Sailboat Position Tracking

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Introduction

Filters take a stream of data and transforms it in a manner specified by the filter. Filters are typically used to filter out noise. Noise is any sort of interference that changes the values of the data. Kalman filtering is a process that takes a noisy signal from some system and measures it in multiple ways. Then it combines the measurements to produce a more accurate output signal than any single measurement could on its own. The Kalman Filter is especially useful when the measured data is noisy (Kalman, 1960). Kalman Filters are applicable to measuring the location of a sailboat using GPS. GPS is a noisy measurement tool that gives the location and speed of the receiver. A sailboat will be used as an example to explain the stages of the filter.

Classical Kalman Filter

The Kalman Filter is valuable for several reasons. First, it only relies on the current state and the previous state to make its predictions. This minimizes the processing memory needed because it only needs to store the previous state for use in calculations. Second, it is the optimal filter under certain assumptions and converges to a bounded uncertainty under those conditions (Cheever). This means there isn't a better choice of filter for many types of systems.

Assumptions

The Kalman filter is the optimal filter for a certain set of filtering problems (Cheever). Some assumptions must be true for the filter to work as intended. First, the system must be linear. When the system is not linear, the Extended Kalman Filter can be used but the guarantee of optimality is lost. Second, the noise in the system must be zero-mean additive Gaussian noise. This is a reasonable assumption to make under most circumstances especially in real-world situations. The static on an AM radio station is this type of noise. Because the real world consists of so many random variables, the sum effect of these on any given process or measurement forms a Gaussian distribution (Solomon, 1987). For systems where the noise is not Gaussian, the Unscented Kalman Filter can be used instead.

Overview

The Kalman Filter algorithm has two stages for each step (Faragher, 2012). The first stage is the prediction stage. In this stage, a distribution of the most likely current state is calculated. A distribution is a blob that describes the possible states of the system. The center of the distribution is calculated the estimated state from the previous step. The prediction stage uses a model of the system. The model for a sailboat could predict where the boat will be and how fast it will be moving based on where it currently is and how fast it's moving.

The second stage is the correction stage. In this stage, a measurement (with some known variance) is taken of the current state. This distribution is combined with the distribution from the prediction stage to form a new distribution with a more accurate center.

In this model, the current state and the noise cannot be directly measured. The Kalman filtering algorithm produces a best estimate of the current state based on a combination of the previously estimated state and the measurements taken.

Prediction

The prediction stage produces a prediction by combining the previous state with the influences of some known external control force. This creates a bestguess prediction of the current state of the system based on the knowledge of the system state and its dynamics.

The external control force can be ignored in some cases when there is no known control force. In the sailboat example, the external control force term can be ignored because there is no external control force that can be reasonably considered by this simple model. This simplifies the model significantly.

For the sailboat example, the assumption is that any noise in the position will create noise in the velocity in the same direction. However, because the two directions are independent, the noise between them will not be related. These noises can be thought of with respect to the mechanics of the boat itself. The velocity of the boat is dependent on the relative windspeed. Therefore, if the wind changes speed during a sample window, the position and velocity in the wind direction will change. This is not a perfect initial estimate, but it does not have to be either. The Kalman Filter will update and correct this noise estimate on each iteration.

Additional uncertainties from factors that the model cannot account for are considered as well. These adjustments could account for uncertainty due to sail adjustments, current, waves, or other factors that affect the motion of a sailboat.

Combining these two models of uncertainty together produces a multi-dimensional distribution that represents an estimate for the current state of the system. This step alone does not give useful information about the current state of a real system. The next step, correction, incorporates real measurements of the state to improve the state estimate.

Correction

The prediction stage produced a multidimensional distribution for the estimated current state of the system. Taking measurements from sensors will give a value that is governed by its own noise distribution. The correction step will combine those two values into a single distribution that is more precise and accurate than either could be alone.

The distributions of noise associated with the estimate and the measurement are independent. The noise associated with the estimate is macro level considering randomness in the terrain and wind. The noise associated with the sensors is from the micro level and considers thermal variation and vibrations. Because the noises incorporated into each are uncorrelated, we can treat the two distributions as independent.

The process of combining the predicted distribution and the estimate distribution is the key to the Kalman Filter. When combining them, the center of the final estimated distribution is an average of the two centers weighted by whichever has lower noise. For instance if the sensors are good and the measurement has low noise, the center of the final distribution will be close to that of the measurement distribution.

For the purposes of this example, we will assume that our prediction model and the GPS accuracy have about the same precision. Then in the correction stage, the final estimate for this step is an unweighted average of the predicted position and velocity and the measured position and velocity. The estimated covariances will become half of the predicted covariance. The Kalman Filter clearly improves estimates by increasing accuracy and reducing uncertainty by combining information from multiple sources.

Iteration

The prediction and correction stages can be completed as often as there are new measurements to correct with. Each iteration only incorporates information from the previous step. Although we are not necessarily interested in the velocity of the boat, by including it our estimate, we gain the historical data about how the boat had been moving. On each iteration, uncertainty is reduced from the combination of the two sources. However, uncertainty increases due to the dynamic nature of the system being estimated. The goal is to reduce uncertainty more than it increases at each time step.

Extensions

While the Classical Kalman Filter is the optimal filter for linear systems, many systems cannot be modeled linearly. Because of this, the Extended Kalman Filter (EKF) was developed to filter non-linear systems. It is very similar to the classical version, but at each sample point in time the non-linear pieces are linearized (Levy, 2016).

The Unscented Kalman Filter (UKF) is another extension of the classical Kalman Filter algorithm. It can handle both nonlinearity and non-Gaussian noise (Wan & Merwe, 2000).

Extensions

Kalman filtering takes two imprecise method methods of determining the state of a system (measurement and extrapolation) and combines them to produce a more accurate estimate than would be possible by either alone. The Kalman Filter is particularly powerful because it only requires information from the previous time step and is the optimal filter for linear systems.

References

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