Lesson 1: Repeating Patterns

Goals (these as learning/content goals—not teacher instructional goals—so that we have a quick view of what math content is being addressed)**:**

- Identify a pattern
- Identify next, near and far elements in a repeating pattern
- Identify position numbers for a repeating pattern
- Identify missing elements in a repeating pattern
- Identify a unit of repeat (optional)
- Identify simple growing patterns;
- Recognize that not all patterns repeat and the difference between repeating and growing patterns
- Identify a rule that can describe *in general* how to build a pattern

1. WHOLE CLASS DISCUSSION:

Finding Next, Near, Far elements

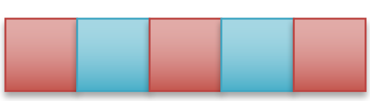
Construct an AB pattern using connecting color cubes using 5 cubes and two colors.
Attribute that varies: color

WHOLE CLASS ACTIVITY AND DISCUSSION:

Have the whole class sit in a large circle. Explain to the children that we will build a pattern together. Ask them if they know what a pattern is. Solicit responses, ask for different opinions, and disagreements. This will be one way to assess what the kids already know. Then tell them that you are going to build a pattern and they should pay close attention because you will follow up with questions.

Build the pattern one cube at a time, verbalizing every choice and move: "First I will put a red cube. Next I will put a blue cube. Now I will put another red cube, and then another blue cube. Last I will put a red cube."

Make sure the pattern is laid out in a spot in the middle of the circle where all the kids have good visibility.



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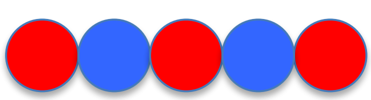


Questions:

- If someone said this was not a pattern, what would you tell him? How would you describe the pattern?
- What do you think comes next? Why?
- Looking at the pattern we have here, what color is the 2nd cube? How can you be sure? What color is the 5th cube? How can you be sure? What did you have to do to figure that out? What if another kid said you were wrong. What would you say?

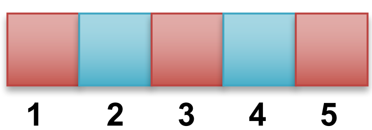
- What color do you think the 10th cube would be? How do you know? How can you find out? How can you be sure?

2. WHOLE CLASS DISCUSSION:

Introduce position number

Tell the children that in order to see the pattern better, we can number the cubes. Have position number cards so that students can help place position numbers with the appropriate cube.

- Now that we have the position numbers on our pattern, what do you notice?
- Let's think about the 10^{th} cube again. Would it be easier to figure out what color the 10^{th} cube would be if we can see the position number on the card?
- Using our position numbers to help us, what would be the color of the 13^{th} cube?
- If someone found out what the 13^{th} cube would be, ask: "Could you explain how you found that out so that other kids could try to do the same thing" (This would be an attempt for them to explain how they arrived at a procedure as well as to articulate a rule that describes the procedure.)



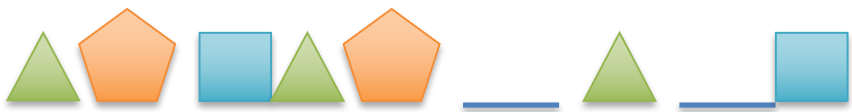
- What kinds of position numbers occur for (red) cubes? How about for blue cubes?
- Do you think this will always happen? Why?
- How would you tell a friend who is not in your class how to build this pattern?

  NOTE: Introduce the term "rule" AFTER students have talked about or identified a pattern (i.e., if they describe a growing pattern as "add 1 each time", we can then talk about how we might call that our rule).

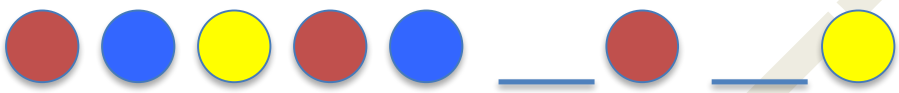
3. WHOLE CLASS DISCUSSION:

What's Missing? What's Repeating?

Show students the following pattern: **(Attributes that vary: color & shape)**



(or – depending on representations):



- Can someone help me put the position numbers in our pattern?
- What shapes are missing? How do you know?
- What is repeating here?

4. WORKING WITH A PARTNER

Explore

<u>CHILDREN SIT WITH A PARTNER AT THEIR TABLES OR DESKS:</u>

<u>SEE HANDOUT</u>***:*** (Partner task): *The students are going to each build a pattern on their handout. Then they will take turns telling each other how to build the pattern and answering questions about the process. It may be important to emphasize that the directions to build the pattern should be as general as possible; i.e.: "it's a red-blue-blue pattern" instead of "red, then blue, then blue, then red, then blue…."*

HANDOUT QUESTIONS:

Build a pattern below. Use 2 colors and 10 stickers

Describe your pattern to your partner without showing him/her your work. Your partner will also describe his/her pattern to you. Build your partner's pattern here:

Compare. Do the two patterns look the same? _____

Ask your partner: "What sticker would come next in my pattern?" ______

5. WHOLE CLASS ACTIVITY AND DISCUSSION:

Class Discussion: Have students share a few examples of their patterns. Try to select patterns that aren't simply alternating patterns (ABAB…) and that vary along different attributes (shape, color, shape and color…).

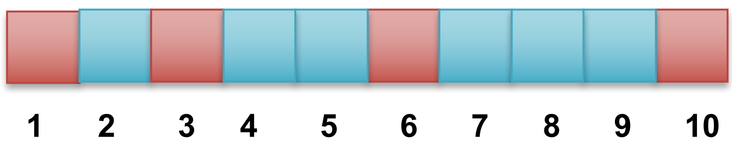
Compare two of the patterns that involve variations on different attributes (i.e., variation on color or shape). Ask the following questions:

- What is the same about the patterns? What is different?
  The goal is to see that the two patterns are structurally similar (i.e., red/blue/red/blue is analogous to square/triangle/square/triangle)
- How would you tell a friend to build each of these patterns? Remember we used the word "rule" before? So, let's see if you are ready to use this word! What is the same and what is different about these "rules"?

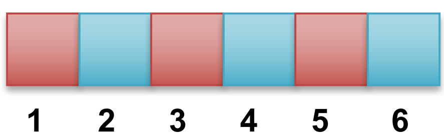
6. <u>WHOLE CLASS DISCUSSION:</u>

Growing or Repeating?

Consider the following cube train **(Attribute that varies: color)**:

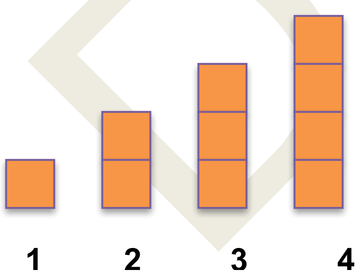


- What do you think comes next? Why?
- How would you tell a friend how to build this pattern? What's the "rule" to build this pattern?
- How is this pattern similar to the following? How are the two patterns different?



Growing Pattern – Next and Near Elements; Reversibility

Construct the following growing pattern as students watch. Make sure to verbalize your moves and decisions about what to do next:



Questions:

- Do you see a pattern here? Describe it.
- What do you think would come next? How do you know?
- What do you think the 8th figure would look like? How do you know?
- From any tower of tiles, how would you get the next tower?

- How would you tell a friend to build this pattern? What's the "rule" to build this pattern?

- If your friend told you that she was pointing to the tower with 12 tiles in it, what tower number is she at?

?

