MATH 61-02: WORKSHEET 2 (§1.4-1.5)

(W1) In class, you have been learning the building blocks of logic. In this problem, we explore some of the *replacement rules* of propositional logic. These are steps that allow us to replace a logical sentence with an equivalent one, knowing that the truth value remains intact in the course of the replacement.

(a) One replacement rule says that we can replace $P \wedge (Q \vee R)$ with $(P \wedge Q) \vee (P \wedge R)$. Verify that these two statements are equivalent using a truth table.

(b) Another replacement rule says that $P \Leftrightarrow Q$ can be replaced by $(P \wedge Q) \vee (\sim P \wedge \sim Q)$. Verify that this is true through a truth table, and explain in words why this would be true.

(W2) Calculus is about limits. However, you've probably never seen a formal definition. In this problem, we look at a mathematically quantified definition of the limit of a sequence.

(a) Let $(x_n) = x_1, x_2, \ldots$ be a sequence of real numbers. We call x the *limit* of the sequence (x_n) if the following condition holds:

$$\forall \epsilon > 0,\ \exists N \in \mathbb{N} \quad \text{s.t.} \quad \forall n > N, \quad |x_n - x| < \epsilon.$$

Parse this statement in English and give some kind of intuitive explanation as to what this definition means.

(b) Using quantifiers, give the negation of the above statement.

(c) This definition of a limit can be expanded to define the limit of a function: Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Then the precise mathematical meaning of $\lim_{x \to c} f(x) = L$ is

$$\forall \epsilon > 0, \quad \exists \delta > 0 \quad \text{s.t.} \quad |x - c| < \delta \implies |f(x) - L| < \epsilon.$$

Can you write down a similar expression for $\lim_{x \to \infty} f(x) = L$?

(d) Use quantifiers to express Fermat's Last Theorem:
No integers a, b, c satisfy the equation $a^n + b^n = c^n$ for any integer exponent n greater than 2.

(W3) In class, you saw the Sheffer stroke, a logical operator ("NAND") defined as follows:

P	**Q**	**$\mathbf{P \uparrow Q}$**
T	T	F
T	F	T
F	T	T
F	F	T

Recall that $P \uparrow P = \sim P$. Work out expressions for both $P \wedge Q$ and $P \vee Q$ using only $P, Q, \uparrow$, and parentheses.