Equal Sign – Part 1

<table>
<tr><td colspan="2"><u>Lesson Objective</u>: Develop a relational understanding of the equal sign</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal for Part 1 – decompose numbers and represent relationships with equations in non-standard form (a = b + c)</td></tr>
<tr><td>A. How many ways can you make 10?

Explore & Record:
1. In what ways can you break about 10 cubes into 2 groups of cubes? What numbers of cubes are in your group?
2. Write down the ways you made 10 by writing an equation in the form “10 = “.

Identify:
1. Through whole-class discussion, compose a list of all the ways to make 10, representing each way in the form a = b + c.
2. For each equation suggested, ask whether students think it’s true and why.
3. Do we have all possible ways to break apart 10? How do you know?</td><td>Have students (individually or in pairs) decompose 10 in different ways (some will decompose into two groups, some three groups) using cubes, cheerios, etc. We decided that having a stacked tower would be optional.

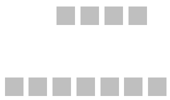
Scaffold students’ thinking about writing an equation in the form a = b + c, rather than a + b = c. This would likely be whole-class discussion initially for grade K.

Keep the list of equations and refer back to it when appropriate for future lessons (e.g., Commutative Property, comparing equations such as 10 = 3 + 7 and 10 = 7 + 3)

This will lay some groundwork for true/false equations.</td></tr>
<tr><td>B. How many ways can you make 11?

Explore & Record:
1. (In partner pairs) In what ways can you break apart 11 cubes into two groups of cubes? Use the following chart to record your work.

Identify:
1. Do we have all possible ways to break apart 11? How do you know?</td><td>Complete one example with students in whole-class discussion. After groups complete the decomposition, have whole-class discussion to generate a collective list of ways to break apart 11.</td></tr>
</table>

Picture of my 11 cubes broken apart	Number of cubes in each group	Equation showing what I found
	4 7	11 = 4 + 7

Equal Sign – Part 2

<table>
<tr><td colspan="2"><u>Lesson Objective</u>: Develop a relational understanding of the equal sign</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal for Part 2 – Finding equivalent quantities and representing their relationship; Identifying equations as true or false</td></tr>
<tr><td>A. Equivalent Quantities – Can you find the same amount?

Play “Equivalent Quantities Game”

Game Rules:
• Pass out cards, where each card has a number or indicated sum (e.g., 3, 2 + 4) and kids find their “partner” (person with card containing equivalent amount).
• The student pair writes an equation that shows the relationship they found.

Questions for discussion:
1. How do you know your cards show the same amount?
2. Write your equation on the board that shows the relationship you found.
3. Are the equations you found <u>true</u> or <u>false</u>? How do you know?</td><td>Cards are designed so that students can construct equations where there is either (1) an operation on each side; (2) no operation on either side; (3) on operation on left or right side

Discuss the equations students generate and how they determined that the amounts are the same.

Discuss what it means for an equation to be true.</td></tr>
<tr><td>B. True or False?

Are the following equations true or false? Why?

<u>2 + 2 = 10</u>
4 = 4
1 + 2 = 1 + 2
5 = 1 + 3
3 + 3 = 6 + 2
18 + 3 = 18</td><td></td></tr>
</table>

Equal Sign – Part 3

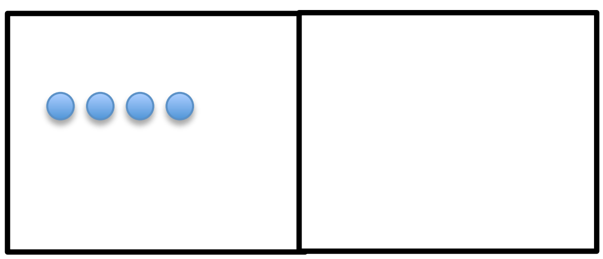
<u>Lesson Objective</u>: Develop a relational understanding of the equal sign

Tasks	Comments

Goal for Part 3 – Finding the number of objects that yields equivalent quantities; Solving open equations by finding missing values;

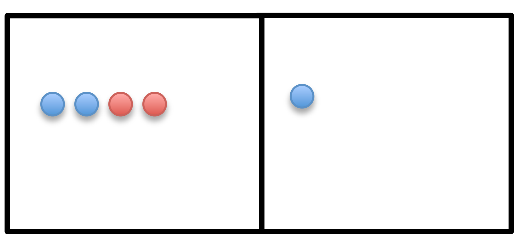
A. Finding Balance

1. How many blue circles should we put in the empty square so that the squares have the same number of circles? How do you know?

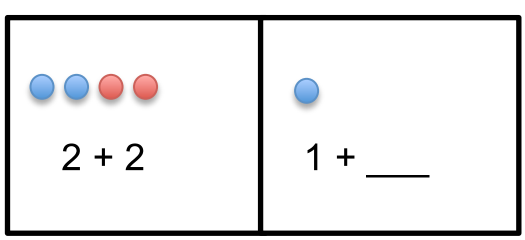


Can we write an equation that shows this?

2. How many red circles should we put in the square on the left so that the squares have the same number of circles? How do you know?



How would you complete the expression to show the number of red circles you needed to add?



Can we write an equation that shows this relationship?

Talk about the circles in terms of their color. For example, "2 blue circles and 2 red circles is the same ***number of circles*** *as 1 blue circle and 3 red circles."*

<table>
<tr>
<td>
3. Use the expressions to draw a picture so that these squares have the same number of circles

<table><tr><td>4 + 2</td><td>1 + 5</td></tr></table>

Write an equation that shows what you found.
</td>
<td></td>
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<tr>
<td>
B. Find the missing value in each equation.