Exploring Non-Patterns (Optional)

(Optional—this task has been very challenging for some children). Using pattern blocks again, construct a sequence of blocks that does not have a pattern. How do you know your sequence of blocks does not have a pattern?

(Optional, depending on time) Identifying Units that Repeat

Continuing to use the pattern you worked with for number 4, explore the idea of repeating units in a repeating pattern.

- How many cubes/stickers do you need to see to know the pattern?
- How many cubes/stickers do you need to see to be sure you know the pattern?

  What is repeating here? What section of cubes do you need in order to create this pattern?

Handout: Repeating Patterns

Lesson 1

Name: ________________________________ Date: ____________

Build a pattern below. Use 2 colors and 10 stickers

My Partner's Name:____

Describe your pattern to your partner without showing him/her your work. Your partner will also describe his/her pattern to you. Build your partner's pattern here:

Compare. Do the two patterns look the same? _______________

Ask your partner: "What sticker would come next in my pattern?"

____

Lesson 2: Growing Patterns (with Positions)

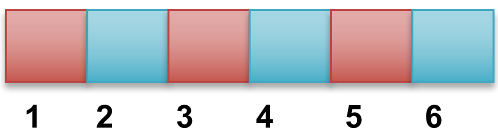
Goals:

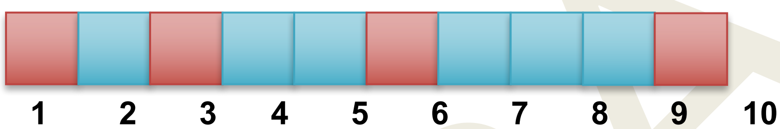
- Review repeating and growing patterns
- Identify functional growing patterns as patterns
- Identify far elements
- "Fix" a broken pattern (missing elements)
- Explore a doubling pattern

1. **WHOLE CLASS DISCUSSION:**

   Review Repeating and Growing Patterns (5-10 minutes)

WHOLE CLASS ACTIVITY AND DISCUSSION:





Put the ABAB repeating pattern and the ABABBA growing pattern somewhere the kids can clearly see both.

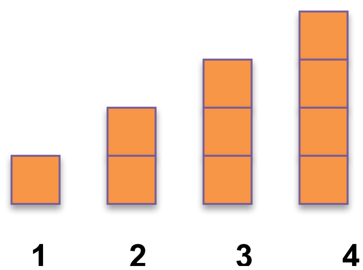
- What are the two patterns we have in front of us?
- What makes the first pattern a repeating pattern? (naming them is not really the goal so if they can only describe each pattern that is fine)
- What makes the second pattern a growing pattern? (naming them is not really the goal so if they can only describe each pattern that is fine)
- What is the same about both patterns? What is different about both patterns?
- How would you tell a friend to build this pattern? What's the "rule" to build this pattern?
- What are the numbers called? (again, being able to name the numbers is not the goal. The goal is to get to discuss their purpose)
- How can they help us?

2. **WHOLE CLASS DISCUSSION:**

   Functional Growing Patterns

(where there is a covariational relationship between position number and elements)

Construct the following type of growing pattern with students. Be explicit about how you are placing the cubes for the pattern (first group, second group, etc.) Let students identify the position numbers by placing the position number under each figure.



Class Discussion:

- This is a different kind of growing pattern. Why would this be called a growing pattern? (once again, the goal is not to get the kids to *name* the pattern but to figure out what makes this arrangement of objects special)
- What is similar, what is different from the growing pattern we just looked at?
- What figure comes next?
- How do you know what comes next?
- How would you tell a friend to make this pattern? What's the "rule" to build this pattern?

3. PARTNER ACTIVITY

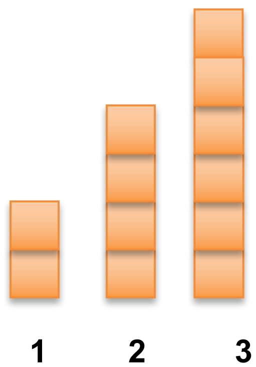
Predicting far and arbitrary elements:

CHILDREN SIT WITH A PARTNER AT THEIR TABLES OR DESKS:
Have the partners build the pattern on their handout. They should start the pattern so that they have plenty of room to add elements to the left. They should have position number cards they can put under their patterns. Guide them through adding next and far elements.
SEE HANDOUT: *Partners work together. In Handout 1, they are working with the opening pattern above. In Handout 2 students are working individually with a y = 2x vertical growing pattern.*
HANDOUT 1 TEXT:

Draw the next figure and write its number. Draw position number 8. Fill in the rest of the position numbers. How would you describe how to make this pattern to a friend? What's the "rule" to build this pattern? If you have 12 tiles, what would be the position number?

4. INDIVIDUAL ACTIVITY:

Functional growing pattern II



Direct students to the pattern on Handout 2

HANDOUT 2 TEXT:

Draw the next few figures. Label the position numbers. What is different about this pattern? How would you describe how to make this pattern to a friend? What is the "rule" to build this pattern? If a figure has 18 tiles in it, what is the position number? How do you know?

5. PARTNER ACTIVITY:

Finding Missing Items

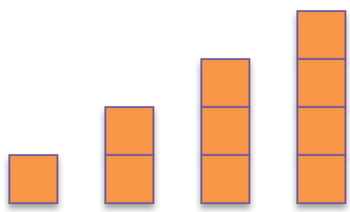
Instruct the kids that they are now going to work in table groups (about 4 students). First, they need to build a pattern like Handout 2. Make about 5 or 6 positions. Then, they are going to play a game. While the group hides their eyes, one person will change one or two things about the pattern. When that person says "Ready!" the others will look up and try to figure out what has been changed.
After the students have the chance to play for a few rounds, bring their attention to a full class discussion.

- How was that activity? Was it easy or hard?
- What were the changes that were easy to see?
- What were the most challenging?
- What helped you to figure out how to fix the patterns?

6. Constructing a Pattern. If there is extra time, they can build patterns at their desk and have friends take a look.

Handout #1 Lesson 2: Growing Patterns

Name: ____ Date: ___________



1 2 3 4 _____ _____ _____ **8**

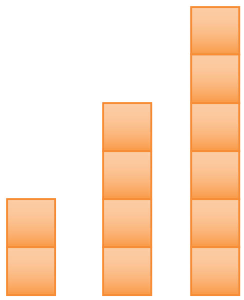
Draw the next figure and write its position number.
Draw the figure for position number 8.
Fill in the rest of the position numbers.

How would you describe how to make this pattern to a friend? What's the "rule" to build this pattern?

If you have 12 tiles, what would be the position number?

Handout #2 Lesson 2: Growing Patterns

Name: ____ Date: ___________



1 2 3

Draw the next few figures. Label the position numbers.
What is different about this pattern?

How would you describe how to make this pattern to a friend? What's the "rule" to build this pattern?

If a figure has 18 tiles in it, what is the position number? How do you know?

Lesson 3: Pennies in a Jar

1. Whole Class Discussion as circle time at front of the room:
Have printed pictures of patterns the children made at the end of Lesson 2. Show the pictures one by one, grouping them (using children's input) into repeating patterns and growing patterns. As mentioned in the previous lesson, the goal is not for students to name the two kinds of patterns correctly. (Circle time at front of room?)

