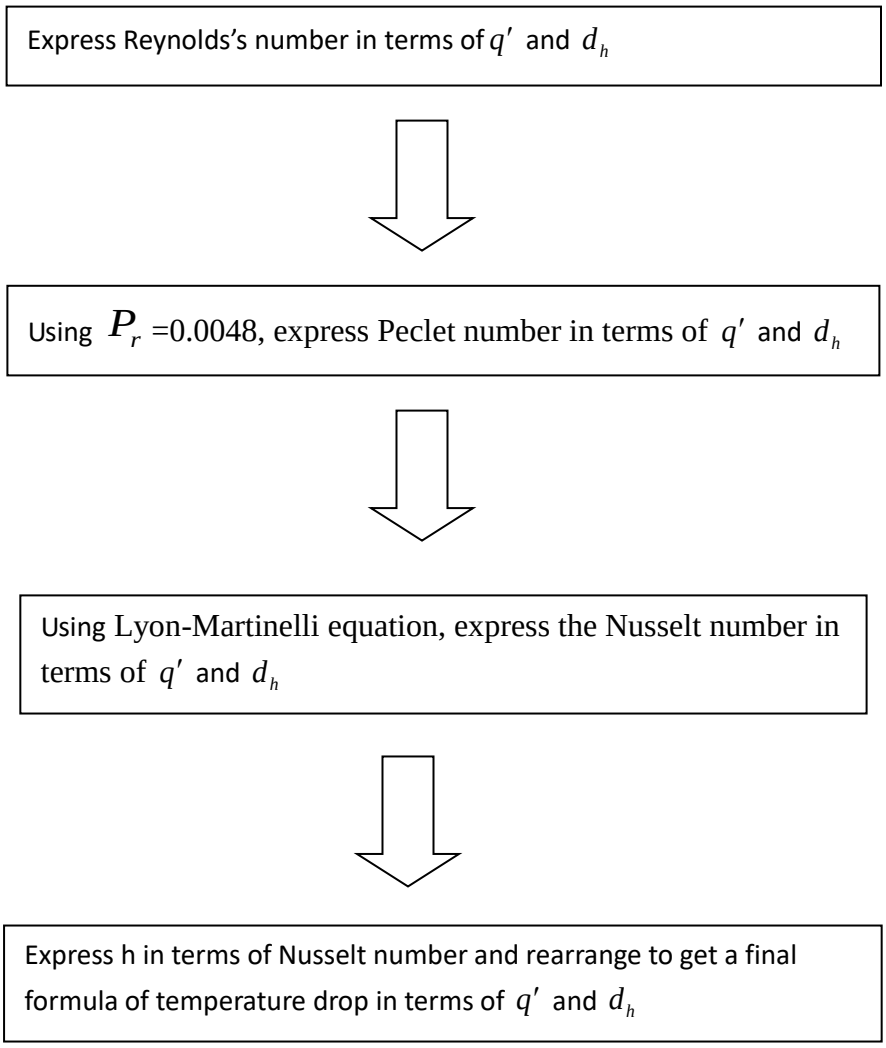
$$\Delta T_{film} = \frac{q'}{\pi Nuk}$$

$$\Delta T_{film} = \frac{q'}{\pi\left(7 + 0.000018357\left(\frac{q'}{d_h}\right)^{0.8}\right)k}$$

Using coolant conductivity k=66.70W/m-k, we get a final expression as:

$$\Delta T_{film} = \frac{q'}{\left(1466.81 + 0.00385\left(\frac{q'}{d_h}\right)^{0.8}\right)}$$

The next page shows a flow chart was what was done.



Temperature drop across Cladding: is given by

$$\Delta T_{clad} \equiv T_{co} - T_{wall} = \frac{q''_{clad}}{k_{clad}} t_c$$

Where ΔT_{clad} is the temperature rise across the cladding, T_{co} is the clad surface temperature on the fuel side, T_{wall} is the clad surface temperature in contact with the coolant q''_{clad} is the heat flux at the center of the clad wall k_{clad} is the thermal conductivity of the cladding and t_c is the thickness of the cladding.

k_{clad} =20.35W/m-k

t_c =0.000559m (same as reference pin cell)

Thus the temperature drop across cladding can be written as:

$$\Delta T_{clad} = \frac{q'}{\pi d_h}\frac{t_c}{k}$$

Simplifying this further;

$$\boxed{\Delta T_{clad} = 8.744\times 10^{-6}\frac{q'}{d_h}}$$

Temperature drop across Fuel: After smearing, the fuel occupies the red region shown in figure 1.7. The temperature between the outer coolant tube to peak fuel has been analyzed using COSMOS and the result is shown in Appendix B. For most purposes, we employ the annular approximation. In the annular approximation, non circular regions are transformed into a circle of equivalent volume. The fuel region looks as in figure 1.8.