3 + 3 = ___

3 + ___ = 7

__ = 4 + 2

10 = 5 + ___

5 + 1 = 6 + ______

4 + 2 = ___ + 3

6 = ___

6 = ____ + 1

4 + ____ = 5 + 1
</td>
<td>Have students use manipulatives or drawings to figure out the missing value.</td>
</tr>
</table>

Equal Sign – Part 4

<table>
<tr><td colspan="2"><u>Lesson Objective</u>: Develop a relational understanding of the equal sign</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal for Part 4 – Determining if equations are true or false.</td></tr>
<tr><td>Are the following true or false? How do you know? How could you explain your answer without adding the numbers on each side?

2 + 3 = 6
1 + 1 = 1 + 1 + 1
3 + 1 + 1 = 3
10 = 0 + 10
16 + 1 = 16 + 3
28 = 28
7 + 0 = 0 + 7
8 = 2 + 6
2 + 3 = 3 + 2
8 + 2 = 10 + 2

1st-grade extension (subtraction):
25 – 6 = 37 – 6
278 – 5 = 349 – 5

(From Rittle-Johnson et al):
Can you put the same number in each blank? Why?
2 + ___ = 6   2 + ___ + 1 = 6 + 1</td><td>Do students reason about the structure or compute to determine if an equation is true or false? For example, for equations like
16 + 1 = 16 + 5, do children compute each quantity or compare them, focusing on 1 and 5?
8 + 2 = 2 + 8, do they see the structure or try to add</td></tr>
</table>

Preliminary thoughts

Rational for sequence of properties

As per Carpenter, Franke & Levi (2003), this sequence involves properties with one (same) variable first, then moves to properties with two (different) variables. This was also the same sequence that we used and tested in LEAP 1-2 projects (order of which was also based on Carpenter et al.)

I think conceptually additive identity would come before additive inverse (e.g., students work with addition before subtraction and will likely be more familiar with addition), unless there is a research-based reason to do otherwise.

Rationale of structure of tasks based on practices of AT in Kaput (2008) and Core actions from LEAP (we might not want to use all of these, but it would be good for us to cite this work as a research-based argument for why we ask certain things – i.e., we need a research-based structure for our tasks):

- analyze information to develop a conjecture about the arithmetic relationship
- express the conjecture in words
- develop a justification or argument to support the conjecture's truth
- explore different types of arguments, including empirical arguments, representation-based arguments, and arguments based on the algebraic use of number
- identify values for which the conjecture is true
- represent the generalization using variables
- examine the meaning of repeated variables in the same equation
- for properties with multiple variables, examine the meaning of different variables in the same equation
- examine the characteristic that the generalization (property) is true for all values of the variable in a given number domain

identify the generalization (e.g., property) in use when doing computational work

Additive Identity – Part 1

Lesson Objective: Generalize, represent, and reason with Additive Identity	
Tasks	**Comments**
Goal 1 - analyze information to develop a conjecture about the property and represent the property in words	
Problem: Charlotte's birthday is coming soon. One day, she got 5 birthday cards in the mail. The next day, she didn't get any cards. 1. How many cards did she get all together? 2. Draw a picture that shows your thinking. 3. Can you write an equation that shows how you got your answer? 4. Do you notice anything special in these problems? (*Follow-up prompts*: • What do you notice about adding zero? • How would you describe this to a friend?	Whole group activity with kids at the rug. Teacher presents the problem and students discuss their thinking about the number of cards Charlotte has and how they got this. Scaffold students in writing the equation and put 5+0=5 on the board. Change the scenario to have 6 cards, 7 cards, and so on to generate a list of equations representing the Additive Identity property. Begin a conversation about what students notice.
Game: *Find the Missing Number* (open equations using Additive Identity) **Discuss:** 1. Do you notice anything special happening in these equations? *Follow-up prompts*: • What do you notice about adding zero? • How would you describe this to a friend?	**Game:** Divide the class into two groups. In one group, give each student a card with an open equation. In the other group, give each student a card with a number on it. Students should find their partner so that their cards match (equation with correct missing number). **Discussion:** Show the matches on the board. Students discuss why their cards match – what the equations say and what '=' means. *Develop a conjecture* for Additive Identity or reinforce conjecture developed with **Birthday Card** problem.
Goal 2 – Identify the values for which the conjecture is true	

Discuss: For what numbers do you think your conjecture is true? (*Follow-up prompts:*) • Do you think it is true for all numbers or just some numbers? • Which ones? • How do you know?	

Additive Identity – Part 2

<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize, represent, and reason with Additive Identity</td></tr>
<tr><td>Goals &Tasks</td><td>Comments</td></tr>
<tr><td colspan="2">Goal 3 - Identify values for which the conjecture is true</td></tr>
<tr><td>Discuss:
For what numbers do you think your conjecture is true?
(Follow-up prompts:)
• Do you think it is true for all numbers or just some numbers?
• Which ones?</td><td>Review the conjecture students developed and their responses in the previous lesson regarding the numbers for which the conjecture is true. List students' ideas of numbers for which conjecture is true to motivate the concept that the conjecture is true for "any number".</td></tr>
<tr><td colspan="2">Goal 4 - Represent the generalization using variables and examine the meaning of repeated variables in the same equation</td></tr>
<tr><td>Problem: Katie started writing an equation on her paper that her teacher was writing on the board. She didn't get to finish. The following is what she wrote.

_____ + 0 = _____

1. What number(s) could Katie have put in the missing places of the equation to make the equation true?

2. What can you say about the numbers that go in the missing places?

Discuss convention of how an unknown number can be represented by a letter.

3. How could we represent any number here?

4. What does your letter mean (represent) to you?

5. If we have to put the same number in the missing places, do you think we should use the same letter or different letters? Why?</td><td>Whole-class discussion (??):

1. Ask students what they think the equation says (i.e., do they see it as "some number plus 0 is the same as some number"). Systematically list different possibilities:
1 + 0 = 1
2 + 0 = 2, etc

(This serves as a way to get kids to generalize a pattern to represent any number and also reviews work with equations.)

