

# Biquadratic Audio Filters

By Charles DePalma, ECE '23

## Introduction

As we designed our guitar effects pedal, we understood consumers would demand flexible, high-quality effects on par with those offered by software effects and analog hardware. This was despite the fact that the pedal uses a small Bluetooth microcontroller with limited processing power. To create an effective tool for artists and sound designers, we would need to supplement our distortion effects with a variety of parameterizable audio filters. But what are these filters, and what do they achieve? How can they be implemented just by writing code? And how can that code be efficient enough to run in real time, without delays or stuttering?

## What are audio filters?

Audio filters are considered one of the fundamental building blocks of music production and sound design. Their role is to preserve some frequencies in the audio, or make them louder, while other frequencies are made quieter or even totally inaudible. While they cannot change the pitch of an instrument, filters can alter the timbre of a note, quiet annoyingly loud low or high frequencies, or even remove noise limited to one region of the frequency spectrum. Synthesizers use filters to radically alter their sounds' timbre, while audio engineers use tools called equalizers that can subtly adjust unwanted frequencies. And many more scientific and engineering disciplines, from aerospace to medicine, rely on technologies that apply filters to data other than audio.

We usually describe filters based on their *frequency response*, which can be visualized as a graph. Frequencies are often listed on the X axis, starting

from nearly 0 Hz and going up to 20 kHz, considered the highest pitch the human ear can hear. The Y value at each point represents how loud that frequency will be in the output.

While we can create extremely complex frequency responses by applying several filters on top of each other, most individual filters come in a handful of predictable shapes. *Lowpass* filters block higher frequencies, creating a smoother, rounder waveform and a less "snappy" sound. *Highpass* filters block lower frequencies, making the result sound tinnier. *Bandpass* filters perform both effects simultaneously, creating a thin-sounding output. *Peak* filters are among the most versatile, allowing an arbitrary range of frequencies to become louder or quieter by any amount. Finally, low and high *shelf* filters allow for adjusting the volume of all frequencies higher or lower than a given cutoff. Figure 1 shows examples of all of these filter frequency responses.



Figure 1. Frequency responses of several filter types implementable with a biquad equation, from top to

bottom: lowpass, highpass, bandpass, peak, shelf.

These filters have other controls, too. The *cutoff* of a filter is the frequency at which it transitions from the affected part of the frequency spectrum to the unaffected part. The *Q*, or quality factor, determines how "sharp" or "steep" the effect is. In practice, no filter can have a perfectly steep cutoff or a perfectly narrow *Q*, so all frequency response graphs will appear slightly rounded.

## Biquadratic Topology

As we've seen, there are an exhausting variety of different filters that can be created, with different shapes, cutoffs, and more. Can a computer handle any of them? Luckily, there exists a generic mathematical formula that can perform almost all of this filtering simply by changing some of its coefficients. It's called a *biquadratic filter*, or a "biquad" for short. There are many versions of the formula for computing a biquad filter, but a popular simplification is:

$$y[n] = a_0x[n] + a_1x[n - 1] + a_2x[n - 2] \\ - b_1y[n - 1] - b_2y[n - 2]$$

What do these terms mean?  $y[n]$  is the current filter output that we're computing.  $y[n - 1]$  is the most recent previous filter output, and  $y[n - 2]$  is the previous output before that. Likewise,  $x[n]$  is the current input data value,  $x[n - 1]$  is the most recent previous input, and  $x[n - 2]$  is the input before that one. The remaining five values  $a_0$ ,  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$  are coefficients that change depending on the shape, cutoff, and *Q* we want to achieve. Their equations are much more complicated, but they only need to be computed once and don't change unless the filter does. The result is one subroutine that can create any filter shape described in Figure 1. There are alternative formulas that can implement similar filters, but most are not as robust and effective as the biquad.

## Efficiency

Much of the reasoning behind using biquads in our design involved their efficiency and ease of computation, especially because our microcontroller

was much less powerful than a desktop computer or dedicated analog circuit. What makes the biquad so efficient?

Part of the reason is its consistency. To an analog designer, a shelf or peak filter is much more complicated than a simple lowpass. But the biquad always performs the same amount of work: five multiplications, two additions, and two subtractions -- which is not very much at all. On the other hand, the coefficients also have to be computed. This only needs to happen once each time the filter is adjusted, making it unlikely to slow down the system. But these computations involve complex equations with sines, cosines, and square roots, and require very high precision, because small inaccuracies can totally throw off the filter's behavior. In our design, we perform these calculations on the user's phone and send the coefficients over Bluetooth.

Another reason involves the *impulse response* of the filter. Some filters are said to have a *finite impulse response*: the designers select a duration, and create a filter that will completely "forget" any data older than that duration. In contrast, a biquad is an *infinite impulse response* filter: if you start it and suddenly send a brief portion of audio through it, it will technically never "forget" it, and never completely return to silence. This might seem undesirable, but the effect is almost impossible to hear.