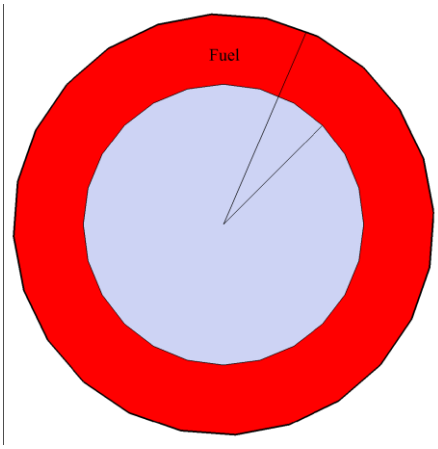


Figure 2.0 A TID Equivalent Cell After smearing

Using simple geometric relations the equivalent diameter (outside diameter) can be written as:

$$d_o = \sqrt{\frac{2\sqrt{3}}{\pi}}\,p$$

The temperature drop across the fuel is given by the formula:

$$\Delta T_{fuel} = \frac{q'}{4\pi k}\left[1 - \frac{\ln\left(\frac{d_o}{d_i}\right)^2}{\left(\frac{d_o}{d_i}\right)^2 - 1}\right]$$ Where d_o is outside diameter and d_i is the inner diameter:

Let's evaluate $\left(\frac{d_o}{d_i}\right)^2$

$$d_o^{\,2} = \frac{2\sqrt{3}}{\pi}\,p^2$$

$d_i^{\,2} = d_c^{\,2}$ Since the inner diameter is the same as coolant tube outside diameter

Thus $\left(\frac{d_o}{d_i}\right)^2 = \frac{2\sqrt{3}}{\pi}\left(\frac{p}{d_c}\right)^2$

From Step 1: volume fraction calculations,

$$\frac{p^2}{d_c{}^2} = \frac{1}{\frac{2\sqrt{3}}{\pi} - \frac{8}{9}}$$

$$\left(\frac{d_o}{d_i}\right)^2 = \left[\frac{\frac{2\sqrt{3}}{\pi}}{\frac{2\sqrt{3}}{\pi} - \frac{8}{9}}\right]$$ Factorizing $\frac{2\sqrt{3}}{\pi}$ top and bottom we obtain

$$\left(\frac{d_o}{d_i}\right)^2 = \frac{1}{1 - \frac{4\pi}{9\sqrt{3}}} = 5.158$$

Therefore the temperature of the fuel can be written as:

$$\Delta T_{fuel} = \frac{q'}{4\pi k}\left[1 - \frac{\ln(5.158)}{5.158 - 1}\right]$$

$$\boxed{\Delta T_{fuel} = 0.00407 q'}$$

The total temperature drop is the sum of the temperature drop across film, cladding and fuel. It is expressed as:

$$\boxed{\Delta T_{Total} = \frac{q'}{\left(1466.81 + 0.00385\left(\frac{q'}{d_h}\right)^{0.8}\right)} + 8.744 \times 10^{-6}\frac{q'}{d_h} + 0.00407 q'}$$

5.3 Step 3: Results and Comparisons

Since the core area and core thermal power are fixed, the average linear power is a function of the pitch. From previous correlation, it is expressed as:

$$q'_{per\ slug} = \frac{1.5\times10^{8}}{h} p_{cell}^{2}$$

The challenge in designing a core with high volume fraction is selecting the linear heat rate. In NOVEX 1.1 high values of linear power will result in a thermal disadvantage. A low linear power means lower pitch, lower hydraulic diameter hence a high pressure drop. Moreover a low linear power means more fuel slugs to achieve the same core thermal power. So it was necessary to see different values of linear power and active fuel height before settling on a particular value. To meet the thermal requirement with the intended fuel volume fraction and a tolerable pressure drop meant a lower average linear power. After careful calculations an average linear power of 16 kW/m and active fuel height of 1.66 m were chosen. Picking these two values it was possible to calculate every other parameter. Thermal calculations and pressure drop were calculated using previously derived equations. The most important results are summarized below.

Material	**Volume Fraction Unit Cell**	**Averaged Over Assembly** **Volume Fraction**
Fuel	60.48%	51.61%
Coolant	12.82%	16.20%
Bond	20.31%	17.34%
Cladding	6.39%	14.85%
Smear Density = 75%		

Table 1.3 Volume Fraction Results

In averaging over an assembly the coolant volume fraction includes the sodium that is flowing between assemblies. To find volume fraction over an assembly the correction factor for the reference pin cell was used in assumption that the design of NOVEX 1.1 will maintain some aspects of the reference cell.

Pressure Drop: The pressure drop was calculated in the same manner as the reference pin cell.

Hydraulic diameter = 5mm

Mass flow rate = 0.168 kg/s

Coolant Velocity = 10.10

R_e Number = 161,219

Friction Factor = 0.0167 (Using McAdams relation)

Core Height = 1.66 + 0.4 + 0.4 + 0.2 =2.66m

Thus the pressure drop is 0.45 MPa. It is within the range of fast reactors.[5]

Thermal Results: Using previous results for thermal analysis, thermal calculations were done. The results are summarized in Table 1.4.