2. The goal is for students to notice that any number can be put in the missing places, but it must be the same number.

5. Introduce mathematical convention that we can use a letter to represent any number.</td></tr>
</table>

<table>
<tr><td></td><td>Discuss choice of letter and same letter. Discuss convention of same number, same letter.</td></tr>
<tr><td colspan="2">Goal 5 - Identify the generalization (e.g., property) in use when doing computational work</td></tr>
<tr><td>Problem: Last week, it snowed some on Tuesday. We didn’t measure how many inches. It didn’t snow any on Wednesday.

1. What can you say about how many inches of snow fell on Tuesday? On Wednesday?
2. How would you represent the number of inches of snow that fell on Tuesday?
3. What can you say about the total number of inches it snowed on Tuesday and Wednesday? How do you know?
4. Write an equation to show what happened.</td><td>Discuss what is known about the number of inches of snow that fell on Tuesday and Wednesday.

Question students to see if they notice “any number plus zero is that number” applies to this problem.

Can they model the situation with an equation such as $a + 0 = a$?</td></tr>
</table>

Game: Find the Missing Number

Group 1:

___ = 9 + 0	0 + 11 = ___
___ = 8 + 0	___ = 0 + 15
6 = 6 + ___	20 = 20 + ___
20 = ___ + 0	___ = 6 + 0

8 = ____ + 0

__ + 8 = 8

25 + 0 = ____

25 = ____ + 25

4 = 4 + ____

4 + 0 = ____

Group 2:

9	11
8	15
0	0
6	20

0	0
4	8
25	0

Additive Inverse – Part 1

<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize, represent, and reason with Additive Inverse</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal 1 - analyze information to develop a conjecture about the property and represent the property in words</td></tr>
<tr><td>Problem:
Last week, it snowed 4 inches on Tuesday. By the next day, Wednesday, all the snow had melted.

1. What can you say about how many inches of snow were left? How do you know?

2. Write an equation to show what happened.

3. Do you notice anything special in these problems? (Follow-up prompts:
• What do you notice about subtracting a number from itself?
• How would you describe this to a friend?</td><td>Whole group activity with kids at the rug. Teacher presents the problem and students discuss their thinking about the number of inches of snow on Tuesday. Discuss the number of inches left after it melts on Wednesday.

Scaffold students thinking by considering specific equations: 4 – 4 = 0. Change the scenario to 5 inches, 6 inches, and so on to generate a list of equations representing the Additive Inverse property. Begin a conversation about what students notice.</td></tr>
<tr><td>Game: Find the Missing Number (open equations using Additive Inverse)

Discuss:
1. Do you notice anything special happening in these equations? (Follow-up prompts:
• What do you notice about subtracting a number from itself?
• How would you describe this to a friend?</td><td>Game: Divide the class into two groups. In one group, give each student a card with an open equation. In the other group, give each student a card with a number on it. Students should find their partner so that their cards match (equation with correct missing number).

Discussion: Show the matches on the board. Students discuss why their cards match – what the equations say and what ‘=’ means. Develop a conjecture for Additive Inverse or reinforce conjecture developed with Snowfall problem.</td></tr>
<tr><td colspan="2"></td></tr>
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<table>
<tr><td colspan="2">Goal 2 – Identify the values for which the conjecture is true</td></tr>
<tr><td>Discuss:
For what numbers do you think your conjecture is true?
(Follow-up prompts:)
Do you think it is true for all numbers or just some numbers?
Which ones?
How do you know?</td><td></td></tr>
</table>

Additive Inverse – Part 2

<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize, represent, and reason with Additive Inverse</td></tr>
<tr><th>Goals &Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal 3 - Identify values for which the conjecture is true</td></tr>
<tr><td>Discuss:
For what numbers do you think your conjecture is true?
(Follow-up prompts:)
• Do you think it is true for all numbers or just some numbers?
• Which ones?</td><td>Review the conjecture students developed and their responses in the previous lesson regarding the numbers for which the conjecture is true. List students' ideas of numbers for which conjecture is true to motivate the concept that the conjecture is true for "any number".</td></tr>
<tr><td colspan="2">Goal 4 - Represent the generalization using variables and examine the meaning of repeated variables in the same equation</td></tr>
<tr><td>Problem: Barbara started writing an equation on her paper that her teacher was writing on the board. She didn't get to finish. The following is what she wrote:

0 = _____ − _____

1. What number(s) could Barbara have put in the missing places of the equation to make the equation true?
2. What can you say about the numbers that go in the missing places?
3. How could we represent any number here?
4. What does your letter represent?
5. If we have to put the same number in the missing places, do you think we should use the same letter or different letters? Why?</td><td>Whole-class discussion (??):

1. Ask students what they think the equation says (i.e., do they see it as "some number minus itself is zero"). Systematically list different possibilities:
0 = 1 – 1
0 = 2 – 2, etc

(This serves as a way to get kids to generalize a pattern to represent any number and also reviews work with equations.)

2. The goal is for students to notice that any number can be put in the missing places, but it must be the same number.

4. Introduce mathematical convention that we can use a letter to represent any number. Discuss choice of letter and</td></tr>
</table>

	same letter. Discuss convention of same number, same letter.
Goal 5 - Identify the generalization (e.g., property) in use when doing computational work	
Problem: Caroline has some cookies in her lunch box. She gave them all to her friend, Ava, during recess. 1. How many cookies does Caroline have now? How do you know? 2. Write an equation to show what happened.	Discuss what is known about the number of cookies Caroline has. If students have trouble, use specific numbers to scaffold their thinking. Question students to see if they notice “any number minus itself is zero” applies to this problem. Can they model the situation with an equation such as $a - a = 0$?

