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Conflict, defense spending, and the number of nations

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Abstract

This paper provides a formal model of endogenous border formation and choice of defense spending in a world with international conflict. We examine both the case of democratic governments and of dictatorships. The model is consistent with three observations. First, breakup of countries should follow a reduction in the likelihood of international conflicts. Second, the number of regional conflicts between smaller countries may increase as a result of the breakup of larger countries. Third, the size of the *peace dividend* (the reduction in defense spending in a more peaceful world) is limited by the process of country breakup.

JEL Classification: D74; F02; H56

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1. Introduction

The end of the cold war has been accompanied by a sweeping process of democratization, creation of new countries and political separatism. However, even though the probability of a confrontation between the two superpowers of the cold war era is greatly diminished, the number of localized conflicts has not decreased.

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Several observers have in fact argued that one should expect more regional conflicts after the end of the cold war.[1]

This paper provides a model that is consistent with both these observations. First, it implies an increase in the number of countries as a consequence of a reduction in the probability of international conflict. Second, it argues that a reduction in the probability of conflict among a few large countries, by increasing, in equilibrium, the number of smaller countries, may increase the number of conflicts between the more numerous, smaller independent political units. A related observation concerns the extent of the "peace dividend," i.e., the reduction in military spending following the end of the cold war.[2] Our model suggests that the worldwide "peace dividend" may be smaller than one might expect. The reason is that the per capita costs of defense may increase in smaller countries than have to deal with potentially more numerous regional conflicts. We analyze both the case of conflicts between democratic governments where decisions are taken by voting and between dictators that maximize their rents.

This paper joins two strands of the analytical literature. One is the economic literature on the size of countries, as in Friedman (1977), Bolton and Roland (1997) and Casella and Feinstein (2002). The present paper, in particular, builds upon Alesina and Spolaore (1997, 2003) where the equilibrium number of countries is derived as emerging from a trade-off: the benefits of economics of scale in the production of nonrival public goods against the costs of heterogeneity in the population. Here we explicitly consider the benefit of size that arises from the possibility of international conflicts and the costs of defense. In Alesina and Spolaore (2004) we discuss the relationship between conflict and political breakup in a two-bloc word. While in that paper we can be more general about the technology of conflicts and wars, in the present paper we simplify in that dimension. On the other hand, in this paper we do not restrict our analysis to the formation and breakup of two blocs, but we can solve for a generic number of political units (countries) as part of a general politico-economic equilibrium. Also in the present paper we explicitly model heterogeneity among individuals as stemming from different preferences over types of government, and we discuss both the case of democracies and of dictatorships.

The second line of research is the literature on conflict resolution and arms races surveyed by Sandler and Hartley (1995), and by various contributions in Hartley and Sandler (1995). Classic references are Schelling (1960), Boulding (1962), Olson and Zeckhauser (1966) and Tullock (1974).[3] Findlay (1996) discusses the stability of empires in a word where armed conflict is explicitly modeled.[4] In particular, our

[1] Hobsbawn (1994) cites the 1991 Gulf War as an example.

[2] For instance, Clemens et al. (1997) cite WEO data according to which a third of 130 countries maintained or increased their military spending as a percentage of GDP between 1990 and 1995. They also calculate that the ten developing countries with the largest increases in defense spending between 1985 and 1992 had an average increase of 2.7 percentage points of GDP.

[3] For recent formal contributions within the field of international relations see Powell (1999).

[4] The relationship between domestic politics and international conflict is studied by Garfinkel (1994) and Hess and Orphanides (1995, 2001). A related literature formally studies the role and consequences of conflict and insurrections for the distribution of property rights. In particular, see Grossman (1991) and Grossman and Kim (1995).

formalization of the technology of conflict resolution follows Tullock (1980) and Hirshleifer (1989, 1995).

This paper is organized as follows. Section 2 describes the basic model as applied to democracies. Section 3 illustrates the domestic equilibrium on the choice of defense and non-defense spending. Section 4 characterizes a voting equilibrium in which the number and size of countries is endogenously determined. Section 5 discusses issues of stability, and specifically, unilateral secessions. Section 6 considers the case in which public spending (including defense), taxation and borders are not chosen by voters but by rent-seeking governments (Leviathans). Section 7 extends the model to allow for a more general matching technology, and discusses other possible extensions of the basic framework. The last section concludes.