Average Linear power (KW/m)	16.00
Nu Number	12.12
Heat Transfer Coefficient (W/m2-k)	161,681
Temp. Drop Across Film (°c)	12.60
Temp. Drop Across Cladding (°c)	56.00

[5] Fast Reactor Database 2006 Update. TECDOC-1531. IAEA.

Temp. Drop Across Fuel(°c)	130.24
Total Temp. Drop(°c)	198.84

Table 1.4 Thermal Results

Parameter	**Value**
Sodium	
Coolant average temperature, °C	427.0
Density, g/cc	0.846
Thermal conductivity, W/m-K	66.70
Specific heat, J/kg-K	1272.00
Viscosity, kg/m-s	2.65E-04
Steel HT-9	
Thermal conductivity, W/m-K	20.35
Metallic fuel (U-TRU-Zr)	
Fresh Fuel thermal conductivity, W/m-K	15.33
After swelling fuel thermal conductivity, W/m-K	11.85

Table 1.5 Select Properties of materials at average coolant temperature

6. Summary of Results

Volume Fraction Comparison:

	PIN	TID
Fuel	34.26%	51.61%
Coolant	28.59%	16.20%
Cladding	28.59%	14.85%
Bond	11.42%	17.34%

Thermal Comparison

	Pin	TID
Temperature Drop Across Film, (°C)	6.83	12.60
Temperature Drop Across Clad, (°C)	30.29	56.00
Temperature Drop Across Fuel, (°C)	161.98	130.24
Total temperature Drop, (°C)	**199.1**	**198.84**

Heat Transfer Coefficient (KW/m2-k)	139.2	161.7
Nu number	5.62	12.12

Pressure Drop Comparison

	Pin	TID
Coolant Mass flow rate (kg/s)	0.126	0.168
Inlet Temperature (°C)	352.0	352.0
Outlet temperature (°C)	502.0	502.0
Re Number	67727	161,219
Pr Number	0.0051	0.0048
Core pressure drop (MPa)	**0.77**	**0.45**
Coolant velocity (m/s)	7.86	10.10

		Pin	TID
Fuel	Fuel slug diameter (cm)	0.5962	0.768
	Cladding thickness (cm)	0.0559	0.0559
	Fuel heated length (cm)	102.0	166.0
	Hydraulic Diameter (cm)	0.273	0.500
	Total Number of Fuel Slugs	97560	90361
Thermal Analysis After Smearing	Temperature Drop Across Film, (°C)	6.83	12.60
	Temperature Drop Across Clad, (°C)	30.29	56.00
	Temperature Drop Across Fuel, (°C)	161.98	130.24
	Total temperature Drop, (°C)	199.1	198.84
	Heat Transfer Coefficient (KW/m2-k)	139.2	161.7
	Nu number (Westinghouse correlation)	5.62	12.12
Power	Linear power, average (kW/m) /Fuel Slug	24.12	16.00
	Power density (kW/l)	289.7	178.05
	Specific power (kW/kgHM)	64.80	151.86
Volume Fractions (Over an assembly before Expansion)	Fuel	34.26	51.62
	Structure	25.73	14.85
	Coolant	28.59	16.20
	Bond	11.42	17.34
	Smeared Density	75%	75%
Pressure drop	Coolant Mass flow rate (kg/s)	0.126	0.168
	Inlet Temperature (°C)	352.0	352.0
	Outlet temperature (°C)	502.0	502.0
	Re Number	67727	161.219
	Pr Number	0.0051	0.0048
	Core pressure drop (MPa)	0.77	0.45
	Coolant velocity (m/s)	7.86	10.10

TID Fissile Mass for the start up core = 4715.4 kg

Pin Fissile Mass for the start up core = 3279.4 kg

7. Future Work

- Evaluate Economics of TID design(Since we have Higher HM loading and significantly higher Fissile loading)

- Make a detailed design of TID assembly and core. In our analysis edge effects and control assemblies were not included.

- Find an optimized design for TID given final design objectives clearly. Given intended volume fraction, core power and thermal limits an optimized design can be found for a TID cell.

- Investigate Other possible designs: Trefoil flow tube, Trefoil fuel slug and NOVEX variations

8. References

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Tang, Y.S. Thermal Analysis of Liquid Metal fast Breeder Reactors. Illinois: The American Nuclear Society, 1978. Print

Todreas, Neil, and Pavel Hejzlar. "Flexible Conversion Ratio Fast Reactor Systems Evaluation." (2008).

Appendix A: Proofs of Important Results

This appendix will go through the derivation for the equations

1. Total number of rods in hexagonal and square assemblies

Hexagonal Assembly:

In section 2 of this report, it was given that formula for the total number of rods in a hexagonal assembly is as follows:

$$N = 3n^2 + 3n + 1$$

Where n is the number of the rings of rods. Note that the center rod is the "zeroth" ring. For example if n =2, N= 19.