What makes each of these a pattern? *For instance, children might say things like "They follow a special order, we know what comes next…". We might follow up with something like: "Is that true for both kinds of patterns?"*

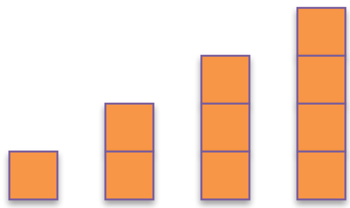
Shift discussion to look only at growing patterns:

- What is similar about the patterns we have here? *For instance, children could say things like: "They get bigger with each one, they grow, the number of cubes goes up each time"*
- What do the numbers tell us about the pattern?
- Last question: Can we predict what will happen in this pattern? What would the 10th object look like?

Growing Patterns to Covariation
Return to the original growing pattern from Lesson 2 or use the pattern from review.
Teacher Commentary:

a. *When we talked about growing patterns last time, we talked about how the patterns grow as you move along the series or the numbers.*
b. *When you predicted what the 10th position looked like, what information did you use?*
c. *Is there a relationship between the position number and what object belongs in that position number?*

If I put 7 cubes in the 10th position, would you guys agree or disagree with my prediction? Why?
Explore the idea that there is a relationship between position number and object configuration. Use this to predict a far value (e.g., 100 has a tower of 100 tiles)



1 2 3 4

2. Whole Class Discussion: A New Pattern of Relationships: Pennies in a Jar
Question: Sara gets 1 penny each day from her Grandma. She keeps them in a jar.

- How many pennies does Sara have after one day?
- How many pennies does Sara have after two days? (continue for 3, 4 and 5 days)

Record the information, with student help, on a whiteboard or blackboard, as follows:

1	1
2	2
3	3
4	4

Discuss what the columns of numbers represent (e.g., number of days) and whether it would be a good idea to include information about this. This may be a good lesson to introduce having a variable as a heading (e.g. Number of Days is represented as D).

Then introduce (finalize) the structure:

Number of Days (D)	Number of Pennies in Jar (P)
1	1
2	2
3	3
4	4
5	5

3. Partner work at tables:

Have students record their thinking on their handouts (see Handout).

HANDOUT TEXT:
Sara gets 1 penny each day from her Grandma.
She keeps them in a jar. Show on this table how many pennies Sara has on each day.
[table pictured here]
How many pennies are in the jar after 10 days?
How did you figure this out?
Is there a pattern you notice in the table?

4. Whole Class Discussion:
Discuss students' solutions, paying attention to how they solved the problem and the different representations they used.

5. Discussion: Explore with Partner Sitting Next to You in the Circle: Do you see any patterns or relationships here [in the function table]? How would you describe the pattern or relationship between the numbers?

Take time to discuss the types of relationships students see. They will likely talk about recursive patterns [Vergnaud's scalar relationship or going down a column]. Develop a shared understanding of how to talk about recursive patterns.

- What happens to the number of pennies every time we add a day? (*The goal here is to get students to think about two quantities co-varying, looking across rows in a table)*
- How would you complete the following sentence:

  Every time the number of days goes up by____, the number of pennies in the jar goes up by _____.

- (grade 2?) How would I figure out how many pennies I would have on day 500?*The goal of this question is to get students to think about whether they can do this problem, or would even want to do this problem, by extending a recursive pattern. Talk about the limits of this approach as a way to motivate the next discussion on function rules.*

6. Whole Class Discussion: Develop a Function Rule
Return to the t-chart. Discuss ***how*** *students got each total number of pennies. This will be important in order for them to notice patterns.*

Number of days (D)	Number of pennies in the jar (P)
1	1
2	2
3	3
4	4
5	5

- Do you see any other relationships (e.g., days→pennies)? *The goal is for students to use information like the above to notice functional relationships. This kind of analysis also builds toward multiplicative thinking.*

Handout: Lesson 3

Name: ______________________ Date: ____________

Sara gets 1 penny each day from her Grandma.
She keeps them in a jar.

Show on this table how many pennies Sara has on each day.

Number of Days	Number of Pennies in Jar
1	
2	
3	
4	
5	
10	

What is the pattern in the table?

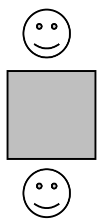
Lesson 4: Brady's Birthday Party

1. Brady's Birthday

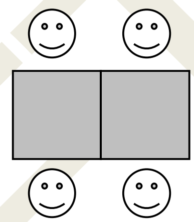
Brady is celebrating his birthday at school. He wants to make sure he has a seat for all of his friends. He has square desks for his friends, but no one can sit on the ends of the desks.

Build the following figures while children watch. Verbalize your actions.

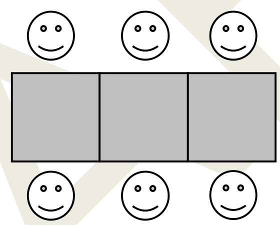
He can seat 2 friends at one desk in the following way:



If he joins another desk to the first one, he can seat 4 friends:



If he joins another desk to the second one, he can seat 6 friends:



Question: Can we find a relationship between the number of desks and the number of friends that can be seated?

A. Explore with Partner: How many friends could be seated at 10 desks? How did you figure this out?

Have students record their thinking on a handout. The choice of 10 might change depending on the class (e.g., 20 for grades 1-2). The goal is to have a relatively large number so that students can't do this mentally, but need some type of representation to model their thinking.

B. Class Discussion:

- How many friends can be seated at 1 desk?
- How many friends can be seated at 2 desks? (continue for 3, 4 and 5 desks)

Discuss students' solutions, paying attention to how they solved the problem and the different representations they used. The growing amount of information should motivate a need for organizing it in some way. Record the information, with student help, as follows:

1	*2*
2	*4*
3	*6*
4	*8*
5	*10*

Discuss what the columns of numbers represent (e.g., number of desks) and whether it would be a good idea to include information about this.

Then introduce (finalize) the representation:

number of desks	number of friends
1	*2*
2	*4*
3	*6*
4	*8*
5	*10*

C. Explore with Partner: Do you see any patterns or relationships here? How would you describe these?

D. Class Discussion:

Take time to discuss the types of relationships students see. They will likely talk about recursive patterns. Develop a shared understanding of how to talk about recursive patterns. Introduce the terminology of "rule".

- If Brady keeps adding desks, what will happen to the number of friends he can seat? How would you describe this?

- What happens to the number of friends every time we add one more desk? (*The goal here is to get students to think about two quantities co-varying, looking across table.)*

- How would you complete the following:

  Every time the number of desk goes up by____, the number of friends goes up by _____.

- Can we add information to our chart even without having people come up to the board? *Have students continue the chart for a few values.*

- How would I figure out the number of friends for 500 desks (or, 100 for grades K-1)? *The goal of this question is to get students to think about whether they can do this problem, or would even want to do this problem, by extending a recursive pattern. Talk about the limits of this approach as a way to motivate the next discussion on function rules. Perhaps ask: "How could we figure out how many friends can sit down if we don't know how many desks there are?"*

3. Develop a Function Rule (this might be too much for grade K and 1)

Class Discussion:

Return to the t-chart. Discuss ***how*** *students got each total number of friends. This will be important in order for them to notice patterns. Write a number sentence that corresponds with each set of values and shows the relationship between the two values (e.g., 1 + 1 = 2; 2 + 2 = 4, etc.) Record the equation beside each set of values.*

number of desks (D)	number of friends (F)	
1	*2*	*1 + 1 = 2*
2	*4*	*2 + 2 = 4 etc*
3	*6*	
4	*8*	
5	*10*	

- Do you see any relationship between the number of desks and number of friends? How would you describe it? *The goal is for students to use information like the above to notice functional relationships. This kind of analysis also builds toward multiplicative thinking.*

Explore with Partner:

- If Brady has 20 desks, how many friends can he seat? How did you get your answer?
- If Brady wants to have 30 friends at his party, how many desks will he need? How did you get your answer?