Game: Find the Missing Number

Group 1:

___ = 9 – 9	11 – 11 = ___
___ = 8 – 8	0 =15 – ___
0 = 6 – ___	0 = 20 – ___
0 = ___ – 20	___ = 6 – 6

$0 = \_\_\_\_ - 8$	$8 - 8 = \_\_\_\_$
$25 - 25 = \_\_\_\_$	$0 = \_\_\_\_ - 25$
$\_\_\_\_ = 4 - 4$	$4 - \_\_\_\_ = 0$

Group 2:

0	0
0	15
6	20
0	20

0	0
4	8
25	0

Commutative Property of Addition – Part 1

<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize, represent, and reason with Commutative Property of Addition</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal 1 - analyze information to develop a conjecture about the property and represent the property in words</td></tr>
<tr><td>

Problem: Snap cube trains
Give partners a set of snap cubes with two colors of cubes. For example:

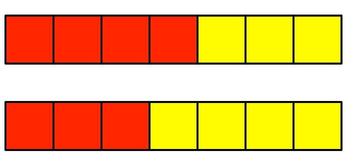
Instruct each partner group to build a red-yellow snap cube train (where all reds go together, all yellows go together) and answer the following:

1. How many cubes do you have all together in your train?

2. Write an equation to show how you got your answer (e.g., 4 + 3 = 7 or 7 = 4 + 3).

3. Find the partner group that has the train of the same length. Draw your trains/equations on the board.

(For example:



4 + 3 = 7

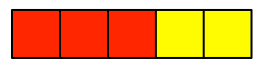
3 + 4 = 7

3. Do you notice anything about your partner group's train? (*Follow-up prompts*:

- What do you notice about what happens when you switch the order of the numbers you are adding?
- How would you describe this to a friend?
- Do you think 3 + 4 = 4 + 3 (or some other example from the Snap Cube trains)? Why?

</td><td>

Whole group activity with kids at the rug. Teacher has 4 red snap cubes and 3 yellow snap cubes. Make a cube train with red cubes together and yellow cubes together:



Each *two* partner groups should have the same total number of cubes, but with the number of cubes per color alternated. (e.g., one group has 4 red cubes, 3 yellow cubes; one group has 3 red cubes, 4 yellow cubes). It is important to NOT give sets whose sum of cubes are the same and for which the number of cubes do not correspond according to the comm prop. I.e., don't give 3 red, 4 yellow to a group and 2 red, 5 yellow to a group. Both have same length train, but the equations they write do not reflect commutative property

Make sure that the sets of cubes don't have the same number of cubes for other partner groups (e.g., If 3, 4 is one combination, don't do 2, 5). The goal is to have trains with clearly distinct

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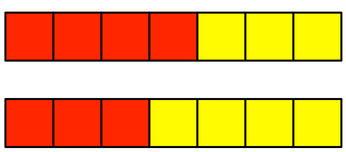
	lengths except for those with the same summands. Develop discussion to the point that students see that 3 + 4 = 4 + 3; write all such pair equations on the board (e.g., 2 + 6 = 6 + 2, etc.)
Game: *Find the Missing Number Value of the Letter* (open equations using Commutative Property of Addition) **Discuss:** 1. Do you notice anything special happening in these equations? (*Follow-up prompts*: • What do you notice about what happens when you switch the order of the numbers you are adding? • How would you describe this to a friend?	**Game:** Divide the class into two groups. In one group, give each student a card with an open equation. In the other group, give each student a card with a number on it. Students should find their partner so that their cards match (equation with correct missing number). **Discussion:** Show the matches on the board. Students discuss why their cards match – what the equations say and what '=' means. Develop a conjecture for Commutative Property or reinforce conjecture developed with **Snap cube trains** problem.
Goal 2 – Identify the values for which the conjecture is true	
Discuss: For what numbers do you think your conjecture is true? (*Follow-up prompts:*) • Do you think it is true for all numbers or just some numbers? • Which ones?	

• How do you know?	

Commutative Property of Addition – Part 2

<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize, represent, and reason with Commutative Property of Addition</td></tr>
<tr><th>Goals &Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal 3 - Identify values for which the conjecture is true</td></tr>
<tr><td>Discuss:
For what numbers do you think your conjecture is true?
(Follow-up prompts:)
Do you think it is true for all numbers or just some numbers?
Which ones?
How do you know?</td><td>Review the conjecture students developed and their responses in the previous lesson regarding the numbers for which the conjecture is true. List students’ ideas of numbers for which conjecture is true to motivate the concept that the conjecture is true for “any number”.</td></tr>
<tr><td colspan="2">Goal 4 - Represent the generalization using variables and examine the meaning of repeated variables in the same equation</td></tr>
<tr><td>Recall Snap Cube Train:

4 + 3 = 7
3 + 4 = 7

Recall
3 + 4 = 4 + 3,
2 + 6 = 6 + 2 (etc., whatever snap cube trains reflect)
What was our conjecture about the order in which we add two numbers?
Can you write an equation of your own that shows this?
3. Problem:
Kimy wrote a problem down that her teacher was writing on the board. She didn’t finish writing it down before the teacher erased it. Here’s what she wrote:</td><td>2. Notice whether they have the correct structure of a + b = b + a. Write all the equations on the board.

For 3, have a conversation to</td></tr>
</table>

5 + ___ = ___ + 5 What could have been written in the blanks to make the equation? *Extend this to any two numbers to develop Commutative Property*	develop the idea that any number could be used and that the representation for any number is a letter.
Goal 5 - Identify the generalization (e.g., property) in use when doing computational work	
Problem: Marcy's teacher asks her to solve "23 + 15." She adds the two numbers and gets 38. The teacher then asks her to solve "15 + 23." How would you tell Marcy to solve this problem?	Do students recognize the commutative property and apply this here?