The problem comes when a finite impulse response filter might have to store many hundreds or thousands of data points to work properly, which will quickly exhaust our microcontroller's memory. A biquad does not. Looking back at its equation, besides the current data value, we use only our two most recent outputs and our two most recent inputs. So, we only need to store four data points per filter, rather than thousands. Overall, few other filters can compete with the biquad.

## References

1. W3C Working Group, 2021. Audio EQ Cookbook. <https://www.w3.org/TR/audio-eq-cookbook/>
2. Cristi, Roberto. Modern Digital Signal Processing. Thomson/Brooks/Cole, 2004.





*iaculis mauris, a  
sed*

*Figure 1. Curabitur pharetra  
consectetur odio fermentum*

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Cras sagittis malesuada est, ac feugiat nulla. Suspendisse luctus mollis diam in gravida. Aenean sollicitudin, arcu non ultricies auctor, magna enim auctor neque, non tincidunt risus diam ac dolor. Maecenas ligula nisl, aliquam id hendrerit id, ultrices id tellus. Vivamus pulvinar neque at magna condimentum dictum. Curabitur congue erat suscipit ex maximus, ultrices lacinia libero iaculis. Etiam vestibulum, tortor ac scelerisque vehicula, justo diam imperdiet massa, interdum accumsan urna eros ut sem. Nullam eget nibh quis urna suscipit volutpat sed eu leo. Ut pharetra consequat velit, nec aliquet lectus venenatis sed. Proin a vestibulum dolor. Fusce hendrerit dignissim nisl eu lacinia. Vivamus aliquet felis vitae enim euismod commodo at ac ipsum. Mauris ac enim tincidunt, rutrum lacus vulputate, volutpat eros.

### Heading 3

Donec mollis, dolor vitae porttitor pharetra, massa ligula volutpat massa, a dictum orci purus vel magna. Suspendisse vel nunc sit amet tortor interdum auctor. In elementum nunc nec erat congue pretium.

Vestibulum commodo sodales pharetra. Vivamus sit amet ligula nibh. In id eleifend enim. Sed quis urna efficitur, faucibus libero euismod, egestas sapien. Nulla vulputate vulputate felis, ut dictum libero interdum sed. Aliquam consectetur, est eu imperdiet ullamcorper, diam lectus hendrerit lectus, quis placerat enim urna non libero. Pellentesque molestie, metus lacinia sodales rhoncus, arcu orci aliquet turpis, vel placerat ligula libero eget mauris. Donec semper pulvinar iaculis. Nam quis sem et purus suscipit pharetra non a nisi. Curabitur suscipit vehicula tortor nec porttitor. sapien. Duis sagittis

euismod tortor, finibus vehicula sem consequat id. Interdum et malesuada fames ac ante ipsum primis in faucibus. Phasellus vitae ante imperdiet, consequat lacus ornare, accumsan dui. Etiam sit amet sapien porta, laoreet magna ac, ornare quam. Sed maximus nisi auctor justo euismod aliquet.



*Figure 2. Curabitur pharetra iaculis mauris, a  
consectetur odio fermentum sed*

Maecenas lacinia eros ac enim tincidunt, vel sagittis erat bibendum. Ut malesuada elementum velit, at porta diam tincidunt in. Maecenas interdum eros id feugiat dignissim. Curabitur eleifend lectus nisl, vitae fringilla lectus pharetra nec. Etiam ut arcu maximus eros feugiat malesuada. Curabitur et neque auctor, blandit massa malesuada, ornare massa. Duis pharetra est et lacus lacinia porta. Quisque rutrum est quis pharetra porttitor. Proin tincidunt auctor metus eget condimentum. Curabitur pharetra iaculis mauris, a consectetur odio fermentum sed. Nulla eu mi rutrum, hendrerit mauris sit amet, condimentum est. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec et erat metus. Morbi id aliquet augue. Sed ut erat et est sodales ornare. Ut nec dui volutpat, sodales dolor pretium, finibus magna. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos.

Suspendisse elit neque, ornare at dui consectetur, aliquam rutrum tellus. Mauris augue velit, iaculis at scelerisque sit amet, egestas vitae justo. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Nullam porttitor tincidunt turpis. Praesent ut pretium ante, vitae porta sem.

### Heading 4

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Cras sagittis malesuada est, ac feugiat nulla.

