Doppler Navigation

By Elsa Ames, ECE '19

Introduction

The Doppler Shift is a phenomenon affecting all waves, and describes an apparent frequency shift of an electromagnetic signal based on the relative motion between the signal source and the observer. In 1842, when the Doppler Effect was proved by Austrian mathematician, Christian Doppler, the primary applications were cosmological, such as identifying reasons certain stars presented with visual red or blue shifts. Following key advancements in special relativity with Albert Einstein's 1905 paper, *Annus mirabilis,* the Relativistic Doppler Effect was developed. Both of these are key scientific advancements that have facilitated a higher accuracy of satellite navigation systems, a greater understanding of distant galaxies and celestial bodies, and dozens of diverse applications in radar, lidar, medical imaging, and proposed interplanetary navigation.

The Classical Doppler Effect

The Classical Doppler Effect shows the compression of waves as the distance between source and observer decreases, and conversely, the expansion of waves as the distance increases. Despite a constant signal frequency, the observer will measure an increased or decreased frequency of the source as the interface changes position within a given plane. In acoustic applications, this can be heard by a change in pitch, while many astronomical measurements show a blue/green shift as objects approach Earth, and a red shift as they move away or as the universe itself expands towards infinity. The equation for the observed frequency, f_o, is given by

$$f_o = f\,(c \pm v_o) / (c \pm v_s)$$

where f is the emitted signal frequency, c is the propagation speed in a given medium, and the velocities are that of the observer and source relative to a fixed point in the medium. The equation can be simplified further for applications where the source is fixed:

$$f_o = f / (1 \pm v_o/c)$$

or the observer is fixed:

$$f_o = f / (1 \pm v_s/c)$$

In many terrestrial applications, the propagation speed is much greater than the velocity of the source, and thus $v_s / c << 1$, and the Doppler Equation can be simplified to the form of

$$f_o = f\,(1 \pm v_s/c)$$

For satellite navigation, it is important to note that the observer's net velocity is related to their ground speed, the Earth's rotational velocity and other orbital mechanic factors based on the satellites positioned orbit (GEO, MEO, LEO, HEO). The Doppler Shift *D*, can be written as follows where

$\mathbf{r}_r$, is the relative velocity vector (with all Keplerian motion factors accounted for) between the signal transmitter on the satellite and the receiver in the line of sight direction. λ_s is the wavelength of the signal $\lambda_s = c / f$.

$$D = (f_o - f) = f\,\mathbf{r}_r/c = \mathbf{r}_r / \lambda_s$$

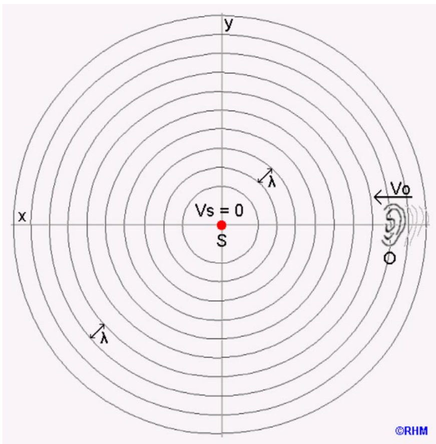

Figure 1. A source emitting sound waves at a constant velocity relative to a stationary observer

The Relativistic Doppler Effect

The Relativistic Doppler Effect accounts for the time dilation of a signal described by special relativity. Navigation satellites are positioned outside of the Earth's gravitational field moving at high speeds (7.8 km/s, for low-earth orbiting satellites) and consequently classical physics is not a sufficient model to achieve a highly accurate position fix. Most navigation satellites rely on atomic clocks to measure time dilation factors with high precision. However, most commercial receivers do not have atomic clocks due to cost. It is more efficient for most of most communication and navigation systems to employ a fourth satellite to account for this clock offset, and apply mathematical solutions to account for physical effects of time expansion and contraction as the signal moves from outer space to a ground receiver of unknown position. A major difference between Classical and Relativistic Doppler Effects is the medium is not the reference point in the latter case, and thus a Lorentz Transformation is required to shift a space time to constant velocity reference frame. The Lorentz Factor $\gamma = 1 / \sqrt{(1- \beta^2)}$ relates the difference in time dilation between the transmitter and receiver clocks, where $t_{r,s}$ represents the frame where the transmitting source is modeled at rest.