Proof:

$$1 + \sum 6n$$
$$1 + 6\sum n$$
$$1 + 6\left(\frac{n(n+1)}{2}\right)$$
$$1 + 6\left(\frac{1}{2}n^2 + \frac{1}{2}n\right)$$
$$1 + 3n^2 + 3n$$
$$3n^2 + 3n + 1$$

QED

Square assembly:

In a square assembly as shown in figure 1.9, as we go from the center square (considered as the Zeroth ring), the number of rods goes like 9, 25 and 49. A formula can be derived that gives the number of rods for a given ring.

$$1+\sum 8n$$
$$1+8\sum n$$
$$1+8\left(\frac{n(n+1)}{2}\right)$$
$$1+8\left(\frac{n^2+n}{2}\right)$$
$$1+4n^2+4n$$
$$4n^2+4n+1$$

Where n is the ring number. For example when n= 0, the number of rods is 1 (the center ring). When n=1, the number of rods is 9.(can be confirmed from the diagram).

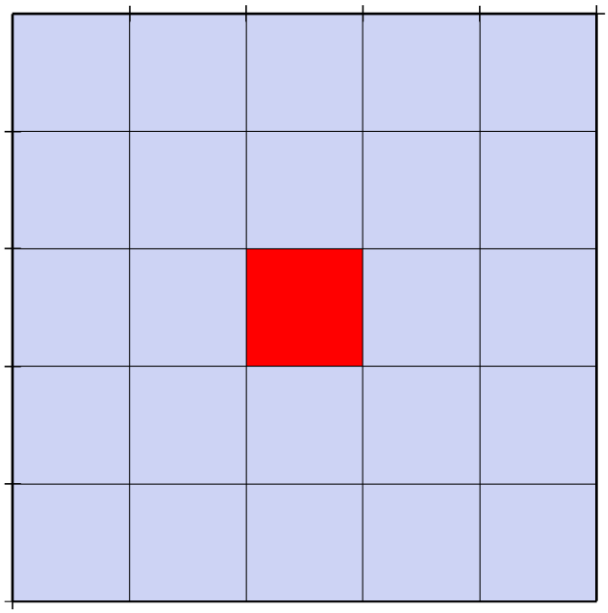

Figure 2.1 Square Unit Cells

2. Equivalent diameter for square and Hexagon

The equivalent diameter of a square is given by:

$$d_{cell} = \frac{2}{\sqrt{\pi}} p$$

Proof: In square assemblies (see figure 1.1) the side of the square is equivalent to the pitch.

Making the area of the square the same as a circle of diameter d , we get

$$p^2 = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4}{\pi} p^2}$$

$$d = \frac{2}{\sqrt{\pi}} p$$

For hexagonal assemblies, the distance across the flats of the hexagon will be equal to the pitch. (See figure 1.9 for illustration)

Making the area of the regular hexagon equal to that of a circle:

$$\frac{\sqrt{3}}{2} p^2 = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{2\sqrt{3}}{\pi}} p$$

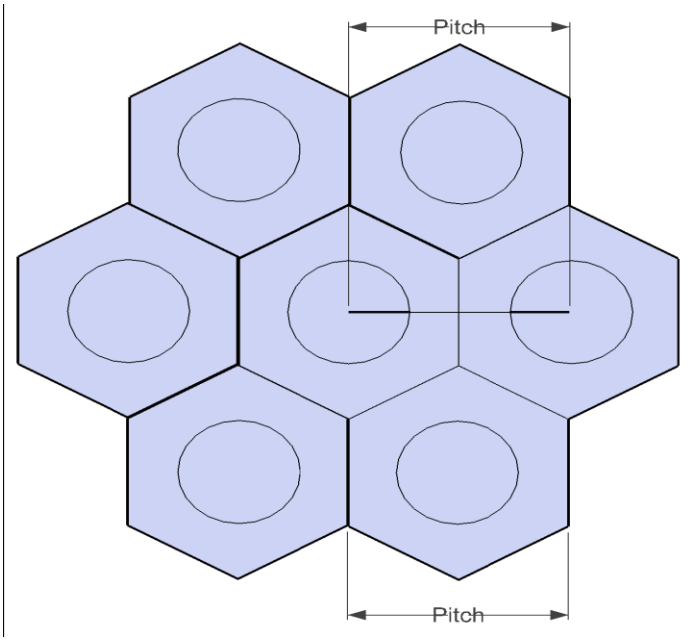

Figure 2.2 Hexagonal Unit cells

Appendix B: Volume Fraction Methodology

Fuel volume Fraction before fuel expansion: Volume fraction of fuel slugs at their as-cast density in a central unit cell. Given as V_f

Fuel volume Fraction after fuel expansion: The fuel mass is conserved. Therefore;

$$V_f' = \frac{V_f}{f_s}$$

Where V_f' is fuel volume fraction after expansion and f_s is the smear density fraction, and is defined as:

$$f_s = \frac{Fuel \text{ volume Before}}{\text{Fuel Volume After}} = 0.75$$

Since the smear density is 75%.