2. The basic model

The world is modeled as a segment of length normalized to 1. The world population has mass 1 and is uniformly distributed on the segment [0,1].[5] A country is defined by two borders and a non-rival public good, which we label the 'government'. Each individual can only use one public good, i.e., one government, and individual utility is decreasing in the distance from the government of the country to which the individual belongs. The distance of individual i from his government is denoted l_i . We assume that this distance captures both a geographical and a preference dimension. That is, being "far" from the government implies being distant both in geographical location and in preferences: if two individuals live far from each other, they are also distant in preferences. Hence, the location of a government captures both a position on an ideological dimension and on a geographical line. As discussed in more detail in Alesina and Spolaore (1997), this assumption ensures that countries are geographically connected. An alternative assumption would be to retain only the preference interpretation of distance and then impose costs on non-geographically connected countries. For the purpose of this paper the "preference" interpretation of distance is not necessary, although it makes the model richer.

Individual utility is given by

$$U_i = z_i - t_i - gl_i, \tag{1}$$

where z_i is the total income available to individual i, t_i are his taxes, l_i is the distance of individual i from his government, and g is a positive parameter. Thus, individual utility is linear in private consumption $(z_i - t_i)$ and linear in distance from the public good. The utility deriving from the public good is highest for $l_i = 0$.

The cost of a government is K, irrespective of the size of the country. This specification captures in the simplest possible way the benefit of "size" of a

[5] In Section 7 we will discuss extensions of these simplifying spatial assumptions.

country.[6] In fact, with a fixed cost of government the average per capita cost of financing is decreasing with the size of the country. In reality, the benefits of country size derive from several fixed costs, including creating and maintaining a monetary system, a bureaucracy, a tax collection system. In addition, in a world of less that perfect free trade, the size of markets is affected by the size of political jurisdictions. In any model with increasing returns in the size of the market economy, and some barriers to international trade, income is increasing in the size of the country.[7] Also, a large country can provide insurance to its regions, needed because of the occurrence of regional idiosyncratic shocks.[8] Thus, in equilibrium the size of countries emerges from a trade-off. Large countries can take advantage of the benefits of size, but are less homogeneous since a larger population has preferences that are more diverse. As the size of a country increases, the per capita cost of government decreases, but the average distance from the government increases.

We now consider the role of international conflict and defense spending. Individual resources z_i are divided into two components:

$$z_i = y + e_i. \tag{2}$$

y is individual income (equal for everybody), which is *safe* from the consequences of conflict; e_i is the expected amount of resources of individual i after a (possible) international conflict is resolved.

Conflict is modeled as follows. Individuals are randomly matched pair-wise. When a pair (i,j) meets, the two individuals generate a pool of resources equal to $2e$ which has to be divided. There are two possible states: conflict (c) and no conflict (nc). In a state of nc resources are distributed peacefully and equally:

$$e_i = e_j = e. \tag{3}$$

We assume that:

A1. If two individuals who belong to the same country meet, they are always in a state of no conflict.

If two individuals, i and j, who do not belong to the same country meet, they can either be in conflict or in no conflict, in which case (3) applies. Conflict occurs with probability p_{ij}. The following assumptions generates a role for 'defense spending':

A2. If conflict occurs, the share of individual i depends on the defense spending of his country, relative to defense spending of the country of

[6] A more general specification would be to impose $K = \alpha + \beta s$ where s is the size of the country. As long as $\alpha > 0$ our results would be qualitatively unchanged. See Alesina et al. (2004a) for a model with this feature.

[7] See Alesina et al. (2000, 2004b) and Alesina and Spolaore (2003, Chapters 6 and 10) for more discussion on this point.

[8] See Sachs and Sala-i-Martin (1992) for an empirical discussion of regional insurance schemes in the United States.

individual j:

$$e_i = \frac{\psi(d_i)}{\psi(d_i) + \psi(d_j)} 2e, \tag{4}$$

where d_i (d_j) is the defense spending in the country of individual i (j) and $\psi' > 0$.

