Newton-Raphson Method and its Application in Power Systems

By Caleb Weinstein-Zenner, ECE '22

Introduction

When it comes to math, finding an answer is almost never easy. Say we want to find what the numerical approximation of $\sqrt{2}$ is. Nowadays, we can use a simple calculator to get the answer or, better yet, google it. But back before computers, before the simple concept of "googling" something, we had to do everything by hand. The average person could not answer the above question without going to the library and (re)learning advanced algebra concepts or (if this person wanted to go the extra mile) even some calculus topics. The point here is that somewhere along the way, we found a way for computers to get an answer so precise that doing the math by hand was no longer necessary. What is that way? It is an algorithm: the Newton-Raphson Method. This paper will cover the method and its application in offshore wind.

What the N-R Method is

The Equation Itself

To start, the Newton-Raphson method is an iterative, non-linear solver. What does that mean exactly? Essentially, it means that for a function *f* and a starting point *x*, the solver will move a certain distance across the function over a time duration *t*. The method in which this occurs is by evaluating the derivative of *f* to determine which direction the solver should move in, relative to the function. The equation for this is shown next, where x_o is the starting point, x_1 is the end point, *f(x)* is the function and *f'(x)* is the derivative of the function (Gershan).

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

An In-Depth Look

By looking at this solver, we can notice a few things. The first is on how far the solver moves in each iteration. To translate from x_0 to x_1, we need some factor by which to do it. Remembering that this is a nonlinear solver, and that the derivative is involved, we know that the second portion of the right-hand side is the translating factor. What this section does is determine the slope of the function relative to the value of the function, creating a tangent line. If this value is much greater than zero, then the magnitude of the translation from start to finish is quite large and the tangent line is more vertical (lots of movement). However, if it is close to zero, then the magnitude of the translation is quite small, and the tangent line is near-horizontal (very little movement). Thus, this solver operates by finding the point on the function that has the most horizontal tangent plane and calling that value of *f* the solution. For a more physical example, it can be thought of as the settling point of a ball in a bowl; that's where the answer resides. Below is a 2-D example of what this looks like.

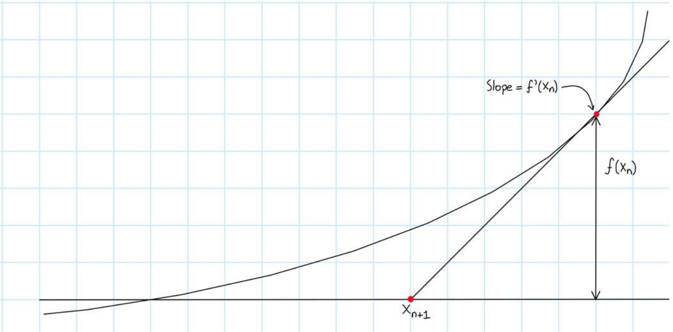


Figure 1. Geometric Representation of Newton-Raphson Method (Gershan)

The Exceptions

There are some exceptions where this method does not behave in the manner we would expect. Take for example an equation that looks like this:

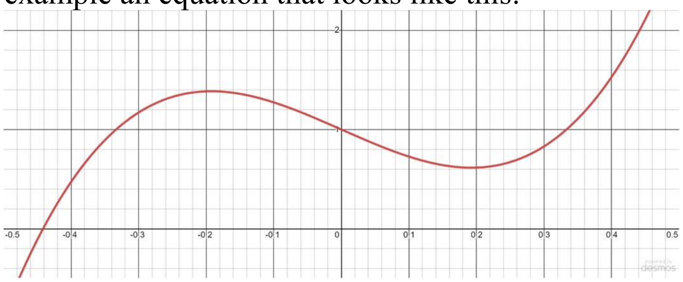


Figure 2. Example of Exception to Newton-Raphson Method Providing Complete Solution (Gershan)

Here there are two locations where the tangent line is flat and a point of inflection at $x = 0$. To see why Newton's method isn't helpful here, imagine choosing a point at random between $x = -0.2$ and $x = 0.2$ and drawing a tangent line to the function at that point. That tangent line will have a negative slope, and therefore will intersect the y-axis at a point that is farther away from the root. In essence, the root of the function will not be the solution acquired from the method.