Over assembly (Over Core) Volume Fractions: Figure 2.1 shows a simplified diagram of an assembly. To define fuel volume fraction over this assembly; Let

$$f_a = \frac{\text{cross sectional area inside}}{\text{cross sectional area of duct + Na}}$$

$$f_a = \left(\frac{W_i}{P_a}\right)^2$$

W_i **is the inner width and** P_a **is assembly pitch.**

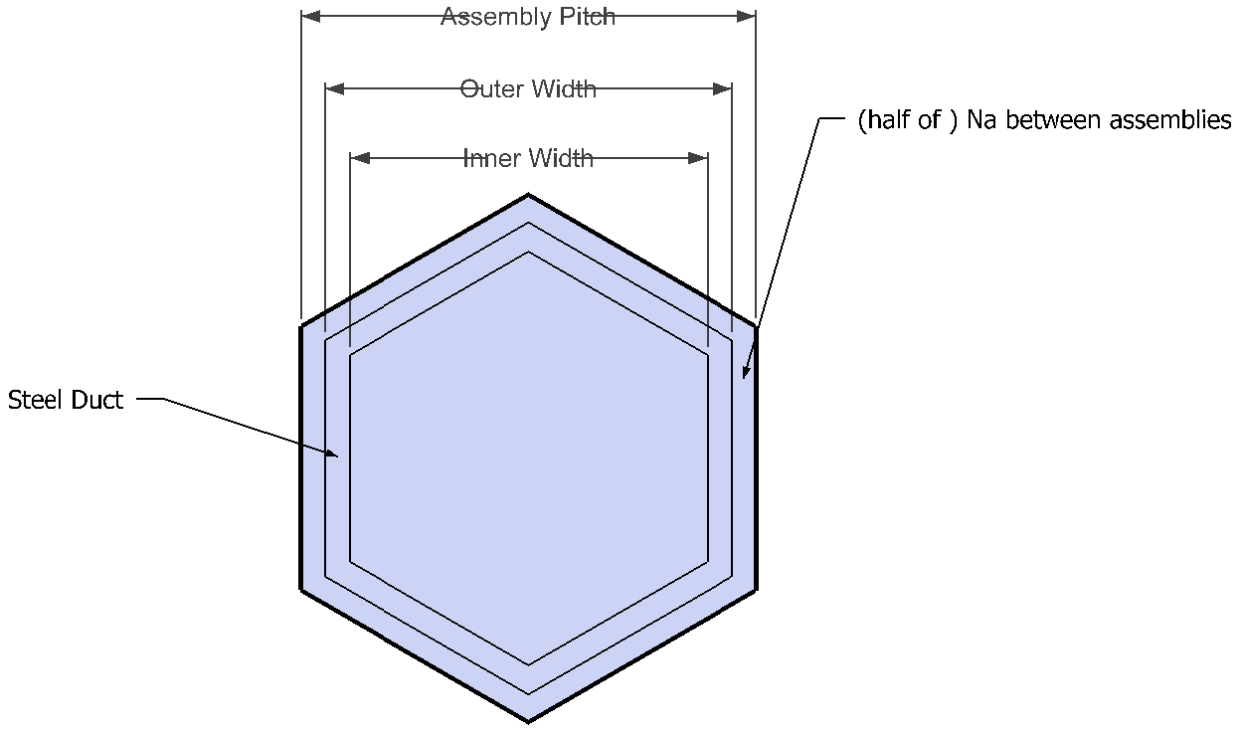

Figure 2.3 Cross section of a hexagonal assembly

To find fuel volume fraction over an assembly; use

$$V_{f\,(over\ \text{assembly})} = V_{f\,(over\ \text{unit cell})} \times f_a$$

This will give homogenized core values.

Note the following assumptions:

1) Duct steel and external Na have to be included in structure and coolant volume fractions
2) We assume duct's interior is filled with central unit cells; no allowance is made for edge cell difference

3) No allowance is made for control assemblies

Appendix C: COSMOS Tutorial

COSMOS is a design analysis system fully integrated with Solid Works. It provides solutions for stress, frequency, buckling, thermal, and optimization analysis. Solid Works is mechanical design automation software that takes advantage of the familiar Microsoft Windows graphical user interface. It is used to quickly sketch out ideas, experiment with features and dimensions, and produce models and detailed drawings.

Analysis Steps

You complete a study by performing the following steps:

• **Create the geometric model in SOLID WORKS**. Creating the geometric model of the object for the study is the first and most important step. SOLID Works has an extensive library where users can already download models.

• **Choose analysis type**. In Our case, the analysis is thermal.