Assumption A1 rules out domestic conflict. In fact we could assume an additional cost for a country for internal ''law and order'' and conflict resolution (courts, legal system, etc.). If this costs were increasing in the size of the country, it would provide an additional argument for the costs of country size, in addition to the ''average distance'' argument emphasized above. If these costs were linear, our results would be completely unaffected. If these costs were decreasing in the size of countries (i.e., economies of scale in law and order) they would provide an additional benefit for large countries. The second assumption borrows from the literature on conflict resolution, and in particular from Tullock (1980) and Hirshleifer (1989, 1995). The idea is that the benefits for the citizens of a certain country in case of international conflict are increasing in the military strength of the country relative to the opponent.[9] The resolution of conflict in our model should not necessarily be interpreted as a ''war.'' The key point is the existence of a link between individuals' payoffs and the relative strength of their respective governments. Our specification is consistent with conflict resolution taking the form of violent confrontations, but can also be interpreted quite generally as military ''muscle flexing'' or the weight in international exchanges and bargaining tables arising from a country's relative strength.[10] Also the sources of potential conflict between individuals and groups belonging to different jurisdictions are modelled quite generally. They may stem from a trade relationship, or from conflicting interests on natural resources and/or other economic and noneconomic issues.[11] For tractability, we make two simplifying assumptions:

A3. $$\psi(d_i) = d_i. \tag{5}$$

A4. $$p_{ij} = p \quad \text{for every} \quad i, j. \tag{6}$$

Assumption A3 is an innocuous functional specification that simplies algebra without loss of generality. By contrast, assumption A4 introduces a drastic simplification, since it implies that all individuals have the same probability of being matched. Hence, it implies that the probability of a match is independent of

[9]We are assuming that spending on defense translates into military strength. Thus, we are abstracting from different 'productives' of defense spending in different countries.

[10]For a game-theoretical analysis of conflict resolution through war or peaceful bargaining in a different analytical setting see Alesina and Spolaore (2004).

[11]For a model of trade, conflict and political borders see Spolaore (2002).

the distance between the pair.[12] This assumption is made for analytical convenience but its realism might be questioned in a world with transportation costs or other obstacles to international exchanges. Assumption A4 will be relaxed in Section 7, where we present a more general setting in which matching probabilities are an explicit function of distance. Finally, the model could also be extended to incorporate explicit direct costs of fighting, in addition to the costs of defense spending, without any qualitative change in the results.

Suppose that the world is divided in N countries, indexed by h, of size s_h, $h = 1, \ldots N$. Then, the value of e_i is given by

$$e_i = [1 - (1 - s_h)p]e + (1 - s_h)p \sum_{h' \neq h} \frac{s_{h'}}{1 - s_h} \frac{d_h}{d_h + d_{h'}} 2e. \tag{7}$$

The first term in (7) represents the payoff of no conflict (e) multiplied by the likelihood of either not being matched with a foreigner, or being matched peacefully with a foreigner. The second terms represent the probability of being matched to a foreigner with conflict $[(1 - s_h)p]$ multiplied by the outcome of conflict, which depends on relative defense spending $d_h/(d_h + d_{h'})$ multiplied by the probability of meeting citizens of the various countries $s_{h'}/(1 - s_h)$. Finally, for country h, extending from borders $\underline{b}$ to $\bar{b}$ the budget constraint is given by

$$\int_{\underline{b}}^{\bar{b}} t_i d_i = K + d_h. \tag{8}$$

Eq. (8) indicates that the total tax revenues have to equal the total of non-defense spending (K) and defense spending (d_h).

3. Voting on government and defense

In this section we will consider equilibrium outcomes when individuals vote by majority rule on the location (type) of government and on the size of defense. We make the following two assumptions:

A5. Voting on the location of the government and the size of defense occurs after the country borders have been established.

This is natural since it implies that policy decisions on the type of government and the amount of defense spending can be taken only after a country is created.

A6. In each country, taxes are the same for everyone.

Two observations emerge immediately from the structure of the model:

[12] This assumption is analogous to the assumption of "panmictic matching" which is standard in the formal biological literature.

(i) For given borders, every citizen has the same preferences on the optimal amount of defense; and
(ii) The government is located in the middle of the country.