Handout: Lesson 4

Name: ________________________________ Date: ____________

How many friends could be seated at 10 desks?

How did you figure this out?

Lesson 5: People and Ears

Goals:

This lesson continues to develop the idea of function tables and patterns in function table data. It includes an explicit focus on developing a correspondence (functional) relationship and describing this in words.

1. Explore with Partner Sitting Next to You in the Circle (with handouts):

A New Pattern of Relationships: People and Ears

Question: Can we find a relationship between the number of people in this room and the number of ears?

Explore with the Partner Sitting Next to You in the Circle: How many ears do 10 people have? How did you figure this out?

Have students record their thinking on their handouts (see Handout #1). The choice of 10 might change depending on the class (e.g., 50 for grade 1, 100 for grade 2). The goal is to have a relatively large number so that students can't do this mentally.

- How many ears does one person have?
- How many ears do two people have? (continue for 3, 4 and 5) people

HANDOUT #1 TEXT:
How many ears do 10 people have?
How did you figure this out?
How many ears does 1 person have?
How many ears do 2 people have?
How many ears do 3 people have?

4. Whole Class Discussion:
Discuss students' solutions, paying attention to how they solved the problem and the different representations they used.
Organize the information, with student help, into a t-chart:

Number of people (P)	Number of Ears (E)
1	*2*
2	*4*
3	*6*
4	*8*
5	*10*

Discuss what the components of the t-chart represent (e.g., first column represents number of people)
Ask the following questions (or similar questions) to make sure students understand how the numbers are connected:

Questions:

- How many ears would there be for 3 people? How can you use your table to find this out?
- How many people would there be if I counted 8 ears? How can you use your table to find this out?

(You may need to ask more questions like this depending on students' understanding)

5. Discussion: Explore with Partner Sitting Next to You in the Circle: Do you see any patterns or relationships here [in the function table]? How would you describe the pattern or relationship between the numbers?

Take time to discuss the types of relationships students see. They will likely talk about recursive patterns [Vergnaud's scalar relationship or going down a column]. Develop a shared understanding of how to talk about recursive patterns.

- What happens to the number of ears every time we add one more person? (*The goal here is to get students to think about two quantities co-varying, looking across rows in a table)*

- How would you complete the following sentence:

  Every time the number of people goes up by____, the number of ears goes up by _____.

- Can you extend this table without drawing people or ears? *Have students continue the table for a few values.*

- (grade 2?) How would I figure out the number of ears 500 hundred people have? *The goal of this question is to get students to think about whether they can do this problem, or would even want to do this problem, by extending a recursive pattern. Talk about the limits of this approach as a way to motivate the next discussion on function rules.*

6. Whole Class Discussion: Develop a Function Rule
Return to the function table. Discuss ***how*** *students got each total number of ears. This will be important in order for them to notice patterns. E.g., 2 = 1 + 1, 4 = 2 + 2, 6 = 3 + 3. Write a number sentence that corresponds with each set of values and shows the relationship between the two values (e.g., 1 + 1 = 2; 2 + 2 = 4, etc.) Record the equation beside each set of values.*

Number of people (P)	Number of Ears (E)
1	*2*
2	*4*
3	*6*
4	*8*
5	*10*

Develop the language of groups of 2 and "doubles". (This kind of analysis also builds toward multiplicative thinking.) Write this beside the values in the tables.

- Do you see any relationship between the number of people and the number of ears?

The goal is for students to use information like the above to notice functional relationships. They should notice and be able to describe in words something like "The number of ears is double the number of people".

- How can you use your relationship to figure out how many ears there would be for 50 people? 100 people? How do you know?

7. OPTIONAL: Whole Class Dicussion: Add non-sequential objects to the table
Add two non-sequential numbers to the number of people, but less than the total number of kids in class.

- What is going to be the number of ears that match the number of people?
- How did you figure that out?
- What about this other number? How did you figure that out?
- If you had to tell a friend how to figure out the number of ears, if they knew the number of people, what would you tell them? What is the “rule” to build this pattern?
- Is this “rule” almost always going to work?

8. Whole Class Discussion: Draw people in household, write sentence to show the relationship between how many people, how many ears.
There are ___ people who live in my house. ____ people have ____ ears altogether.

9. Exploring reversibility; using the table to problem solve

Leave the T-chart on the whiteboard. Draw a set of ears or write a number for the number of ears. Ask the kids to figure out how many people there are:

[DRAWING]

Confirm answer by drawing in the missing people or writing the numeral for the corresponding number of people.
How did they figure that out? How could you use the T-chart to find the answer?
What if there were a lot of ears?
Confirm answer by drawing in the missing people or writing in the numeral for the number of people.

Review the different ways the kids found their answer: grouping by twos, counting by twos, using the table…
*It would be easy to make ears/people (looking ahead to lesson 4) on overhead material so that there’s an easy way to “draw” a picture. However, there is no need to draw if the kids are comfortable with the written numbers.

Handout: Lesson 5

Name: ________________________________ Date: ____________

How many ears do ___ people have?

How did you figure this out?

Fill in the table:

Number of People	Number of Ears
1	
2	
3	

Lesson 6: Relationships of Two

Goals:

1. **Whole Class Discussion with Partner Groups:**

   Questions:

   - What are things you have two of? (*Students might identify things such as eyes, hands, arms, pencils, etc. Select something (e.g., arms) that everyone in the room has 2 of).*
   - *(If students are working in groups of 2-4)* How could you figure out how many [e.g., *arms*] are at your table?
   - How could we figure out how many total [e.g., *arms*] there are in the room? In the school? *(This should begin to motivate the need for a rule, rather than counting all the arms. The goal of this question is to get students to think about what they would need to consider in answering this question, not necessarily to answer it.)*

2. **Partner Activity with Handout #1 a&b**

   Questions (*these need to be provided orally at appropriate points with the handout)***:**

   A. With your partner, find out how many [e.g., *arms*] there would be for 1 person, 2 people, 3 people, and 4 people.

   B. Organize your information in a table. How would you represent the two quantities [e.g., number of people, number of arms] with variables?

   C. Do you see any patterns or relationships in the numbers in your table?

   D. Can you find a relationship between [*the two quantities*]? How would you describe or represent this relationship? Can you represent your relationship using variables?

   E. How would you find [near and far data values]? Did you use your table or rule? Why?

3. Whole Class Discussion: Share your Pattern

Select partner groups to share their work with the class. Discuss what is similar (or different) about each of the patterns/relationships/tables.

Questions:

A. Is the relationship we see here (*referring to all or one example of students' work)* similar to any relationship or pattern we've seen before? *(Look to see if students notice similarity with People and Ears relationship (Lesson 5).)*

Briefly review the relationship between number of people, number of ears (Lesson 5). Do the children remember the relationship, the rule and the tools they used to explore the relationship?

