MATH 61-02: WORKSHEET 9 (§7.1-2)

(W1) (a) For a vertex v in a graph $G = (V, E)$, let $\deg(v)$ be the *degree* of v, which is the number of times v appears as the endpoint of an edge. (So loops count double.) What is the smallest possible graph with at least one loop where every vertex has odd degree?

(b) Let K_n be the complete graph on n vertices (the simple graph where each vertex is connected by one edge to each other vertex). Sketch K_4, K_5, and K_6. For general n, what is $|E(K_n)|$ and what is the degree of each vertex? (Note: we discussed this in class.)

(c) Prove that $\sum_{v \in V} \deg(v) = 2|E|$ for any graph.

(d) Is it possible for a graph to have 11 vertices, all of which have degree 3?

(e) Is it possible for a graph to have 19 vertices, each of which have degree 1, 5, or 9? (Hint: consider the degree sum mod 4.)

(W2) The practice test for the second midterm involved the relation $R = \{(x, x), (x, y), (y, y), (y, x), (z, z)\}$ on $S = \{x, y, z\}$; you were supposed to check that it is transitive. Draw the graph associated to the relation, and use the adjacency matrix to verify transitivity.

(W3) Consider the Fig 7.7 in the book (p253), which shows airline connections between cities.

(a) Ordering the cities alphabetically, write the adjacency matrix A for the graph.

(b) Show by hand that it is possible to get from Seattle to Miami in exactly 10 flights.

(c) Using a computer (or by hand if you want), compute A^8 and A^{10}. How many ways are there to get from Seattle to Miami in 10 flights? How about Chicago to Atlanta in 8 flights? Explain the connection.