The first observation derives from the fact, embodied in (7), that every individual has the same probability of meeting a foreigner and all individual payoffs depend identically on the country's aggregate level of defense.[13] The second result derives from a straightforward application of the median voter theorem over the choice of location of the government, noting that this choice, by assumption, does not influence individuals' taxes, nor their desired amount of defense. Assumption A6 implies that taxes cannot be a function of the (unique) parameter which varies across individuals: the distance from the government. This assumption can be justified in two ways. First, to the extent that individual location captures a preference dimension, unobservability of preferences would imply that taxes linked to preferences are generally unfeasible.[14] Second, Alesina and Spolaore (1997) derived this assumption as a result of a realistic voting process on the distribution of the tax burden. With the same tax for everyone, individuals close to the government are better off than those far from it. If taxes were decided by majority vote, those individuals who are far from the government would favor tax compensation schemes in their favor. Such schemes might also enforce larger countries, by keeping border individuals "in," with tax advantages. Alesina and Spolaore (1997) consider linear taxation schemes, where the tax rate is a linear function of the distance from the government. They show that, under some weak assumptions, if voting on taxes occurs *after* country borders are decided (exactly as assumption A5 requires), then the voting equilibrium implies the same tax for everyone and the government located in the middle. The intuition is that for given borders, 50 percent of the voters (those with a distance from the government above average) would like to maximize compensations. The other half would want to minimize them. The tie is broken if one assumes even infinitesimal implementation costs of these transfer schemes. In summary, under realistic assumptions on the order of voting, a majority would favor equal taxes. Thus, A6 could be derived as a result, rather than imposed as an assumption. Since our focus here is not on compensation schemes, we simply impose A6 from thestart.[15]

With taxes equal for every citizen the budget constraint for country h of size s_h implies

$$t_h = \frac{K + d_h}{s_h}. \tag{9}$$

[13]This result would not hold if, for instance, individuals close to the borders had a higher change of engaging in conflicts with foreigners. In the latter case, border individuals would prefer a higher spending on defense. In Section 6 we will discuss a relaxation of our matching assumption such that individuals closer to the borders might in fact face a higher probability of non-peaceful conflict with foreigners.

[14]We do not explore here a connection with the literature on revelation mechanisms.

[15]See Le Breton and Weber (2003) and Alesina and Spolaore (2003, Chapter 4) for more discussion about the role of compensation schemes in preventing secessions.

Using (1), (7), and (9) we can derive the following first order condition which determines the desired amount of defense by each individual of country h:

$$\frac{1}{s_h} = p \sum_{h' \neq h} s_j \frac{d'_h}{(d_h + d'_h)^2} 2e. \tag{10}$$

Eq. (10) shows that the marginal costs of an extra unit of defense spending (equal to $1/s_h$ from (9)) must equal the marginal benefits, in terms of a higher 'prize' in case of conflicts, which is the second term in (10), obtained from (7).

4. The equilibrium number of countries

We now characterize an equilibrium number of countries when not only type of government, taxation and public spending but also borders are determined democratically. Unfortunately the assumption that political borders are determined via majority voting is unlikely to have held in actual societies through most of history, and is still far from reflecting actual border formation—although one would hope that it is a better approximation today than it would have been in the past. We derive this equilibrium as a useful benchmark to provide insights on the democratic formation and redrawing of borders.[16] The voting equilibrium can then be profitably compared with the perhaps realistic equilibrium solution developed in Section 5, in which taxation, public spending, defense and borders are the outcomes of decisions taken by rent-maximizing "Leviathans."

The first requirement which we impose on an equilibrium configuration of borders is the following:

Requirement 1. No individual (or group of individuals) can be forced to belong to an existing country if he prefers to belong to a different one.

This feature of equilibrium is a benchmark, in which countries cannot impose restrictions on individuals who want to join or exit. Remember that individuals are not physically mobile. Thus "joining a country" means moving the border of that country. This requirement implies a condition of indifference at the border: the individual at the border must be indifferent on the choice of which country to join.