- What was the pattern we looked at the last time we were together?
- Can we put some examples in a table?
- Does someone remember the "rule" that described that pattern? *The children should already be familiar with this wording. If they don't feel comfortable, we can use the wording of "how would you tell a friend to build this pattern?"*
- Can we use that rule to figure out how many hands there are in a group of people? Why? How? *Likely, the children will say things such as "it's a doubling relationship, or a 2's relationship, or everytime you add a person, you add two hands".*

B. Can you think of any other relationships (of 2's) like these?

C. If I wanted a new table that described the relationship between the number of people and, say, the number of [*legs, kidneys – something they haven't used and that they have 2 of*], what would that table look like?

Discuss how the tables would look the same.

Handout 1a: <u>Lesson 6</u>

Name: ________________________________ Date: ____________

1. Organize your information in a table:

2. Do you see any patterns or relationships in your table? Describe them.

Handout 1b: Lesson 6

1. Can you find a rule that describes your pattern?

4. Can you show your rule in variables (using letters)?

Lesson 7: How many legs on a dog?

Goals:

- Solve a specific example of number of dogs/number of legs
- Use different tools (function tables, words, algebraic notation) to explore and represent the relationship between co-varying quantities (i.e., the number of dogs and number of legs)
- Generalize the relationship between the number of dogs and number of legs and represent in words and variables
- Use the relationship to solve problem scenarios

1. WHOLE CLASS DISCUSSION

Review "Relationships of Two" generated in Lesson 6

Briefly review the relationships between number of people and number of X (e.g., ears, eyes, arms, feet, legs). Do the children remember the relationships, the rule, and the tools they used to explore the relationships?

- What was the "rule" we came up with for the relationships between the number of [ears] and the number of people?
- How did we represent this rule? (in words and variables)
- Was there another way that we could represent this rule?
- How do you think this rule would change if people had 3 [ears]?

Finding ways of showing a new function rule: y=4x (y = x + x + x + x)

NOTE: The number of dogs to start with is more than 2, to avoid finding the answer by doubling the number of legs. It is not 4 to avoid the confusion of number of dogs and number of legs being the same.

Let's say we have a group of 3 dogs. With your partner, figure out how many legs the group of dogs has.

Questions for Discussion:

- How could we figure out how many legs a group of 3 dogs has?
- What do you need to know?
- Is there anything that will help you figure it out?

Explore whether the relationship between number of dogs and number of dog legs can be generalized to a mathematical rule.

- What if there were a different number of dogs? How could you show a friend how to figure out how many legs there are?

2. PARTNER ACTIVITY (HANDOUT #1) - Developing a function table:

Questions for Discussion:

- How many legs are there on 1 dog? What about 2 dogs? 3 dogs? 4 dogs?
- How did you organize your information in a table? What were your table headings? (*select student examples to look at together)*

3. WHOLE CLASS DISCUSSION – Meanings in a table

(Select students' tables and discuss the following)

- What do the parts of your table represent?
- What is the left side showing? The right side?
- Why did you use these headings in your table?

(Chose a table where students use variables to represent the quantities and ask the following)

- What do your variables represent?
- What can the value of your variables be?
- Why did you use different variables for each quantity?
- Can you tell me why you put the numbers in as you did? What do they mean?
- How many dog legs would there be for 3 dogs? How do you know? How can you use your table to figure this out?

(Extending a table to find unknown information)

- How would you use your table to find the number of dog legs for 10 dogs?
- If your friend counted 16 dog legs, how could you use your table to figure out the number of dogs there must be?
- If your friend said he had 5 dogs and counted 21 dog legs, do you think he counted correctly? How can you use your table to figure this out?

4. PARTNER ACTIVITY (HANDOUT #2) - Explore patterns and relationships in data:

Questions for Discussion

- What patterns or relationships did you find in your table? Can you describe them?

- How would you describe the change in the number of dog legs each time we add one more dog?
- Do you think this will always happen? Why?

5. **PARTNER ACTIVITY (HANDOUT #3) - Finding a rule:**

Questions for Discussion

- What rule did you find to represent the relationship between the number of dogs and the number of dog legs for any number of dogs? *(get students to share their rules in various formats; note those that use words and those that use variables)*
- How did you describe your rule in words?
- How did you represent your rule using variables?
- What do your variables represent?
- Why did you use different variables?
- What could the value of your variables be?
- Is this relationship always going to be true? How do you know?

6. WHOLE CLASS DISCUSSION: Explore how students use the generalization they developed

- How would you use your rule to find the number of dog legs for 10 dogs?
- If your friend counted 16 dog legs, how would you tell your friend how to figure out the number of dogs there must be?
- If your friend said he had 5 dogs and counted 21 dog legs, do you think he counted correctly? How do you know?

LESSON 7 – HANDOUT #1

Name______________________________________

1. How many legs there are on 1 dog? How about 2 dogs? 3 dogs? 4 dogs?

2. Organize your information on the number of dogs and the total number of dog legs in a table. Label your table headings.

LESSON 7 – HANDOUT #2

Name____

1. Do you see any patterns or relationships in your table? Describe them.

2. How would you describe the change in the number of dog legs each time we add one more dog? Do you think this will always happen? Why?

LESSON 7 – HANDOUT #3

Name____

1. Can you find a rule that describes your pattern? Describe your rule in words.

2. Write your rule using your variables (using letters).

3. Is this relationship always going to be true? How do you know?

Lesson 8: Find your Rule (y = mx)

Goals:

- Recognize a functional relationship represented in different ways

Functional relationships from Lessons 3 through 7:

Lesson 3:	*Pennies in a jar:*	*y = x*
Lesson 4:	*Attached Tables:*	*y = 2x (or x + x)*
Lesson 5:	*People and ears:*	*y = 2x (or x + x)*
Lesson 7:	*Dogs and legs:*	*y = 4x (or x + x + x + x)*

1. "Find your Rule" game: Teacher leading WHOLE CLASS

Propose that the class is going to play a game. In the other lessons, the students created a table with examples that matched a relationship from the situation and ended up with the rule. Today, the teacher is going to give the students different rules, tables, and stories, and the students have to try to match the ones that go together. Students should find students who match their function and form a group. So, e.g., the students that end of with the cards below should end up together. See last page for the example cards that should be distributed. Use this example below to get the conversation started.

Why do these cards go together?

How can we explain why these cards go together?

y = z

Number of dogs	Number of tails
1	1
2	2
3	3

"The number of dogs is the same as the number of tails on the dogs"

2. Start playing the game

Pass out the card types for different functions, where each student gets a card for some function represented in a different way (for younger kids, don't have to use the rule card). Repeat the rules of the game to the kids. As you facilitate and support the activity, make sure to emphasize that they to be able to explain WHY they go together; how are they sure that they go together?

3. Justifying the groupings

After the game, have a discussion on why different groups are together (why they think the representations are equivalent, how would they explain why they go together, how they would convince someone that they go together).

$y = z + z + z + z$

where z is the number of dogs and y is the number of legs

Number of dogs	Number of legs
1	4
2	8
3	12

The number of dog legs is the number of dogs added together four times

$y = z + z$

where z is the number of dogs and y is the number of eyes on the dogs

Number of dogs	Number of eyes
1	2
2	4
3	6

The number of eyes is the number of dogs plus the number of dogs

$y = z$

where z is the number of dogs and y is the number of noses on the dogs

Number of dogs	Number of noses
1	1
2	2
3	3

The number of dogs is the same as the number of noses on the dogs

$y = z + z + z + z + z$

where z is the number of feet and y is the number of toes

Number of feet	Number of toes
1	5
2	10
3	15

The number of toes is the number of feet added together 5 times

Lesson 9: Cutting String

A. Whole-Class Discussion:

Charlotte has a piece of string that she cuts one time. How many pieces of string will she have after she makes the cut?

