MATH 61-02: WORKSHEET 8 (§6.3-6.4)

(W1) Prove that if the interval $[0,1]$ is partitioned into nondegenerate intervals (i.e., points don't count as intervals), then the partition is countable. (That means you're trying to show the surprising result that that *you can't partition an interval into uncountably many intervals, even though it has uncountably many points.*) Explain why this implies the same result for partitions of $\mathbb{R}$.

Hint: for a given partition, first list all the intervals of length $> 1/2$...

(W2) Suppose that you're walking on a road and every mile you go you come to another fork in the road. Suppose the roads go on forever. Consider the set P of all paths (that is, all choices of L or R at every fork; so one path is LLLLLLL... and another path is LRLRRLRRRLRRRR....). Prove that $\aleph_0 < |P| \leq c$, where $\aleph_0 = |\mathbb{Z}|$ and $c = |\mathbb{R}|$. (Bonus: prove $|P| = c$.)

(W3) Prove that $|(0,1)| = |[0,1]|$, or in other words, there is a bijection between the open interval and the closed interval. (Hint: by Schroeder-Bernstein, it suffices to find injections both ways.)

(W4) Extra credit: look up the Cantor set (see Exercise 6.4.8 in the book). Show that it's uncountable.