Proposition 1. *Countries of equal size, with the government located in the middle and with the same amount of defense spending, satisfy Requirement 1.*

The proof is immediate, remembering that everyone pays the same tax. Proposition 1 does not imply that the only type of equilibria which satisfies Requirement 1 has countries of equal size. We return on this point in Section 5 in the context of our discussion of stability. Note, however, that given our assumption of

[16] By focusing on democratic decisions over borders, we are assuming that conflict is *not* about borders, but about economic and/or noneconomic issues that arise between groups belonging to different jurisdictions *after* borders have been determined. For a different model in which conflict is over territory see Alesina and Spolaore (2004).

uniform distribution of individuals, equilibria with equally-sized countries are the natural candidates. In turn, if all the countries have equal size, the natural candidates for an equilibrium is the symmetric one, with each country spending the same amount on defense. Using (10), the symmetric equilibrium with N countries of equal size s (so that $N = 1/s$) implies that each country spends d^* in defense:

$$d^* = \frac{s(1-s)pe}{2}. \tag{11}$$

Several observations are in order. First, the equilibrium amount of defense is increasing in the probability of conflict. Not surprisingly, it is also increasing in the amount of the payoff from conflict e. Second, defense spending is zero when there is only one country in the world, since, by definition, there is no conflict. Third, defense pro capita, which is

$$\frac{d^*}{s} = \frac{(1-s)pe}{2} \tag{12}$$

is decreasing with country size. Larger countries have, in equilibrium, a lower per capita defense bill. Fourth, since defense is, from the point of view of global efficiency, pure waste, individual utility would be maximized if $p = 0$ and $d^* = 0$. In fact, if $p > 0$ and $d_h = d^*$ for every country using (7) $e_i = e$ for every i, exactly as in the case of $p = 0$. When $p > 0$, however, the "price" of the payoff e is the per capita cost of defense given in (12). Obviously, we have a suboptimal Nash equilibrium on defense spending.

Empirically, the relationship between country size and defense per capita is influenced by two critical factors which our model does not capture: the existence of military alliances, and the fact that smaller members of an alliance can free ride on the defense capabilities of the larger member(s).[17] Both considerations are very important, and we do not mean to downplay them. However, we see our model as a stepping stone upon which to build these additional realistic features.

We now consider border redrawing by majority rule. We allow the existing countries to eliminate a country or create a new country if the border change is approved by majority rule in each of the countries whose borders are affected by the change. We restrict voting only on proposals of border redrawing which satisfy Requirement 1.

Requirement 2. Given a configuration of countries that satisfies Requirement 1, in at least one country a majority should oppose any proposal to redraw borders so that a new country is created or eliminated. Only proposals that satisfy Requirement 1 are admissible.

In other words, this requirement implies that in an equilibrium at least one country would veto any border rearrangement. Thus, we are trying to capture situations like referenda on the choice of joining politico-economic unions (like the European Union), or post-war international conferences to settle border disputes,

[17]For a survey of the literature on this point, see Sandler and Hartley (1995).

or, more generally, border rearrangements which are the result of some form of international agreement rather than the result of unilateral secessions, which we study in the next section.

Proposition 2. *The number of equally-sized countries which satisfies Requirement 2 is given by the integer that is closest to*

$$\sqrt{\frac{g-pe}{2K}}. \tag{13}$$

Proof. See Appendix.

For the sake of simplicity in exposition, from now we will abstract from the integer condition and assume that $\widetilde{N} \equiv \sqrt{(g-pe)/2K}$ is the equilibrium number of countries.

Corollary. *The equilibrium number of countries is decreasing with the probability of conflict p.*

This is one of the critical results of the paper: it implies that a sharp decrease in the probability of conflict would result in the break-up of countries. Two forces underlie this inverse relationship between p and $\widetilde{N}$. First, if p increases, an individual would like to belong to a larger country in order to reduce the probability of "being matched" with foreigners. Second, since defense spending increases in p and defense per capita is decreasing in larger countries, the benefits of size increase. If we view the end of the cold war as a large drop in p, the model predicts that the creation of new countries should go hand in hand with the end of the cold war.

One could expect that a large fall in the probability of conflict p (e.g., the "end of the cold war"), should bring about a more peaceful world and a "peace dividend," namely a reduction in the per capita cost of defense. However, the emergence of several local conflicts cast doubt on the first implication, and the "peace divident" has been slow in materializing. Our model is consistent with both these rather sad observations.