Game: Find the Value of the Letter

Group 1:

$2 + a = 4 + 2$

$11 + 0 = b + 11$

$c + 3 = 3 + 4$

$1 + 2 = 2 + d$

$2 + 6 = 6 + a$

$5 + 1 = b + 5$

$3 + 2 = e + 3$

$4 + 5 = 5 + r$

$10 + 1 = m + 10$

$8 + 0 = n + 8$

$25 + 2 = 2 + s$

$9 + s = 1 + 9$

$b + 4 = 4 + 3$

$8 + m = 2 + 8$

Group 2:

4	11
4	1
2	1
2	4

1	0
25	1
3	2

Candy Box Problem – Part 1

<u>Lesson Objective</u>: Generalize, represent, and reason with arithmetical operations (focused on addition, but possibly exploring subtraction with some students)	
Tasks	**Comments**
Problem: Jack has a box of candies. He's not sure how many pieces of candy are in his box. His mother gives him 2 more pieces. **Explore & Record** 1. What do we know about the number of candies Jack has? What do we *not* know?	1. Have a **whole class discussion** of what children know about the situation.
2. Can you draw a picture to represent this situation?	2. Ask children to work **individually** to represent the situation on their own. Discuss children's drawings and what children intend that they represent.
3. What could you tell me about Jack's total number of candies if he had 3 (4, 5, 6) in his box? Can you write an expression to show how you got your answer?	
4. Can we record what we know so far about the number of pieces of candy Jack has in his box and how many he has altogether?	4. To keep track of the exploration, construct a table comparing how many candies Jack has in box and how many total candies he has when his mother gives him 2 more pieces. Ask children to offer possible mathematical expressions/equations to represent the number of pieces of candies. This should help them represent the general form later.
5. How would you describe the number of pieces of candy Jack has?	5. Encourage children to **generalize** using their own words.
6. How can we represent the number of candies in Jack's box? How can we represent Jack's total number of candies?	6. Explore how to represent an unknown quantity. Refer back to the table so that students can generalize the pattern in the expressions. Discuss the letter students choose and why. Discuss what they think the letter represents.

Candy Box Problem – Part 2

<table>
<tr><td colspan="2"><u>Lesson Objective</u>: Generalize, represent, and reason with arithmetical operations (focused on addition, but possibly exploring subtraction with some students)</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td>Problem: Angela has a piggy bank with some pennies in it, but she doesn’t know how many. She has 1 more penny in her hand.

1. What do we know about the number of pennies Angela has in her bank? What do we not know?

Explore & Record

2. Draw a picture to represent Angela’s total number of pennies.
How many pennies do you think are in the bank? Why? How certain are you?

3. How can we represent the number of pennies in Angela’s bank?
How can we represent Angela’s total number of pennies?
What does the letter you used represent?
What are possible values of your variable/letter? Why?</td><td>Have students work on this in partner groups using “turn and talk” (since kindergartners will have trouble reading/writing)

Discuss 1. Briefly with students before they go into partner groups.

Students do 2. Individually. Discuss students’ representations.

3. (turn and talk) Discuss the representation. Discuss use of letters, what the letter represents to students, and why they chose that letter.</td></tr>
<tr><td>Problem: Ben has some trucks in his collection. He is not sure how many he has.
1. How would you represent the number of trucks Ben has?
2. Ben’s mom gives him 3 more trucks for his birthday. Write an expression to represent the total number of trucks Ben now has.
3. What does the variable in your expression represent?</td><td>Work in partner groups.</td></tr>
</table>

Exploring Evens and Odds – Part 1

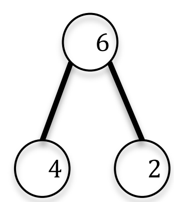
<table>
<tr><td colspan="2"><u>Lesson Objective:</u> Generalize properties of even and odd (parity), make conjectures about addition of even and odd numbers, reason about numbers based on their parity.</td></tr>
<tr><th>Tasks</th><th>Comments</th></tr>
<tr><td colspan="2">Goal 1 – Recognize and define “even” and “odd.” Generalize properties of even and odd (parity)</td></tr>
<tr><td>
Numbers can be represented by pairs of objects:
In our lessons and in your class, we have been looking at how to make numbers by combining two numbers together and how a whole can be thought of as made up of parts. [like when they make number bond diagrams]

Today, we are going to look at how numbers can be made up of pairs.
-Has anyone hear the word “pair” before? How many is a pair?
-(pair of shoes, pair of earrings, pair of mittens)
[Students should arrive at the idea that a pair is the same as two]

So, if I take a number like 8, can we make it into pairs?
-Count out 8 magnets, put them on the board in loose arrangement,
-Put them into pairs, describing what you are doing as you do so,
-Check for understanding of showing a number as pairs,
-Demonstrate how to fill in the worksheet for the number 8.

Hand out worksheet, give each student a number to figure out.
Now you are going to find the pairs in a number.
You can use [snap cubes] if you want or record directly on the worksheet. (worksheet is at the bottom of this document).

Checking work with each other:
-Show me you are finished by (standard classroom
</td><td>
Math in Focus (MIF): uses “number bonds” to represent composition and decomposition of numbers. Used in first grade and second grade. Unknown about kindergarten (ask Oscar)



Materials
-magnetic white board
-small magnets (like foam mosaic)
-worksheet
- a set of number cards with one value on each card (in the range from 4 to 15; use the range of 2 to 9 for K) (the number cards are just a way to pick which numbers the students will model)

Start as whole group to introduce their task, it moves relatively quickly into describing the task and handing out the student worksheet.

Final definitions from Keith, 2006:
</td></tr>
</table>

<table>
<tr>
<td>practice), and wait patiently until everyone is done.
-Some of you created pairs out of the same number. Let’s get you together to check your work before we review it as a group.

Review student answers by looking over worksheets together & modeling some of the numbers on the board.

Define “even” and “odd” classes of numbers
Numbers that can be made into pairs with no leftovers are called “Even numbers”
Numbers that have a leftover when they are made into pairs are called “Odd numbers”

How would you test if something is an even number?

How would you test if something is an odd number?</td>
<td>Even numbers:
-Even numbers won’t have any left over when divided by two.
-Even is when you have an amount that two people can share and each person will have the same amount. It will be fair.