Make the cut while children watch. Verbalize your actions.

If she cuts her piece of string two times in this way, how many pieces of string will she have after she makes the cuts?

Make the cuts while children watch. Verbalize your actions.

If she cuts her piece of string three times in this way, how many pieces of string will she have after she makes the cuts?

Make the cuts while children watch. Verbalize your actions.

Question: Can we find a relationship between the number of cuts and the number of pieces of string Charlotte has after she makes the cuts? Take a piece of string and pair of scissors back to your table and work with a partner.

B. Explore with Partner: {Handout text}

- Organize your information about the number of pieces of string Charlotte has after 1, 2, and 3 cuts in a T-chart.
- Find a relationship between the number of cuts and the number of pieces of string.
- Find out how many pieces of string Charlotte would have after 10 cuts. …100 cuts

C. Whole-Class Discussion:

- How did you organize your information? If you used a table (t-chart), how did you label the quantities you are comparing?
- How would you use your table (t-chart) to find the number of pieces of string Charlotte would have if she made 4 cuts?
- How would you use your table to show your friend how many cuts were made if you counted 3 pieces of string? *(reversibility)*
- What relationships did you find? *(Discuss different types of recursive, co-variational, or correspondence relationships students found)*

- What happens to the number of pieces of string every time Charlotte makes one more cut? (*explore co-variation)*
- How did you figure out the number of pieces of string Charlotte has after 10 cuts? Did you use your table or the relationship you found? *(Discuss how students might have extended the table or might use their relationship.)*
- How many pieces of string would Charlotte have if she made 100 cuts? How did you find this out? Is it easier to use your relationship or extend your table to figure this out? Why?
- How many pieces of string would I have if I made *v* cuts?

F. Explore with Partner:

- Write a rule in your own words that represents the relationship between the number of cuts and the number of pieces of string.
- Write a rule using your variables to represent this relationship.

G. Whole-Class Discussion:

- What is your rule? How did you represent it in words?
- How did you represent your rule in variables?
- What do your variables represent?

Handout #1: Lesson 9 – Cutting String

Name: ________________________________ Date: ____________

Organize your information about the number of pieces of string Charlotte has after 1, 2, and 3 cuts.

Find a relationship between the number of cuts and the number of pieces of string. Describe your relationship.

How many pieces of string would Charlotte have after 10 cuts? 100 cuts?

Handout #1: Lesson 9 – Cutting String

Name: ________________________________ Date: ____________

Write a rule in your own words that represents the relationship between the number of cuts and the number of pieces of string.

Write a rule using variables to show this relationship.

Lesson 10: Describing an Unknown: Candy Boxes

Goals (Treat these as learning/content goals (not teacher instructional goals) so that we have a quick view of what math content is being addressed)**:**

- Create representations of candy box problem
- Create function table
- Work with variables

5. Whole Class Discussion

Comparing the Amounts

NOTE: Lesson is written as y=x+3 relationship, however, adjusting that to y=x+1 (for example) may be appropriate for K or 1.

Show two identical boxes of candies, said to be John's and Maria's. Put 3 extra candies on top of Maria's (possibly in ziplock bags or taped to the boxes to avoid dropping them repeatedly), and explain:

John and Maria each have a box of candies.

The two boxes have exactly the same number of candies in them.

Maria's box has 3 extra candies on top of it.

What can we say about how many candies they have?

Questions for the class:

- What do you know about how many candies John and Maria each have?
- What do you know for certain?
- What can you say about the *difference* between their amounts?

6. Individual Activity with Handout:

Representing the problem

Distribute the class handout and ask the children to show on their handout what they know about the amount of candies that John and Maria have.

Go around and observe varieties of answers in handouts. Jot down names of four or five children whose examples will later be discussed. Some possible representations include:

- Iconic instantiations (e.g., drawings of 6 candies for John, 9 candies for Maria).

- Numerical instantiations (e.g., John has 6, Maria has 9).
- Line segments representing diverse amounts.
- Symbolic (e.g., Question marks used for the amount in each box).

Put examples from children on board. Focus on:

- How children have represented differently the amounts for John and Maria.
- How the difference between John's amount and Maria's amount is represented.

If children are at a place in the lessons where they feel comfortable with the use of letters to represent quantities, show John's quantity as Z and ask how much Maria has. Use the representations on their handouts as a starting point for this discussion.

7. Whole Class Discussion:

Table of Possible Values

Refer to students' suggested amounts for John and Maria not as predictions but as possible values. *We want to downplay the notion that we're trying to guess the single correct value*.

Present on the classroom blackboard or whiteboard the table of possible values shown below. Explain that their job is to tell us how many candies they think could be in each box.

Student Name

John's total candies	**Maria's total candies**	**Difference between their amounts**

- Write each student's name in the first column, on the respective row. That way, you can refer to the different rows as "Paul's value", "Susan's value" etc. As "values" are entered make sure to note whether some students think others' "values" are not sensible or possible and ask them to explain their disagreements.

- For each row in the table, ask the students to state how many candies, in total, John would have, and how many candies Maria would have. Write these amounts in the appropriate cells, sometimes as a total, sometimes as the guessed amount for John and that amount plus 3 for Maria. Example:

Student Name	John's total candies	Maria's total candies	Difference between their amounts
Jonathan	*3*	*6*	*3*
Isabel	*5*	*5+3*	*3*
Jason	*10*	*13*	*3*

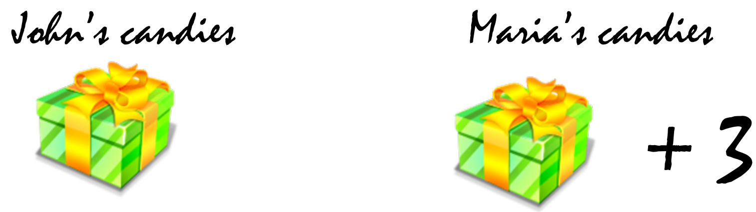
- If there is an inconsistency in the number presumed to be in the box, allow a student to change the answer. Strike through the original number suggested (leaving it on the overhead) and place the new value next to it.

- After the table has been tidied up, ask students to state whether they see any pattern. Carefully try to assess the pattern they are trying to articulate. Eventually, if not right away, some students will come up with the idea that Maria will always have 3 more than John, and John will always have 3 less than Maria. Refer to this as the difference in their amounts. But if the students use other expressions that are consistent with the mathematics, you can use those also.

8. Whole class discussion

Generalize through Notation

Depending on what children have done on their handouts as well as on the grade level, this conversation may be obvious at this point, or not. If possible, use an example from the class in generalizing the problem, then develop that into a substitution of their notation with a variable.



Explain that since we don't know yet how many candies are in each box, we are going to say that there are *z* candies and ask:

> *If z is the number of candies in a box, what can we write about the total number of candies that John has? [z]*
>
> *What can we write about the total number of candies that Maria has? [z+3]*

Wait for answers, discuss them, and guide the children to propose and/or to adopt notation with a letter or some other symbol standing for the amount in a box.

Handout: Representing John and Maria's Candies

Lesson 10

Name: ________________________________ Date: ____________

John and Maria each have a box of candies.
John has a box of candies.
Maria has a box of candies and ____ extra candies on top of hers.
The two boxes have exactly the same number of candies in them.

Show on this paper how many candies John and Maria have.

If there are Z candies in the boxes, how would you show how many candies John has?