Let's begin with the amount of world conflict. Define the mass of observable conflict M. We have that

$$M(p) = p(1-\widetilde{s}(p)), \tag{14}$$

where $\widetilde{s}(p)$ is the equilibrium size of countries. Thus, from Proposition 2

$$\widetilde{s}(p) = \sqrt{\frac{2K}{g-pe}}. \tag{15}$$

Proposition 3. *The mass of international conflicts is increasing in p, if and only if*:

$$\widetilde{s}(p) < \frac{g-pe}{g-pe/2}. \tag{16}$$

Proof. From (14)

$$\frac{\mathrm{d}M(p)}{\mathrm{d}p} = 1 - \widetilde{s}(p) - p\,\frac{\mathrm{d}\widetilde{s}(p)}{\mathrm{d}p}, \tag{17}$$

substitute $\widetilde{s}(p)$ and $\mathrm{d}\widetilde{s}(p)/\mathrm{d}p$ using (15) to obtain (16). □

The intuition of Proposition 3 is that a reduction in p has two effects. For a given size of countries, it reduces the mass of international conflict. This direct effect is larger the smaller is s, namely the larger the "mass" of international "matches," relative to domestic "matches." The second, and indirect effect, is that a reduction in p reduces the size of countries, thus it increases the mass of international interactions that can, potentially, lead to conflict. As Eq. (16) shows, for $\widetilde{s}$ small the direct effect dominates, for $\widetilde{s}$ large it does not. Therefore, starting from a world with a few large countries, a reduction in p which leads to the formation of many new countries may actually increase the mass of observed conflicts.

A similar intuition underlies the effect of a reduction of p on defense spending per capita. From (12) it follows that

$$\frac{\partial(d^*/s)}{\partial p} = \frac{(1-s)e}{2} - \frac{pe}{2}\frac{\mathrm{d}s}{\mathrm{d}p}. \tag{18}$$

This first term is the direct positive effect of a change in p on defense per capita: a lower p leads to lower defense. The second term, with the opposite sign, is the indirect effect due to the consequences of a change in p on the size of countries. Eq. (18) leads to the following:

Proposition 4. *A reduction in p determines a reduction of defense per capita, if and only if*

$$\widetilde{s}(p) < \frac{g - pe}{g - pe/2}. \tag{19}$$

Thus, a reduction of p may actually lead to an increase in defense spending per capita because countries become smaller. More generally, even when (19) holds, so that lower p means lower defense, the model emphasizes a channel (through the size of countries) which reduces the effect of p on d^*/s.

5. Stability

In this section, we consider the issue of stability of the equilibrium both to small perturbations and to unilateral secessions. We begin perturbations in which one border is moved slightly, so that a small mass of population changes country.

Requirement 3. (ε-stability)**:** A configuration of countries is stable if after a small ε perturbation of the border between two countries, the original equilibrium is re-established. Consider a configuration of N countries of equal size. We know that this configuration of borders satisfies Requirement 1. But is it stable? Suppose, without loss of generality, that starting from the equilibrium with N countries of size $s = 1/N$, country 1 is reduced to size $s'_1 = s - \varepsilon$ and country 2 to size $s'_2 = s - \varepsilon$. Would the original equilibrium be restored? Namely, would the mass of individual ε want to return to country 1? When the size of two countries changes the following adjustments occur:

(i) the type (location) of government in countries 1 and 2;
(ii) the defense spending of countries 1 and 2; and
(iii) the defense spending of all the other $(N-2)$ countries.[18]

Intuitively, the third adjustment is "second order," particularly for N large. In fact, looking at the first order condition (Eq. (10)) one immediately verifies that the $(N-2)$ countries not affected by the border perturbation change their choice only marginally because the sizes of the other two countries have been reshuffled. We can obtain analytical results under the simplifying assumption that the third type of adjustment is zero, namely the $(N-2)$ countries not affected by the border change maintain their defense spending fixed. In this case we can show the following:

Proposition 5. *Consider a configuration of N equally sized countries. If all the $(N-2)$ countries not affected by the border change maintain their level of defense fixed, the smallest size of countries which is stable $\hat{s}$, is a function of p, $\hat{s}(p)$ such that:*

$$\hat{s}(0) \leqslant \hat{s}(p) \quad p > 0$$
$$\frac{\partial \hat{s}(p)}{\partial p} \geqslant 0. \tag{20}$$

Proof. See Appendix A.