Odd Numbers:
-Odd numbers cannot be divided into two groups that are equal without splitting or having one left over.
-With an odd number you will have one left over after dividing by two when all numbers are kept whole.</td>
</tr>
</table>

<table>
<tr>
<td colspan="2">Goal 2 – Generalize the result of adding of two even numbers</td>
</tr>
<tr>
<td>Leave examples of even numbers on the board, remove the examples of odd numbers and ask, “What happens if we take two of our even numbers and add them together? Can we make a conjecture about what the results will be? How do you know?”

As a group, do some examples that stay within the 1 – 20 answer range; write them as number sentences on the board; model with magnets

Explore this situation with the students:

There is a snowball dance and every snowman needs a partner to dance with. A sleigh of 6 snowmen arrive at the dance first. How many snowman can arrive in the next sleigh so that everyone has a partner? Is there more than one answer to this question?

What if 5 snowman arrive in the second sleigh, will</td>
<td>Capture the students conjectures on the board below the list of numbers.</td>
</tr>
</table>

everyone have a partner?	

Exploring Even and Odds – Part 2

Lesson Objective: Generalize properties of even and odd (parity), make conjectures about addition of even and odd numbers, reason about numbers based on their parity.	
Tasks	**Comments**
Goal 3 – *Generalize about properties of adding: two odd numbers; an even and an odd number*	
Review the work of Day 1: Ask for share-outs of the work that was done in the last session: -Showing numbers as pairs -Definitions of evens and odds (model an even number and odd number) -what happens when we add two even numbers together. -The Snowball Dance **Does anyone have any questions about what we did yesterday? Is there anything that you curious about based on what we discovered?** Hopefully, there will be a few questions about adding odd and odd together or adding even and odd together. **What if we add two odd numbers together, what do you think is going to be the result?** Can we make this a conjecture? Why do you predict that results? Will it always be true? How do you know? Hand out worksheet for students to do some sums (below) Do the answers on their worksheet match the conjectures that were made?	

What if we add an odd number and an even number together, what do you think is going to be the result? Can we make this a conjecture? Why do you predict that results? Will it always be true? How do you know? Hand out worksheet for students to do some sums (below) Do the answers on their worksheet match the conjectures that were made?	

Extensions to Even/Odd

Tasks	Comments
Lesson Objective: Apply the generalizations they have made to new contexts	
Goal 6 – *Apply generalizations about the properties of sums of evens and odds in a "thinking problem"*	
Give the students a worksheet with the problem, read the problem aloud, then have them work on it for 8 minutes (suggest they can get cubes or sticks if they want); come back to the rug to discuss. **PROBLEM:** **1st Grade Version:** Toby had two bags of candy. None of the bags had an even number of candies in it. Toby counted 16 pieces of candy in total. Please consider how to decide if he counted correctly. You can draw, use cubes or talk with a partner. Did he count right? How do you know? **Kindergarten version:** Toby had two bags of candies. Both bags had an even number of candies. Is it possible that there are 13 candies total? How do you know?	

WORKSHEETS START HERE

Name:____________________ Date:______________

Worksheet 1 for Part 1

My number is: _________

Color in the same number of squares as this number:

__________cut here______________________________________

Name:____________________ Date:______________

Worksheet 1 for Part 1

My number is: _________

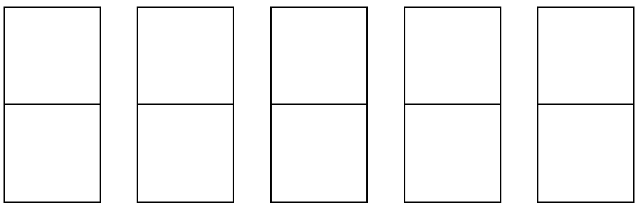
Color in the same number of squares as this number:

Name:________________________ Date:________________

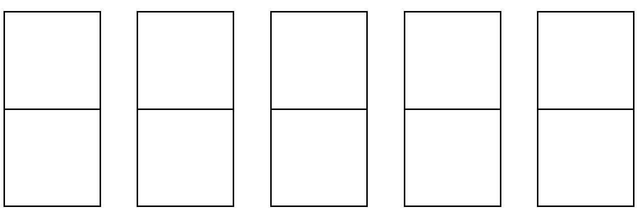
Worksheet 2 for Part 2: odd + odd

Do these sums, model the answer by coloring in the total number of squares:

3 + 5 = _____



5 + 1 = _____



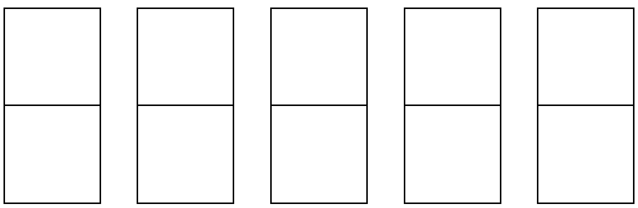
What happens when two odd numbers are added together? Use numbers, pictures, or words to explain.

Name:________________________ Date:________________

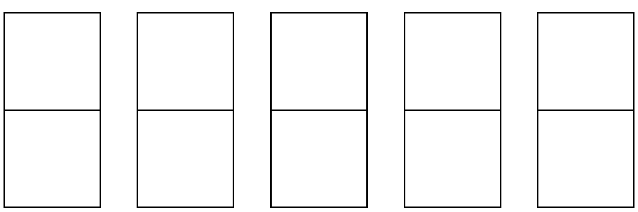
Worksheet 2 for Part 2: odd + even

Do these sums, model the answer by coloring in the total number of squares:

5 + 4 = _____



4 + 3 = _____



What happens when an odd number is added to an even number? Use numbers, pictures, or words to explain.

Name:_________________________ Date:_________________

Kenji has M&Ms, lollipops, gumballs, and Hershey Kisses. He doesn’t have an even number of any of the candies. Kenji counted 31 pieces of candy.

Did he count right? How do you know?

Name:_________________________ Date:_________________

Shari had three bags of candy. None of the bags had an even number of candies in it. She counted 16 pieces of candy in total.

Did he count right? How do you know?

Name:________________________ Date:________________

Toby had two bags of candies. Both bags had an even number of candies. Can there be 13 candies total?