If there are Z candies in the boxes, how would you show how many candies Mary has?

Lesson 11: Pennies in a Jar

Goals (Treat these as learning/content goals (not teacher instructional goals) so that we have a quick view of what math content is being addressed)**:**

- Create representations of pennies in a jar problem
- Create function table
- Work with variables

1. Whole Class Discussion:

Pennies in a Jar (y=x+3)

Question: Do you remember one of our very first stories, the story about the girl and her grandmother? Let's imagine a different story about this girl and her penny jar.

Sara has a jar where she keeps all of her pennies. One day, she finds 3 pennies on the sidewalk and takes them home.

NOTE: Use 3 pennies for first and second grade; 2 pennies for kindergarten?

- How many pennies does Sara have in the jar? (suggestions from the kids)
- Do we have enough information to know how many pennies are in Sara's penny jar?
- If Sara started with one penny in her penny jar, how many would she have all together?
- If she started with two pennies in her penny jar, how many would she have with the pennies she found? If she started with 3? If she started with 4?

Record the information, with student help, on a whiteboard or blackboard, as follows:

0	3
1	4
2	5
3	6
4	7

Discuss what the columns of numbers represent (e.g., number of pennies to start) and whether it would be a good idea to include information about this. Good opportunity to introduce having a variable as a heading (e.g. Number of Pennies in the Jar is represented as J).

Then introduce (finalize) the structure:

Number of pennies Sara already had (P)	Total number of pennies in Sara's jar (T)
1	4
2	5
3	6
4	7

2. **Individual Activity with Handout:**

   A different example of y=x+b

   Introduce a new context:

   Sara is walking home and finds 2 pennies. She brings them home and puts them in her jar.

   Have students record their thinking on their handouts (see Handout).

HANDOUT TEXT:
Sara finds 2 pennies and brings them home.
Show on this table how many pennies she has total.
[table pictured here]
How many pennies are in the jar if Sara started with 10 pennies?
How did you figure this out?
Is there a way to tell a friend how to figure out the number of pennies?

3. **Whole Class Discussion:**

   Share solutions from handout

   Discuss students' solutions, paying attention to how they solved the problem and the different representations they used.

   Do you see any patterns or relationships here [in the function table]? How would you describe the pattern or relationship between the numbers?

Take time to discuss the types of relationships students see. They will likely talk about recursive patterns. Develop a shared understanding of how to talk about recursive patterns.

- What happens to the total number of pennies when the amount Sara has in her jar changes? (*The goal here is to get students to think about two quantities co-varying, looking across rows in a table)*
- How would you complete the following sentence:

  Every time the number of pennies Sara starts with goes up by____, the total number of pennies goes up by _____.

- *Extreme case (1st, 2nd grade?)*: How would I figure out how many pennies I would have if Sara had 50 pennies in her jar? *The goal of this question is to get students to think about whether they can do this problem, or would even want to do this problem, by extending a recursive pattern. Talk about the limits of this approach as a way to motivate the next discussion on function rules.*

4. Whole Class Discussion:

Develop a Function Rule

*Return to the t-chart. Discuss **how** students got each total number of pennies. This will be important in order for them to notice patterns.*

- Do you see any other relationships (e.g., days→pennies)? *The goal is for students to use information like the above to notice functional relationships. This kind of analysis also builds toward multiplicative thinking.*
- Can we come up with a rule that describes how to find the total number of pennies in Sara's jar?

Handout: Lesson 11

Name: ______________________________ Date: ____________

Sara keeps pennies in a jar. She finds ____ pennies.

Show on this table how many pennies Sara might have.

Number of pennies Sara already had (P)	Total number of pennies in Sara's Jar (T)
0	
1	
2	
3	
4	

How many pennies are in the jar if Sara started with 10 pennies?

Is there a way to tell a friend how to figure out the number of pennies?

Lesson 12: Age Difference

1. Whole Class Discussion: Two friends who are different ages

Keisha and Janice are sisters. Keisha is two years older than Janice.

(The age difference might vary for different grades, but avoid an age difference of 1 so that kids aren't just counting.)

Do we know how old these girls are?

What do we know? What can you say about their ages?

If Janice is 3, how old would Keisha be?

How old is Keisha when Janice is 4? 5? 6?

Big Question: Can we find a rule that represents how Keisha's age is related to Janice's age?

(Next, students work with partners to organize their information about ages and find a relationship in the data)

2. Partner work at tables (Handout #1):

- Organize your information about Keisha's and Janice's ages when Janice is 3, 4, 5 and 6 years old.
- Find a relationship between Keisha's age and Janice's age. Describe your relationship in words.

Have students record their thinking on their handouts (see Handout #1).

3. Whole Class Discussion:
Discuss students' solutions, paying attention to how they organized their information, how they labeled the quantities in their table, and the relationship they found.

- How did you organize your information? If you used a table (t-chart), how did you label the quantities you are comparing?
- How would you use your table (t-chart) to find Keisha's age if Janice is 4 years old?
- How would you use your table to show your friend how to find Janice's age if Keisha is 7 years old? *(reversibility)*

- What relationships did you find? *(Discuss different types of recursive, co-variational, or correspondence relationships students found)*
- What happens to Keisha's age every time Janice gets one year older? (*explore co-variation)*

4. Partner work:

- What is Keisha's age when Janice is 20 years old?
- What is Keisha's age when Janice is 80 years old?

5. Whole-Class Discussion:

- How did you figure out Keisha's age when Janice is 20? Did you use your table or the relationship you found? *(Discuss how students might have extended the table or might use their relationship.)*
- How did you figure out Keisha's age when Janice is 80? Is it easier to use your relationship or extend your table to figure this out? Why?
- What would Keisha's age be when Janice is *k* years old?

6. Partner work:

- Write a rule in your own words that represents how Keisha's age relates to Janice's age.
- Write a rule using your variables to represent this relationship.

Have students record their thinking on their handouts (see Handout #2).

7. Partner work (see Handout #3)

Handout text:

Devan and Eric are brothers. When Devan is 2 years old, Eric is 6 years old.

Show Eric's age on the table.

What if Devan is 7 years old. How old will Eric be?

8. Whole-Class Discussion about Handout#3:

- Do you see any patterns or relationships here [in the function table]? How would you describe these?

Take time to discuss the types of relationships students see.

- Can you use your table (chart) to find a rule to represent the relationship between Eric's age and Devan's age (similar to how we did with Keisha's age and Janice's age?)
- Do you think your relationship will always be true, regardless of Eric's and Devan's ages (that is, will they always be 4 years apart in age)?
- What happens to Eric's age when Devan's age changes? (*The goal here is to get students to think about two quantities co-varying, looking across rows in a table)*
- How would you complete the following sentence:

Every time Devan's age goes up by____, Eric's age goes up by _____.

- What about when these boys are adults? How old will Eric be when Devan is 40 years old? How old will Devan be when Eric is 50 years old? How did you figure this out?

Handout #1: Age Difference

Name: _________________________________ Date: ____________

Organize your data about Janice's age and Keisha's age.

Find a relationship between Janice's age and Keisha's age. Describe your relationship.

Handout #2: Age Difference

Name: _________________________________ Date: ____________

Write a rule in your own words that represents how Keisha's age relates to Janice's age.

Write a rule using your variables to represent this relationship.

Handout #3: Age Difference

Name: ______________________ Date: ____________

Devan and Eric are brothers. When Devan is 2 years old, Eric is 6 years old.

