How Does Your Smartphone Know Where it is?

UAV LiDAR Mapping

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Introduction

The modern smartphone experience relies on the phone's ability to locate itself in the world. The phone knowing its location is essential for Google Maps, weather reports, geo-tagging, fitness tracking, lost phone recovery, and even whether the screen should be in landscape or portrait mode. To solve this problem, which is often called the Localization Problem, requires multiples sensors. These sensors include GPS, IMU. The principals that allow a phone to localize itself are also used by many other devices, for example: aircrafts, automobiles, and ships.

Relevant Technologies

GPS – Global Positioning System

GPS is a system that utilizes a network of satellites to locate the device. The satellites send out an electromagnetic signal down to earth. While the signal travels very quickly, it is not instant. A device with a GPS receiver can take the difference in arrival times to determine its location. GPS is very popular, but has accuracy limitations to about 20 ft. The signal from the satellites is not strong enough to reach indoors and becomes easily obstructed by trees or metal posts. When a GPS signal bounces off of a surface, and then arrives at the device, an improper location can be determined due to the extra time introduced by the bouncing. GPS is also sensitive to atmospheric conditions. Moisture and temperature can slightly affect the travel time of the signal, reducing the accuracy. There is a method to reduce these atmospheric disturbances: Real Time Kinematic GPS. By adding an additional receiver, which is known to be stationary, the atmospheric conditions can be calculated and canceled out by the device. A smartphone does not have access to RTK GPS, but it needs a little more precision than GPS affords to provide a good experience with driving navigation.

IMU – Inertial Measurement Unit

To supplement the GPS system, the device can have another array of sensors called the IMU. The IMU describes the orientation, called "attitude," of the device. This attitude is expressed in angles: Roll, Pitch, Yaw.

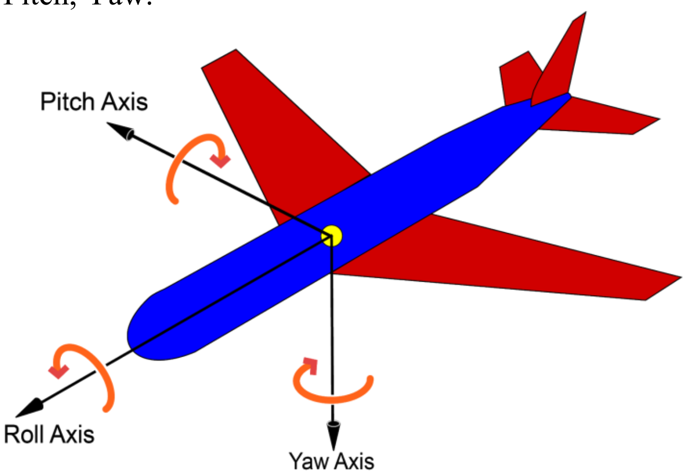

Figure 1. Pitch, Roll, and Yaw. By Yaw_Axis.svg: Auawisederivative work: Jrvz (talk) - Yaw_Axis.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=9441238

The IMU is also capable of detecting accelerations in the device, including the constant acceleration caused by gravity. In order to make use of these readings to provide a better prediction of the device's location, some mathematics are required.

Mathematical Principals

Dead Reckoning

Imagine that the accelerometer has measured the phone move 10 feet in one of its axes. Assuming the original position was known, where is the new position? In addition to how far the device traveled, the direction in which it traveled also needs to be known. The merging of these two values is called "dead reckoning". In 2 dimensions, it is easy to perform this calculation. We can solve the problem graphically by drawing a vector in the direction we travelled, with a length of the distance we traveled. The head of the vector is our new position:

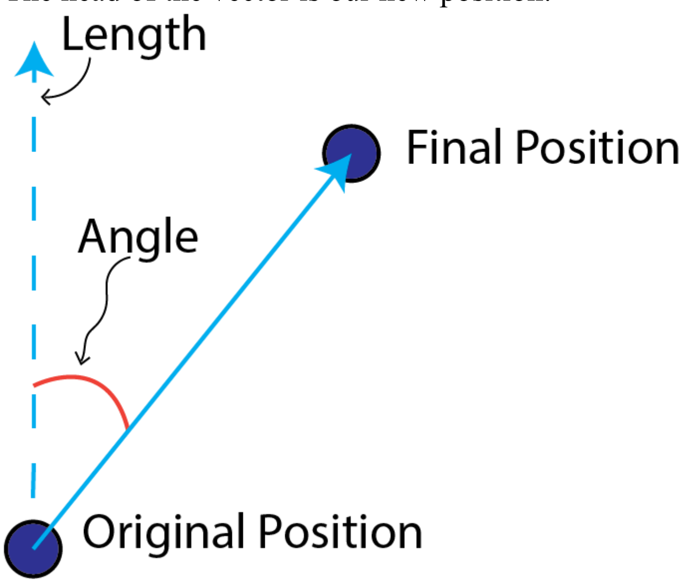

Figure 2. "Dead Reckoning" by Benjamin Holen, CC BY-SA 3.0

This is good for ships at sea, but modern devices move in 3 dimensions. To account for this, more sophisticated mathematics are required.