• **Define material properties**. This is specifying what material is being used and specifying the necessary material properties. For example if we are doing analysis of cladding we can define it as Steel put the density and specific heat capacity. COSMOS also has extensive library of materials a user can choose from..

• **Specify the loads**. These are equivalent to boundary conditions. In thermal analysis this refers to heat flux, heat power, and temperature and so on. Depending on the load, one can specify it on the vertex, edge or face of a model.

• **Mesh the model**. By default COSMOS does mesh the model but a user can customize it to his/her need.

• **View and list the results.**

Appendix D: Thermal Analysis in COSMOS

The annular approximation is sufficient enough in calculating the temperature difference between an outer cladding tube and the smeared fuel. However since the corner of the hexagon is hotter than the circle, it was important to know by how much it differed from the approximated value. There were two objectives in using COSMOS. One was a more accurate temperature calculation and second was evaluation of the annular approximation model.

1. Temperature Calculation: Two bench marks were set before starting any calculation in COSMOS. This was done to see how accurate COSMOS values compared to analytical calculations.

Benchmark one (Reference Pin cell): For a cylindrical fuel slug with a given linear power and thermal conductivity, the temperature difference is just a function of the two. Analytically it was calculated to be 161.98°c. The COSMOS result is 161.6°c. There is a difference of 0.38. The COSMOS result is shown in figure 2.2.

Benchmark Two (Annular TID cell): For an annular fuel with inner radius r_i, outer radius r_o, linear heat rate q' and thermal conductivity k, the annular approximation gives the temperature difference. Analytically it was calculated to be 130.24°c. The COSMOS result is 129.7°c. There is a difference of 0.54. The COSMOS result is shown in figure 2.3.

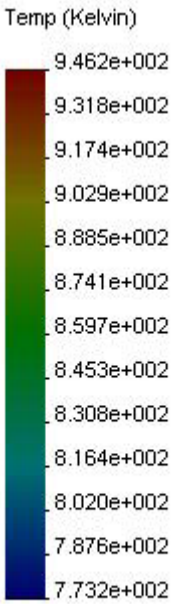

Figure 2.4 Bench Mark Pin Results

Figure 2.5 Annular Bench Mark TID Results

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After the two benchmarks were set, the next step was using COSMOS to do thermal analysis.

A. **TID HEX AFTER SWELLING**: COSMOS was used to find the temperature difference between outer cladding tube and corner of a hexagon. The COSMOS result is 131.1°c. It makes since the corner of the hexagon is hotter than the circle by a few degrees. (In this case about 0.86°c)

The result is shown in figure 2.3.