Show Eric's age on this table.

Devan's Age	Eric's Age
1	
2	
3	
4	
5	

What if Devan is 7 years old. How old will Eric be?

Lesson 13: How does your height change with a hat?

Goals (these are learning/content goals—not teacher instructional goals—so that we have a quick view of what math content is being addressed)**:**

- Create representations of height problem
- Create equations of height problem
- Create table of possible values

1. Whole Class Discussion:

Comparing the height with and without a hat

Remind children that some have already worked with the hat problem and that we will now share with the rest of the class what they did. Have a volunteer come up and put on the hat. Tell students that the height of the hat is 1 foot. Discuss how his/her overall height changes with the hat:

What happens when I put this hat on (name)?

Does it make them seem taller or shorter?

What is the difference between the height of (name) with the hat on, and (name) with the hat off?

2. Individual Activity:

Representing the problem

Distribute the handout (page 1).

3. Whole Class Discussion of Students' Productions and Data Table:

Sharing of individual representations

In a large group discussion, highlight a few of the example representations produced by children. Focus on the more schematic representations. Also focus on the algebraic expressions and the tables produced by the children.

Table of Possible Values

Now let's consider what happens to the height of different people wearing the hat. With the children, complete a data table such as the one below on the whiteboard or blackboard.

Try to steer the conversation in such as way that the children begin to focus on the fact that every time, you add 1 foot (the height of the hat) to your height to get the total height. You might use a typical function table (t-chart) or something like the following:

Kid's Height (K)	Hat's Height (H)	Total Height (T)
	1	____ + 1 =
	1	____ + 1 =
	1	____ + 1 =
	1	____ + 1 =
	1	____ + 1 =
	1	____ + 1 =

The goal following filling out of the data table with the class would be to have a discussion about the relationship between a kid's height and height with a hat. Get students to represent their rules using words (e.g., the kid's height with a hat is going to be 1 foot more than his/her height) and, if possible, variables.

Children might use the following types of variable representations (only go as far as seems possible with the children):
K+1=T
T-1=K
T-K=1

Of course, it is likely that many kids will not be able to arrive at such expressions. We will explore how far we can go with them.

Handout 1: Lesson 13 - Representing your Height with a Hat

Name: ________________________________ Date: ____________

Show your height in any way you want:

Show your height with a hat:

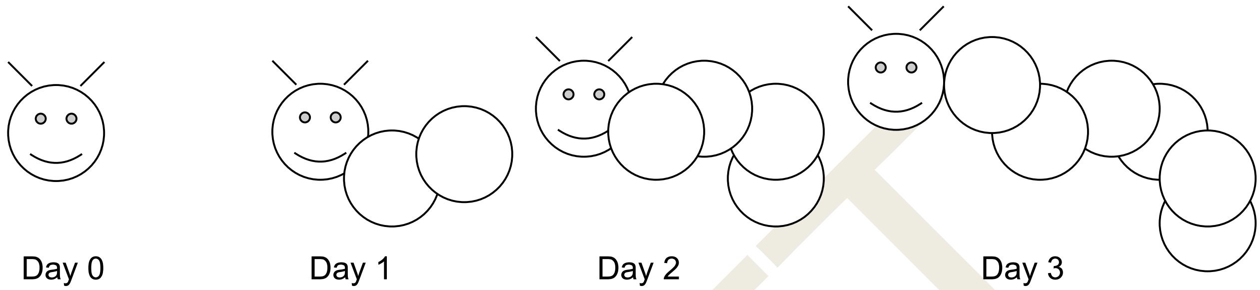
Use this table to show different people's heights with and without the hat.

If you are K inches tall, what would be your height with a hat on?

Lesson 14: Growing Caterpillar

1. Whole Group Discussion:

Cate the Caterpillar is getting bigger each day:



How would you describe how Cate is growing?

What is going to happen on each additional day?

Could we predict how many body parts Cate will have after 10 days?

If needed, a table can be used to document this information. For example:

Number of Days	Number of body parts
0	1
1	3 (2+1)
2	5 (4+1 or 2+2+1)
3	7 (6+1 or 2+2+2+1)

2. Individual/Pair work with handout

a) If Cate keeps growing, how many body parts will she have on:

Day 4? ________   Day 5? ________   Day 6? ________

Record your responses in the table below and fill in any missing information:

Number of Days	Number of body parts
0	
1	
2	

3	
4	
5	
6	
10	
___	
___	
___	
D	

b) What is the relationship between the number of days and the number of body parts?

c) Represent your relationship using variables.

e) What do your variables represent?

3. Whole Group Discussion:

Show some of the students' written responses. Fill out the table as a large group.

Number of Days	Number of body parts
0	
1	
2	
3	
4	
5	
6	
10	
D	

Ask questions such as:

- What is the relationship between the number of days and the number of body parts?
- Represent your relationship using variables.
- How many body parts will Cate have on day 20? How do you know?
- How many body parts will Cate have on day 100? How do you know?
- If Cate continues to grow, on what day will she have 81 body parts? How do you know?
- If Cate continues to grow, on what day will she have 101 body parts? How do you know?

Lesson 14: Growing Caterpillar

Name: ______________________ Date: ______________________

If Cate keeps growing, how many body parts will she have on:

Day 4? _______ Day 5? _______ Day 6? _______

Record your responses in the table below and fill in any missing data:

Number of Days	Number of body parts
0	
1	
2	
3	
4	
5	
6	
10	
D	

a) What is the relationship between the number of days and the number of body parts?

b) Represent your relationship using variables.

Lesson 15: Find your Rule (y = mx+b and y=x+b)

Goals:

- Recognize a functional relationship represented in different ways

Functional relationships from Lessons 3 through 7:

Lesson 5: People and ears: $y = 2x$ (or $x + x$)

Lesson 10: Candy Boxes: $y = x + 3$

Lesson 14: Growing Caterpillar: $y = 2x+1$ (or $x + x + 1$)

1. "Find your Rule" game: Teacher leading WHOLE CLASS

Propose that the class is going to play a game. In the other lessons, the students created a table with examples that matched a relationship from the situation and ended up with the rule. Today, the teacher is going to give the students different rules, tables, and stories, and the students have to try to ***find the ones that fit together****. Students should find students who* ***fit*** *their function and form a group. So, e.g., the students that end of with the cards below should end up together. See the excel sheet for the cards. Use this example below to get the conversation started. Avoid the word "match" since it makes them think of matching colors.*

Kindergarten version (and Grade 1 if it seems necessary):

Have them start ONLY with the verbal description of the problem and the tables. Match the algebraic expression at the end as a group activity.

Questions to frame the activity:

Why do these cards go together? Make each "type" of card/representation a different color on cardstock.

How can we explain why these cards go together?

$z + z$

Number of dogs	Number of eyes
1	2
2	4
3	6

The number of eyes is the number of dogs plus the number of dogs

2. Start playing the game

Pass out the card types for different functions (make each "type" of card/representation a different color on cardstock), where each student gets a card for some function represented in a different way (for younger kids, ***don't have to use the rule card, see above—use the cards with the rule on it at the end as part of the group activity****). Repeat the rules of the game to the kids. As you facilitate and support the activity, make sure to emphasize that they to be able to explain WHY they go together; how are they sure that they go together?*

3. Justifying the groupings

After the game, have a discussion on why different groups are together (why they think the representations are equivalent, how would they explain why they go together, how they would convince someone that they go together).