Quaternions and Vector Rotation

3D dead reckoning follows the same principle as 2D, we want a vector in the direction of travel with the length of travel. The length is easy and is provided by double integrating readings from the accelerometer. The direction is the challenging part, as roll, pitch, and yaw come in the forms of angles and not vectors. "Quaternions" come to the rescue. A quaternion is a set of 4 numbers, expressed as an addition:

$$a + b\,\mathbf{i} + c\,\mathbf{j} + d\,\mathbf{k}$$

Figure 3. Quaternion Form

Quaternions can be added together, just like vectors, but follow a special rule for multiplication called the Hamilton Product:

$$\begin{aligned} & a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 \\ & + (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)i \\ & + (a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2)j \\ & + (a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)k. \end{aligned}$$

Figure 4. Hamilton Product

By taking the readings (roll, pitch, yaw, distance) the following quaternions can be constructed:

$$Q = \begin{bmatrix} cos(\text{roll}/2)cos(\text{pitch}/2)cos(\text{yaw}/2) + sin(\text{roll}/2)sin(\text{pitch}/2)sin(\text{roll}/2) \\ sin(\text{roll}/2)cos(\text{pitch}/2)cos(\text{yaw}/2) - cos(\text{roll}/2)sin(\text{pitch}/2)sin(\text{yaw}/2) \\ cos(\text{roll}/2)sin(\text{pitch}/2)cos(\text{yaw}/2) + sin(\text{roll}/2)cos(\text{pitch}/2)sin(\text{yaw}/2) \\ cos(\text{roll}/2)cos(\text{pitch}/2)sin(\text{yaw}/2) - sin(\text{roll}/2)sin(\text{pitch}/2)cos(\text{yaw}/2) \end{bmatrix}$$

$$R = \begin{bmatrix} 0 \\ 0 \\ 0 \\ distance \end{bmatrix}$$

Figure 5. Quaternion Rotation Matrices

The following Hamilton product provides the vector needed for dead reckoning:

$$R' = QRQ^*$$

Figure 6. Quaternion Rotation Operation

This operation is called a rotation, and devices like smartphones do it all the time to translate vectors in one reference frame to another. Because this vector rotation can be reduced to matrix multiplications, it is very efficient when performed by a computer. This efficiency and robustness have contributed to the extensive use of the quaternion rotation.

Additional Applications

The same vector rotation can be used to solve a different, but related problem. For the purposes of land surveying, many use an aircraft equipped with a LiDAR system. LiDAR allows for a very precise measurement from the aircraft to the ground. The trouble is that planes do not fly perfectly level, and the plane's orientation needs to be accounted for to make good use of the LiDAR data. This is the same problem as dead reckoning, except that the accelerometer distance is replaced with the LiDAR distance:

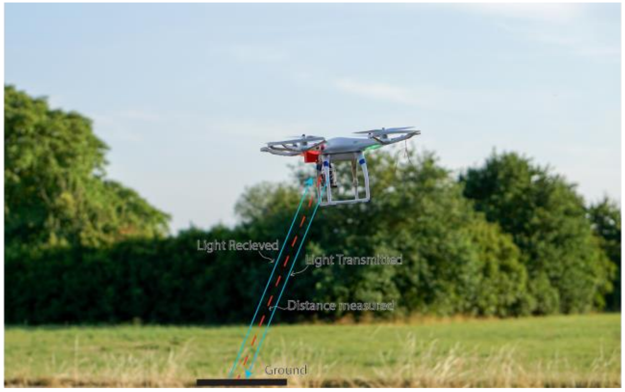

Figure 7. Image by Kira Hoffmann, Pixabay License Annotations by Benjamin Holen, CC-BY-SA 3.0

Conclusion

The Principles shown here are the very same that guide rockets in space and control the autopilot on commercial jets. Quaternions are a powerful tool for manipulating objects in 3 dimensions, and along with modern sensors allow for devices to surmise their location with excellent accuracy.

References

1. Hamilton, W. R. (1969). *Elements of quaternions*. New York, NY: Chelsea Publ.

2. Hamilton, W. R. (1853). *Lectures on quaternions: Containing a systematic statement of a new mathematical method; of which the principles were communicated in 1843 to the Royal Irish academy; and which has since formed the subject of successive courses of lectures, delivered in 1848 and subsequent years, in the halls of Trinity college, Dublin; with numerous illustrative diagrams, and with some geometrical and physical applications*. Dublin: Hodges and Smith.

3. Haren, B. (Director). (2016, January 18). *Fantastic Quaternions* [Video file]. Retrieved from https://www.youtube.com/watch?v=3BR8tK-LuB0

4. (Director). (2018, September 6). *What are quaternions, and how do you visualize them? A story of four dimensions.* [Video file].

5. EvenMoreAuthorOrAuthors, D. (1989). Free for the Taking. *The Journal Of Cost*, *22*, 210-212. doi:678.1234598

6. Conway, J. H., & Smith, D. A. (2009). *On quaternions and octonions: Their geometry, arithmetic, and symmetry*. Natick, MA: Peters.