$$t_r = \gamma\, t_{r,s}$$

In the satellite navigation case where both the source and observer are in motion, the observed frequency is given by the following equations where $\beta = v / c$

$$f_o = f / (\gamma - \beta\gamma)$$

$$f_o = f \sqrt{((1 + \beta) / (1 - \beta))}$$

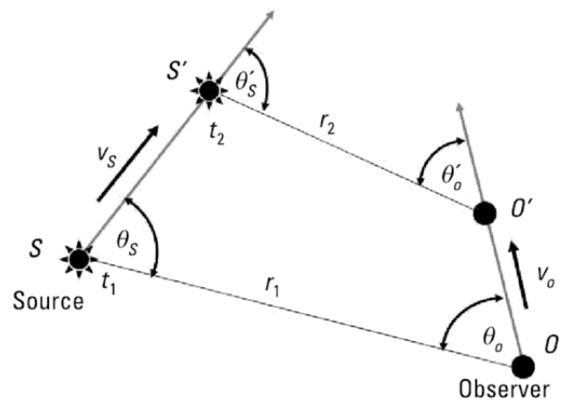

Figure 2. A case where transmitting body moves from S to position S' while the observer, O moves to position O'. Both bodies move position in the medium and relative to one another.

Thus the ratio between the received, fo and transmitted frequencies fr can thus be written:

$$f_o / f = (1 - (v * \cos\theta)/c\,) / \sqrt{(1 - (v^2/c^2))}$$

$$f_o / f = (1 - (\mathbf{r}/c)) / \sqrt{(1 - v^2/c^2)}$$

And the Doppler shift (following binomial series expansion) can be written as a simplified approximation as:

$$D = f_o - f$$
$$D = f * r / c = r / \lambda$$

Practical and Theoretical Doppler Applications

Numerous fields of scientific and military research continue to employ the principles of Doppler Effect to advance navigation and cosmological sciences. One interesting radar application utilizes the Micro-Doppler Effect to identify very small vibrational and rotational motion of a given body at a distance. Another potential application is the use of pulsars as unique radio beacons to traverse interplanetary space. All of these systems, including the Doppler Navigation System created by Team Caribbean Green, rely on measuring the expansion and contraction of waves as relative distance changes to determine location of an observer in space. The underpinnings of all of these technological advancements arise from fundamental physics principles of waves and light developed and refined over the last two centuries.

References

1. Downs, G. S. (1974). Interplanetary navigation using pulsating radio sources. *NASA Technical Reports Server*. Doi:19740026037

2. Zhang, J., Zhang, K., Grenfell, R., & Deakin, R. (2006). Short Note: On the Relativistic Doppler Effect for Precise Velocity Determination using GPS. *Journal of Geodesy,80*(2), 104-110.

3. Chen, V., Li, F., Ho, S., & Wechsler, H. (2006). Micro-doppler effect in radar: Phenomenon, model, and simulation study. *IEEE Transactions on Aerospace and Electronic Systems, 42*(1), 2-21. doi:10.1109/taes.2006.1603402

4. Hasselkamp, D., Mondry, E., & Scharmann, A. (1979). Direct observation of the transversal Doppler-shift. *Zeitschrift Fur Physik A Atoms and Nuclei, 289*(2), 151-155. doi:10.1007/bf01435932

5. CHEN, V. C. (2019). *MICRO-DOPPLER EFFECT IN RADAR*. Library of Congress: ARTECH Hous