MATH 61-02: WORKSHEET 1 (§1.1-1.2)

(W1) Let $\mathcal{P}(X)$ denote the power set of a set X.
Show that it is not in general true that $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
On the other hand, give an example where they are equal.

Answer. I'll give a counterexample for the first part. Let $A = \{a\}$ and $B = \{b\}$. Then $\{a, b\} \in \mathcal{P}(A \cup B)$ but it is not in either $\mathcal{P}(A)$ or $\mathcal{P}(B)$.

On the other hand, suppose $A = [2]$ and $B = [3]$. Here $A \subseteq B$, so $A \cup B = B$. Then any subset of A is also a subset of B, so we have $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. Also, $A \cup B = B$. So we have $\mathcal{P}(A \cup B) = \mathcal{P}(B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

(W2) For any natural number n, define $A_n = (\frac{-1}{n}, \frac{1}{n})$, $\quad B_n = [n, n + \frac{1}{n})$, $\quad C_n = \{x \in \mathbb{R} \mid x < n < x^2\}$, and let $\mathcal{A} = \{A_n\}$, $\mathcal{B} = \{B_n\}$, $\mathcal{C} = \{C_n\}$ be the corresponding collections of sets, indexed over $\mathbb{N}$.
(a) On some number lines, sketch A_n, B_n, and C_n for $n = 1, 2, 3$.
(b) Find $\bigcup \mathcal{A}$ and $\bigcap \mathcal{A}$.

Answer. (a) I won't draw these, but I'll say a bit about C_n. First, C_1 is the set of real numbers that satisfy $x < 1 < x^2$, and this is impossible—if a number is less than 1, then so is its square—so $C_1 = \emptyset$. Next, C_2 is the solutions to $x < 2 < x^2$. This means $x < 2$, and also $2 < x^2$ means $x > \sqrt{2}$. Putting these together we find $C_2 = (\sqrt{2}, 2)$. Likewise, $C_3 = (\sqrt{3}, 3)$.

(b) $\mathcal{A}$ is a nested collection, whose first set is $(-1, 1)$ and whose sets get smaller and smaller, squeezing down to zero. The union of the sets in $\mathcal{A}$ is the open interval $(-1, 1)$. The intersection of the sets in $\mathcal{A}$ is $\{0\}$, because that one value is in every set $(-1/n, 1/n)$, but no other positive number is less than $1/n$ for all n.

(W3) Let our universal set be $U = [6]$. Let $A = \{1, 2, 4, 5\}, B = \{1, 3, 5, 6\}, C = \{4, 5\}, D = \{1, 2, 6\}$, $E = \{2, 3, 6\}$. Use these sets and parentheses, unions, intersections, and complements to express the following sets. (It is possible to create all six.) As an example, the set $\{2, 6\}$ is equal to $D \cap E$, among other possible expressions.
(a) $\{1, 4, 5\}$
(b) $\{2, 4\}$
(c) $\{2\}$

Answer. Multiple answers are possible for each part, but here are some sample answers:
(a) E^c
(b) B^c
(c) $(D \setminus B) \cap E$