Suspendisse luctus mollis diam in gravida. Aenean sollicitudin, arcu non ultricies auctor, magna enim auctor neque, non tincidunt risus diam ac dolor. Maecenas ligula nisl, aliquam id hendrerit id, ultrices id tellus. Vivamus pulvinar neque at magna condimentum dictum. Curabitur congue erat suscipit ex maximus, ultrices lacinia libero iaculis. Etiam vestibulum, tortor ac scelerisque vehicula, justo diam imperdiet massa, interdum accumsan urna eros ut sem. Nullam eget nibh quis urna suscipit volutpat sed eu leo. Ut pharetra consequat velit, nec aliquet lectus venenatis sed. Proin a vestibulum dolor. Fusce hendrerit dignissim nisl eu lacinia. Vivamus aliquet felis vitae enim euismod commodo at ac ipsum. Mauris ac enim tincidunt, rutrum lacus vulputate, volutpat eros.

### Sub-Heading

Donec mollis, dolor vitae porttitor pharetra, massa ligula volutpat massa, a dictum orci purus vel magna. Suspendisse vel nunc sit amet tortor interdum auctor. In elementum nunc nec erat congue pretium. Vestibulum commodo sodales pharetra. Vivamus sit Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Suspendisse elit neque, ornare at dui consectetur, aliquam rutrum tellus. Mauris augue velit, iaculis at scelerisque sit amet, egestas vitae justo. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Nullam porttitor tincidunt turpis. Praesent ut pretium ante, vitae porta sem.

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Cras sagittis malesuada est, ac feugiat nulla. Suspendisse luctus mollis diam in gravida. Aenean sollicitudin, arcu non ultricies auctor, magna enim auctor neque, non tincidunt risus diam ac dolor. Maecenas ligula nisl, aliquam id hendrerit id, ultrices id tellus. Vivamus pulvinar neque at magna condimentum dictum. Curabitur congue erat suscipit ex maximus, ultrices lacinia libero iaculis. Etiam vestibulum, tortor ac scelerisque vehicula, justo diam imperdiet massa, interdum accumsan urna eros ut sem. Nullam eget nibh quis urna suscipit volutpat sed eu leo. Ut pharetra consequat velit, nec aliquet lectus venenatis sed. Proin a vestibulum dolor. Fusce hendrerit dignissim nisl eu lacinia. Vivamus aliquet felis vitae enim euismod commodo at ac ipsum. Mauris ac enim tincidunt, rutrum lacus vulputate, volutpat eros.

Donec mollis, dolor vitae porttitor pharetra, massa ligula volutpat massa, a dictum orci purus vel magna. Suspendisse vel nunc sit amet tortor interdum auctor. In elementum nunc nec erat congue pretium. Vestibulum commodo sodales pharetra. Vivamus sit amet ligula nibh. In id eleifend enim. Sed quis urna efficitur, faucibus libero euismod, egestas sapien. Nulla vulputate vulputate felis, ut dictum libero interdum sed. Aliquam consectetur, est eu imperdiet ullamcorper, diam lectus hendrerit lectus, quis placerat enim urna non libero. Pellentesque molestie, metus lacinia sodales rhoncus, arcu orci aliquet turpis, vel placerat ligula libero eget mauris. Donec semper pulvinar iaculis. Nam quis sem et purus suscipit pharetra non a nisi.

Curabitur suscipit vehicula tortor nec porttitor. Donec vitae condimentum libero, quis efficitur sapien. Duis sagittis euismod tortor, finibus vehicula sem consequat id. Interdum et malesuada fames ac ante ipsum primis in faucibus. Phasellus vitae ante imperdiet, consequat lacus ornare, accumsan dui. Etiam sit amet sapien porta, laoreet magna ac, ornare quam. Sed maximus nisi auctor justo euismod aliquot

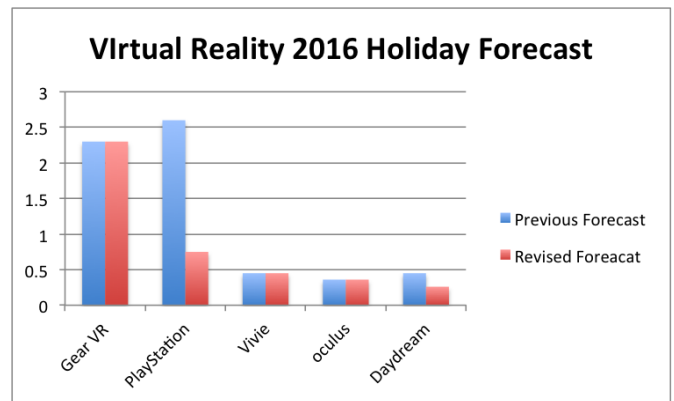


Figure 3. Curabitur pharetra iaculis mauris, a consectetur odio fermentum sed

### Sub-Heading

Maecenas lacinia eros ac enim tincidunt, vel sagittis erat bibendum. Ut malesuada elementum velit, at porta diam tincidunt in. Maecenas interdum eros id feugiat dignissim. Curabitur eleifend lectus nisl, vitae fringilla lectus pharetra nec. Etiam ut arcu maximus eros feugiat malesuada. Curabitur et neque auctor, blandit massa malesuada, ornare massa. Duis pharetra est et lacus lacinia porta. Quisque rutrum est quis pharetra porttitor. Proin tincidunt auctor metus eget condimentum. Curabitur pharetra iaculis mauris, a consectetur odio fermentum sed. Nulla eu mi rutrum, hendrerit mauris sit

amet, condimentum est. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec et erat metus. Morbi id aliquet augue. Sed ut erat et est sodales ornare.