3 + 2

■ ■
■ ■
■

4 + 1

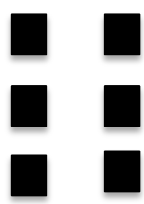
■ ■
■
■
■

3

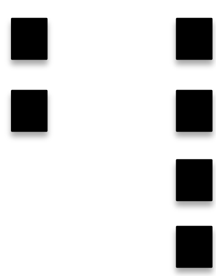
■
■
■

3

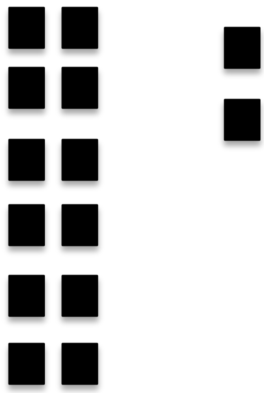
6



2 + 4



12 + 2



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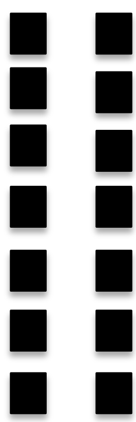
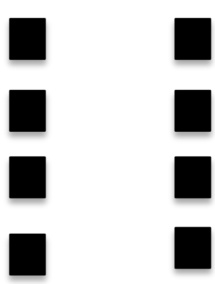
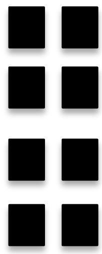


4 + 4



8 + 0



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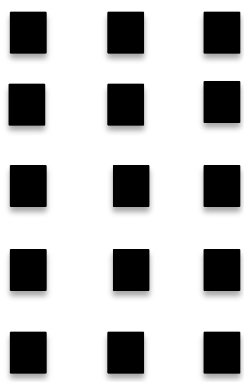
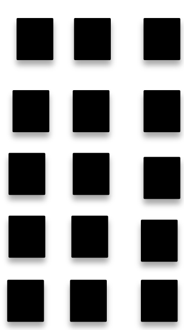
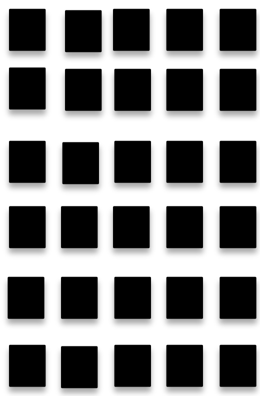


15 + 0



30 + 0



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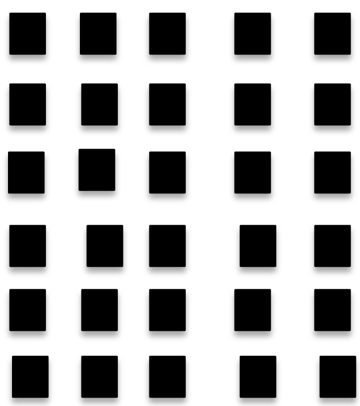
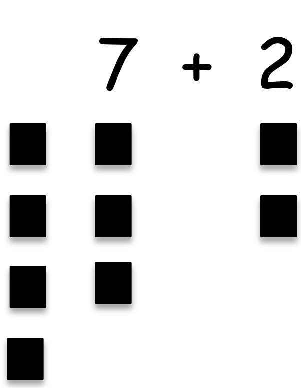


7 + 2

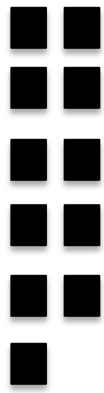


8 + 1

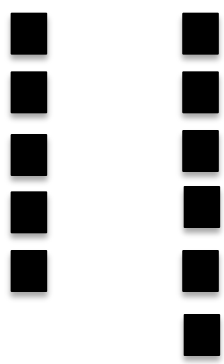
10

0 + 10

11 + 0



5 + 6



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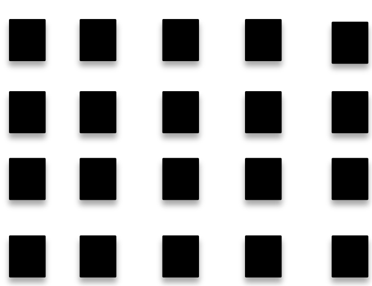
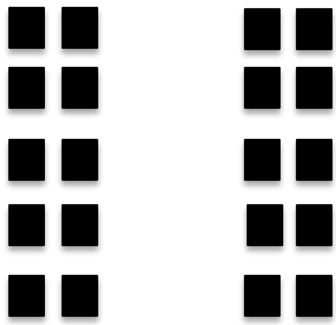
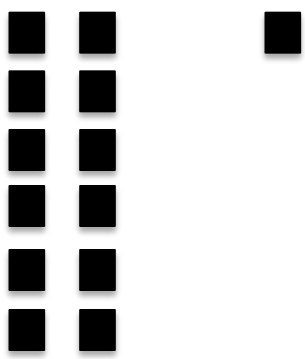