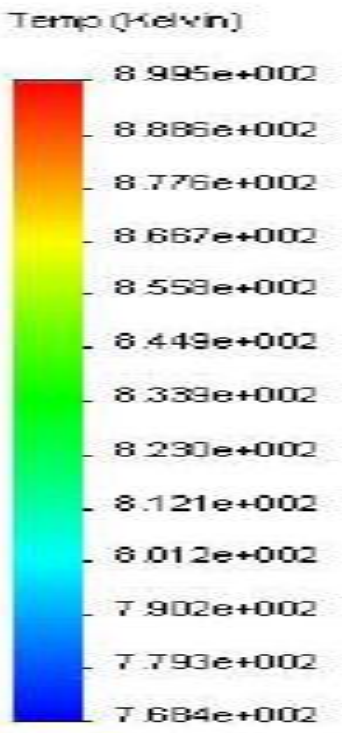


Figure 2.6 TID HEX After Smearing Result

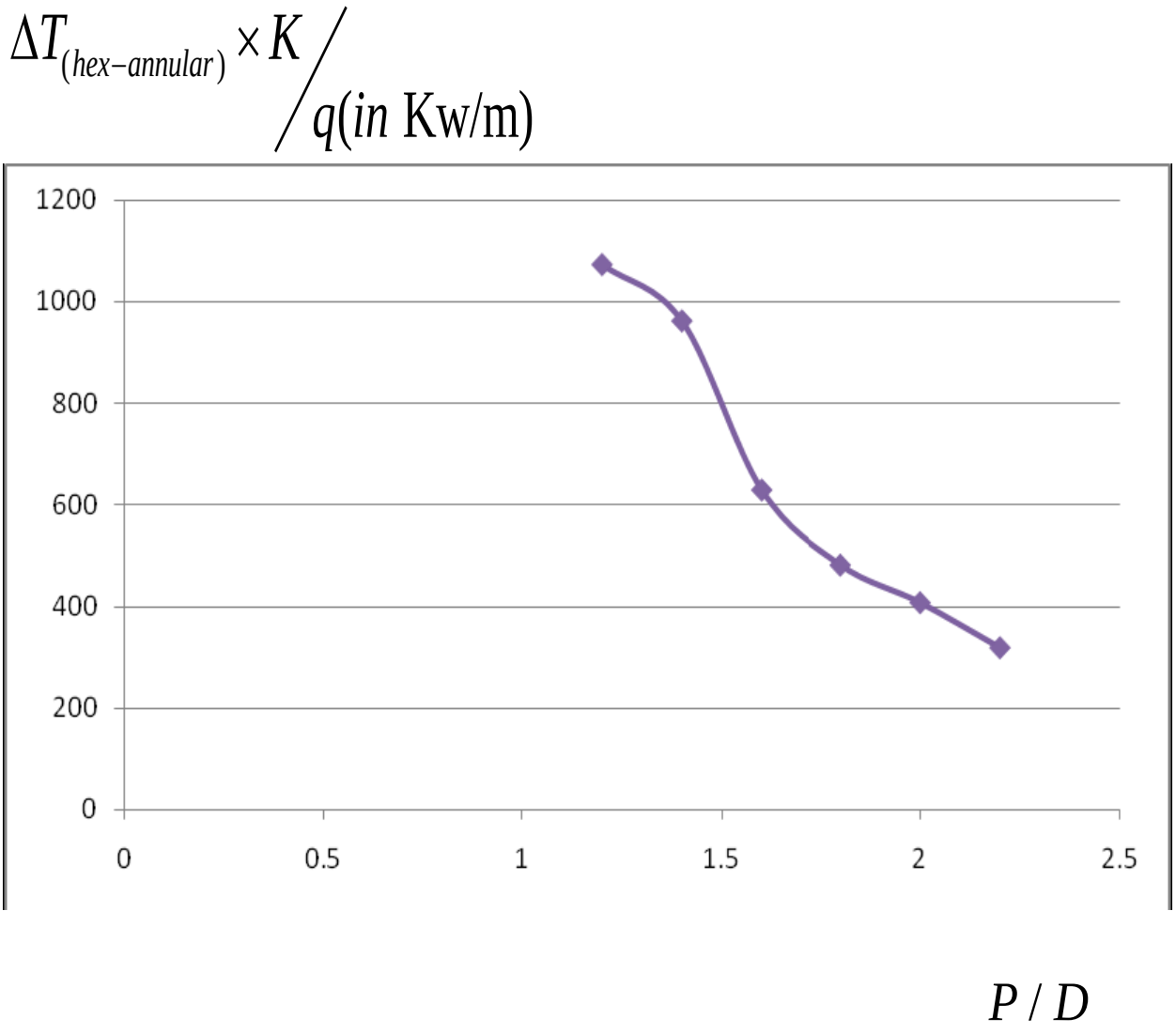
2. Evaluation of the Annular Approximation Model: This was an idea recommended by Professor Neil Todreas. The evaluation is meant to assess how good the annular approximation is. In other words, it was done to know under what conditions the annular approximation holds or is more accurate. To evaluate that P/d_c was varied and the difference between COSMOS and analytical calculation was calculated. The table below shows the 6 P/d_c values taken and the corresponding $T_{hex} - T_{annular}$ value. Following that the graph shows what happens.

P/d_c	$T_{hex} - T_{annular}$	Analytical value
1.2	2.9°c	48.2°c
1.4	2.6°c	72.2°c
1.6	1.7°c	92.4°c
1.8	1.3°c	108.52°c
2.0	1.1°c	121.37°c
2.2	0.86°c	130.24°c

Table 1.9 Evaluation of Annular Approximation

GRAPH

A graph of P/D_c (on the x-axis) versus $\Delta T_{(hex-annular)} \times K \Big/ q(in\ \text{Kw/m})$ on the y-axis was drawn. It shows that as the P/D_c ratio increases, the annular approximation model tends to be more accurate.



Graph 1.1 Evaluation of the annular approximation

Appendix E: Other Possible Designs

E1: NOVEX 1.2

Same linear power per slug: If one starts with the same linear power per slug, the thermal requirement will not be met. For the same linear power and thermal conductivity, an annular fuel operates at a lower temperature than a solid fuel. However at high fuel volume fraction, the TID thermal exceeds the pin thermal limit. (Remember that for a solid fuel the temperature difference across is independent of the fuel radius.) So if one wants a design with the same linear power per slug, fuel volume fraction has to be around 50%.

Same Pin Pitch: The main challenge in this approach is the pressure drop. The design in the report has a lower coolant fraction and if one wants to keep that, the hydraulic diameter will be too small. This would mean a very high pressure drop.

Corrugated Flow Tube: Round fuel slugs but corrugated flow tubes can be used. This can be explored in future work.

Trefoil fuel tube: Round fuel slugs but trefoil flow tubes can be used. This can be explored in future work.

Appendix F: Trefoil Design Analysis

NOVEX 1.2 was the second design (design analysis) in this project. NOVEX 1.2 uses a trefoil slug (not the conventional round fuel slug.) Figure 1.8 shows a trefoil slug and figure 1.9 shows how it looks like before smearing.