Ut nec dui volutpat, sodales dolor pretium, finibus magna. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos. Suspendisse elit neque, ornare at dui consectetur, aliquam rutrum tellus. Mauris augue velit, iaculis at scelerisque sit amet, egestas vitae justo. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Nullam porttitor tincidunt turpis. Praesent ut pretium ante, vitae porta sem

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Cras sagittis malesuada est, ac feugiat nulla. Suspendisse luctus mollis diam in gravida. Aenean sollicitudin, arcu non ultricies auctor, magna enim auctor neque, non tincidunt risus diam ac dolor. Maecenas ligula nisl, aliquam id hendrerit id, ultrices id tellus. Vivamus pulvinar neque at magna condimentum dictum. Curabitur congue erat suscipit ex maximus, ultrices lacinia libero iaculis. Etiam vestibulum, tortor ac scelerisque vehicula, justo diam imperdiet massa, interdum accumsan urna eros ut sem. Nullam eget nibh quis urna suscipit volutpat sed eu leo. Ut pharetra consequat velit, nec aliquet lectus venenatis sed. Proin a vestibulum dolor. Fusce hendrerit dignissim nisl eu lacinia. Vivamus aliquet felis vitae enim euismod commodo at ac ipsum. Mauris ac enim tincidunt, rutrum lacus vulputate, volutpat eros.

### **Sub-Heading**

Donec mollis, dolor vitae porttitor pharetra, massa ligula volutpat massa, a dictum orci purus vel magna. Suspendisse vel nunc sit amet tortor interdum auctor. In elementum nunc nec erat congue pretium. Vestibulum commodo sodales pharetra. Vivamus sit amet ligula nibh. In id eleifend enim. Sed quis urna efficitur, faucibus libero euismod, egestas sapien. Nulla vulputate vulputate felis, ut dictum libero interdum sed. Aliquam consectetur, est eu imperdiet ullamcorper, diam lectus hendrerit lectus, quis placerat enim urna non libero. Pellentesque molestie, metus lacinia sodales rhoncus, arcu orci aliquet turpis, vel placerat ligula libero eget mauris. Donec semper pulvinar iaculis. Nam quis sem et purus suscipit pharetra non a nisi.

Curabitur suscipit vehicula tortor nec porttitor. Donec vitae condimentum libero, quis efficitur sapien. Duis sagittis euismod tortor, finibus vehicula sem consequat

id. Interdum et malesuada fames ac ante ipsum primis in faucibus. Phasellus vitae ante imperdiet, consequat lacus ornare, accumsan dui. Etiam sit amet sapien porta, laoreet magna ac, ornare quam. Sed maximus nisi auctor justo euismod aliquet.

## **Conclusion**

Maecenas lacinia eros ac enim tincidunt, vel sagittis erat bibendum. Ut malesuada elementum velit, at porta diam tincidunt in. Maecenas interdum eros id feugiat dignissim. Curabitur eleifend lectus nisl, vitae fringilla lectus pharetra nec. Etiam ut arcu maximus eros feugiat malesuada. Curabitur et neque auctor, blandit massa malesuada, ornare massa. Duis pharetra est et lacus condimentum. Curabitur pharetra iaculis mauris, a consectetur odio fermentum sed. Nulla eu mi rutrum, hendrerit mauris sit amet, condimentum est. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec et erat metus. Morbi id aliquet augue. Sed ut erat et est sodales ornare.

## **References**

1. AuthorOrAuthors, A. (2017). The Best Things in Life Are Free. *The Journal Of Buying Things*, 105, 593-598. doi:123.456789
2. AnotherAuthorOrAuthors, B. (2009). Free As the Crow Files. *The Journal Of Flying Things*, 10, 43-68. doi:987.654321
3. SomeMoreAuthorOrAuthors, C. (1986). Downloading for Free. *The Journal Of the Paid Internet*, 16, 101-168. doi:765.123489
4. EvenMoreAuthorOrAuthors, D. (1989). Free for the Taking. *The Journal Of Cost*, 22, 210-212. doi:678.1234598
5. LastButNotLeastAuthorOrAuthors, E. (1913). Panhandling. *The Journal Of the Beggar's Opera*, 45, 723-734. doi:345.12