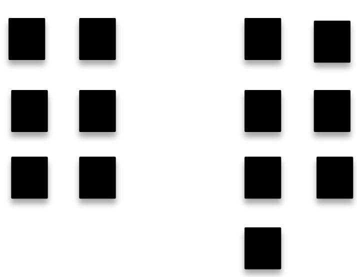
10 + 10



12 + 1



6 + 7



1 + 1

15 - 15

0

30 - 0

30

4 - 1

5 - 2

10

15 - 5

10 - 4

5 + 1

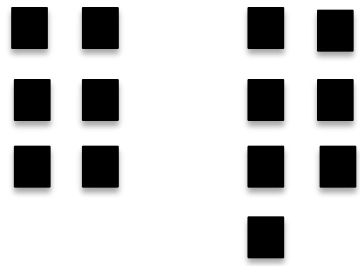
8

8

29

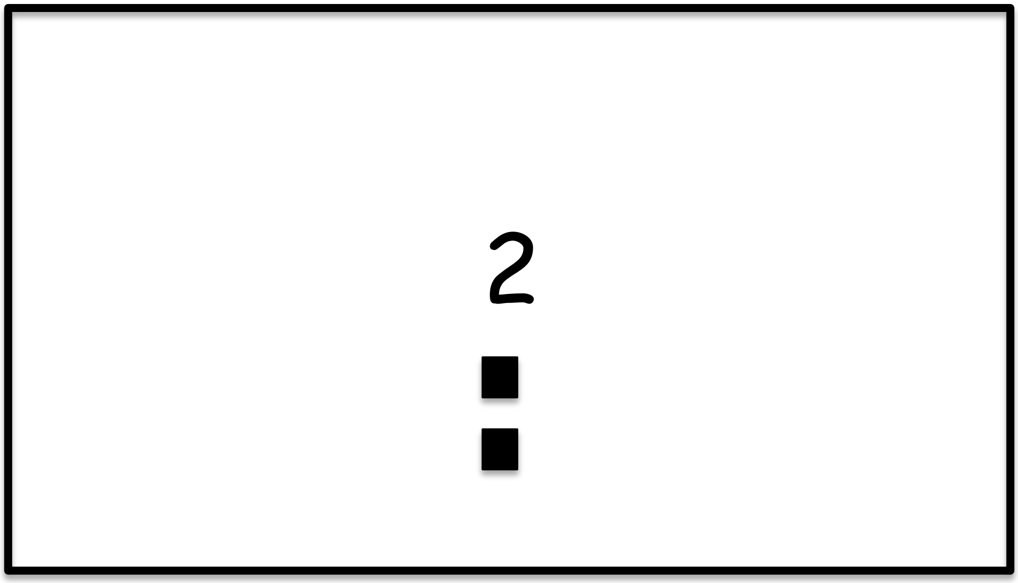
29

6 + 7



13

1 + 1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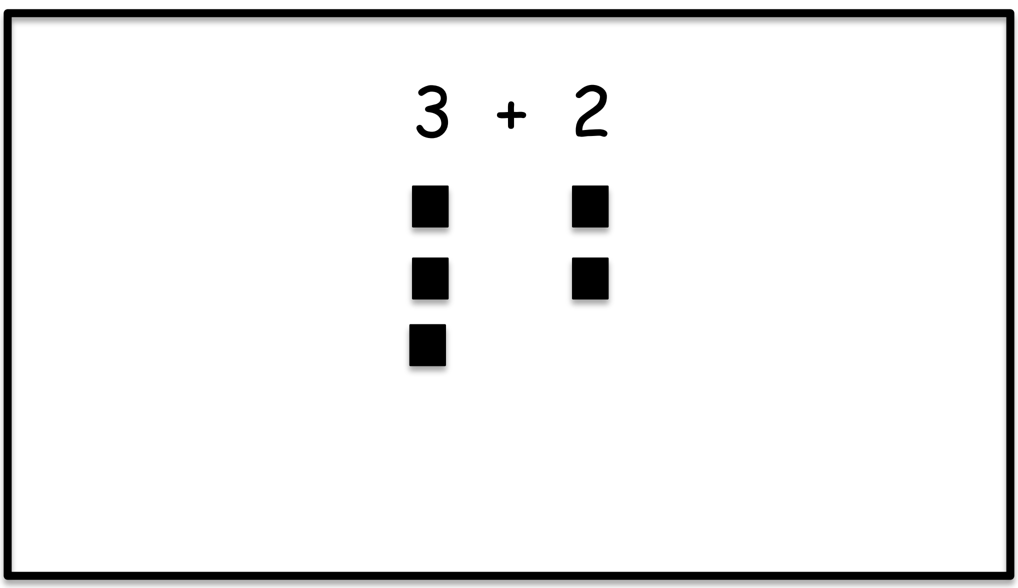
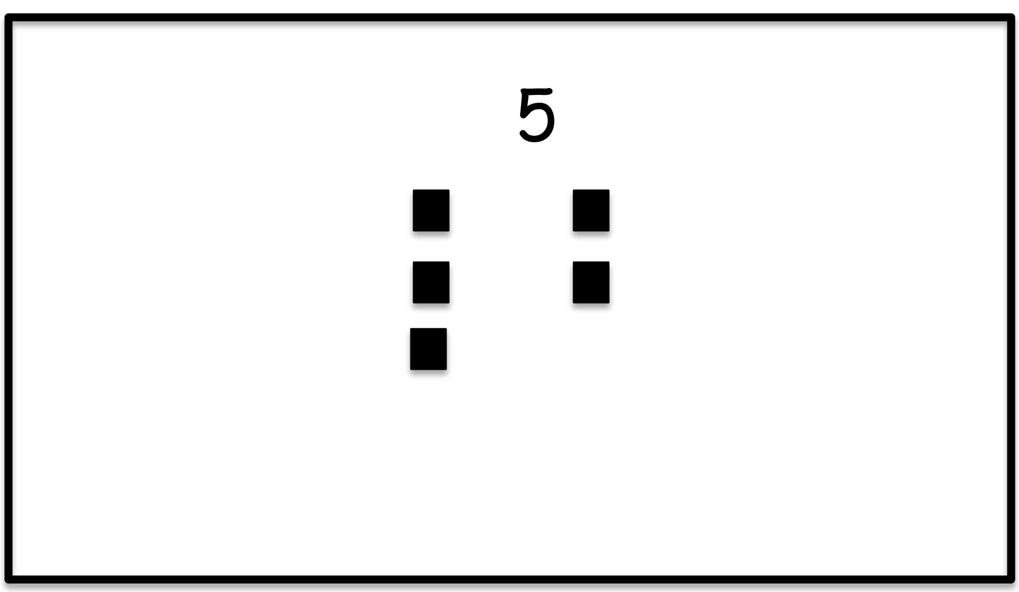

3 + 2


5

2+___

7

5

2+___

10-5

3

___-2

4+4

10