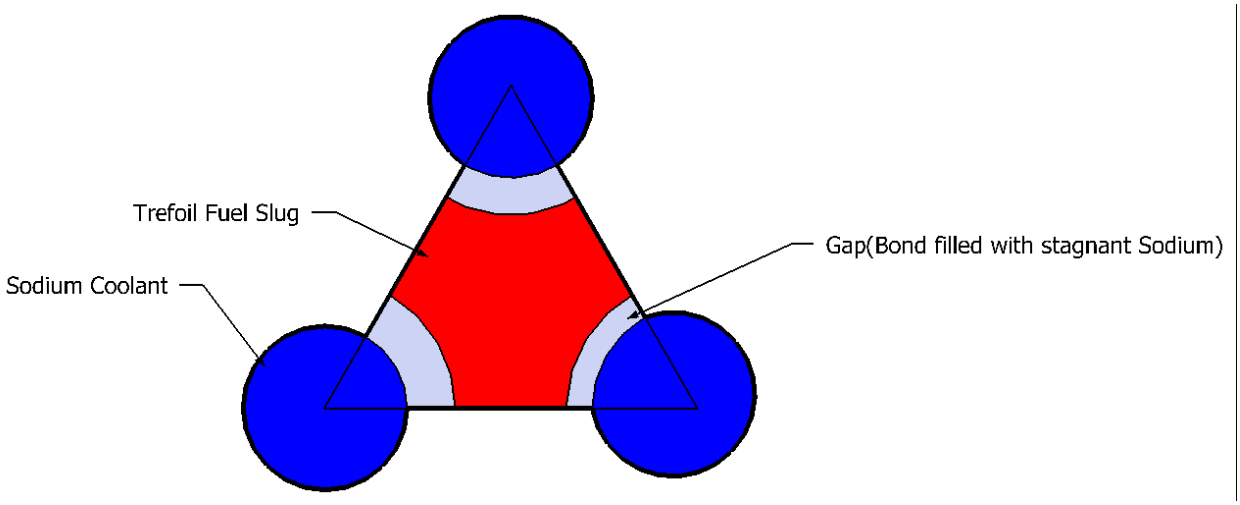


Figure 2.7 A Trefoil Slug

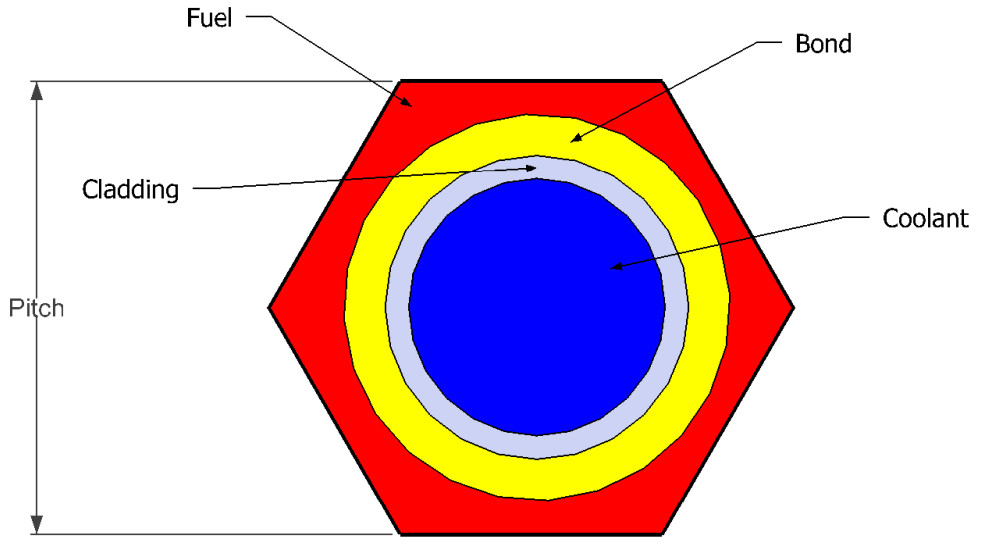


Figure 2.8 NOVEX 1.2 Design before smearing

The significant advantages of using a trefoil slug are:

A) Wide choice of Lattice P/D

B) More Uniform Sodium Bond Gap

C) More secure trapping of fuel in place

The only disadvantage that might come to the mind of the reader might be the fabrication of the slug shape. To check on this issue the writer of this report communicated with Yoon Ching, Associate Laboratory Director and distinguished fellow at Argonne National Laboratory. His reply was positive. Below is an excerpt from his email.

Dear Abiy,
I guess metal fuel can be fabricated in any shape or form, although in varying degrees of difficulty. I don't know what particular advantages will come about with your proposed configuration, but the 4.567m pitch seems awfully big. Maybe a typo?

Yoon

Since after smearing, the smear density is the same, the maximum limit of the fuel volume fraction will remain the same. As a result it doesn't offer any advantages in terms of fuel volume fractions, pressure drop and thermal. But as mentioned above it can provide a secure trapping of fuel in place and create a more uniform sodium bond.

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