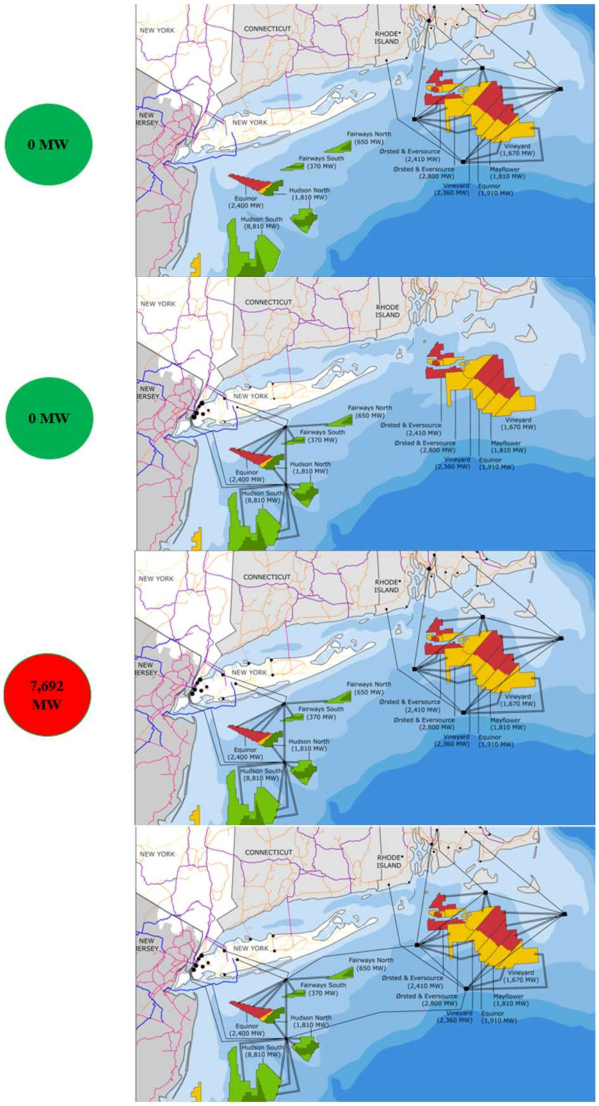
This problem also occurs with functions that have many local minima near the global minimum. In this case, there will be multiple points where the tangent line will be flat, so if the global minimum is the correct solution, then Newton's method may not settle in the correct minima. This will not give the correct solution / root. With these exceptions in mind, we can look at applications of the Newton-Raphson method.

Application of N-R

Now we can look at its application in the field of power systems engineering. Within power systems, there are substations (nodes), power loads, power generators, and power lines that are all used to describe a model of transmission across a given geological area (Mohan). The power lines are what connect loads to generators, other loads, or just standard nodes. To understand a system, the voltage, current, active power, and reactive power are all needed to be known at every node, load, and generator (Mohan). This is done by running a power flow.

To not get too deep in the weeds on this topic, there is a function that can, under normal circumstances, use any two of the previous attributes to calculate the other two (Mohan). This function uses derivatives, so it is a non-linear function, thus requiring a non-linear approach to solving. On top of this, typically, a power systems engineer will know two attributes at all nodes/loads/generators due to them being inherent to the object: a generator is built with a specific power output, so active and reactive power are known, etc. This allows for the function to be used at every node in the model (creating a matrix for all known values). So, when running a power flow, the Newton-Raphson Method is used at every node to calculate reactive/active power, voltage, or current to get a complete understanding of the system (Nirupama).

In some scenarios, the power flow can fail, and a stable value cannot be produced from the Newton-Raphson Method. Why? This goes back to the exceptions of when the method can converge. If the equation at enough of the nodes fails, where the equations don't have roots or they have multiple roots and the wrong ones are calculated, then the power flow fails (Mohan). Related to real life, this happens when there is too much power generated in a system and not enough load to use that power. Or in the case of research previously done by the author, due to the density of generated power. The figures below show the amount of overloaded power (most likely calculated in error) in the case of wind turbine farms being integrated in large quantity to a small area near the US Northeast shoreline.



Figures 3-6. Newton-Raphson Method giving incorrect information as power generation density increases

What happened with the last simulation? The Newton-Raphson Method failed to converge on a solution:

Figure 7. Power flow fails to converge on a solution

An important distinction between the last two maps: The second-to-last map technically has a less dense area of power generation, as there are no lines connecting the top region of nodes to the bottom region. Thus, there are more errors as compared to the top two maps, but the power flow still converges on a solution.

The main takeaway from this example is that while the Newton-Raphson Method is a vital tool for power flow analysis, it is on occasion inept for that use due to the ever-evolving strategies used for power generation and (especially accurate in this example) errors in the model it is being used on. In the case that it is inept, there are other tools to use, like the Gauss-Seidel Method, but in general Newton-Raphson is the standard due to its relative lack of complexity compared to the other (and possibly because Newton is slightly more well-known than Gauss).

Conclusion

The Newton-Raphson Method is a powerful tool that, while only one use was discussed in this paper, has many uses due to its ability to find the roots of functions that are non-linear. It has its limitations that keep it from being the be-all-end-all solution to any problem involving a curve, but even still is heavily used.

References

Mohan, N. (2012). Power Flow in Power System Networks. In *Electric Power Systems: A First Course* (1st ed., pp. 78–91). Wiley.

Gershan, A, et al. Newton Raphson Method. *Brilliant.org*. Retrieved 12:03, January 30, 2022, from https://brilliant.org/wiki/newton-raphson-method/

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