Thus, the minimum stable size is (weakly) increasing in p. If p is high, small countries are not stable, in the sense that if a perturbation makes one country larger, the citizens of the neighboring smaller countries would want to join the bigger country, because defense is too expensive in the smaller country. Thus, the implication of Proposition 5 is that, even leaving aside Requirement 2, the minimum stable size of countries (weakly) increases in p: with a higher probability of conflict, the minimum stable size of countries is larger.[19]

We can now extend our analysis of Requirement 2 to the case where we explicitly take into account the issue of ε-stability. Namely we modify Requirement 2 as follows:

Requirement 2′. Given a configuration of countries that satisfies Requirement 3, in at least one country a majority should oppose any proposal to redraw borders so that a new country is created or eliminated. Only proposals which satisfy Requirements 1 and 3 are admissible.

[18]Note that the location-type of government in these $(n-2)$ countries does not change, because their borders do not change.

[19]Of course, it is possible that small increases in p leave the equilibrium size unchanged.

Numerical simulations described in Appendix A show the following.

Results.

(a) The maximum number of equally-sized countries that satisfies Requirement 3 is decreasing in p.
(b) The number of equally sized countries N that satisfies Requirement 2′ is the integer closes to $\sqrt{(g-pe)/2K}$ (or, for small values of pe, the largest integer smaller than $\sqrt{(g-pe)/2K}$).

For large values of $\sqrt{(g-pe)/2K}$, we can ignore the integer condition and approximate the number of countries that satisfies Requirement 2′ by $\widetilde{N} \equiv \sqrt{(g-pe)/2K}$. This number, therefore, not only approximates the equilibrium number of countries that satisfies Requirements 1 and 2, but also the stable number of countries that satisfies Requirements 3 and 2′.

We now turn to the issue of stability to unilateral secessions. A unilateral secession occurs when a (connected) set of individuals belonging to an existing country unilaterally forms a new country.

Definition. A country of size s is secession free, if no group of citizens would want to unilaterally break away and form a new country.

First of all, note that if a secession occurs, three adjustments have to occur:

(i) the location of government and defense spending of the new country;
(ii) the location of government and the defense spending of the country which has been split, and
(iii) the defense spending of all the other countries not affected by the secession.

While for the case of the ε-stability the third adjustment was "second order," this is not the case for (potentially large) unilateral secessions. Thus, we cannot derive results analogous to Proposition 5.

Note that the individuals with the highest incentives to break away are those located far away from the government near the borders of the original country. In fact, for given country size, and given size of a secession, those who gain the most are those who were far from the original government and are much closer to the government of the new country. Clearly, the possibility of secession imposes an upper limit on country size. If a country is too large, a fraction of its citizens at the border would break away because they are so far from the government that they are willing to bear the costs of a higher defense bill and total tax per capita, and lower total defense in a conflictual world.

The first question which we ask is whether the configuration of $\widetilde{N}$ countries of equal size $\widetilde{s} = 1/\widetilde{N}$ is secession free. Numerical simulations described in the Appendix show the following results:

Result. The size $\widetilde{s}$ is secession free.

The simulations imply a search over a grid of values for possible secession sizes. We then check whether or not all the individuals in the proposed break-away region are better off after the secession. A country is secession free if we cannot find a size for a possible secession in which all the individuals who are breaking away are better off. Note that this procedure implies (rather realistically) that nobody can be forced to unilaterally break away from an existing country against his will.

A second interesting question is the following. Leaving aside Requirement 2, what is the relationship between p and the minimum size which is secession free? By numerical simulations, described in Appendix we obtain the following:

Result. The maximum size that is secession free is increasing in p.

In other words, when the probability of a conflict increases, larger countries that would not have been secession-free for a smaller p become secession free. Conversely, a sharp drop in p would induce certain regions to secede, which would not have seceded with a higher p. Therefore, even leaving aside the equilibrium number of countries $\widetilde{N}$ obtained applying Requirement 2, this result on secessions establishes that one should observe secessions when, for whatever reason, the probability of conflict decreases.

This observation is quite important because it underlies the generality of the direct relationship between country size and probability of international conflict. In fact, this relationship emerges simply as a result of the secession-free requirement, regardless of any other requirement which (like Requirement 2) identifies a specific equilibrium number of countries.

6. Conflict in a world of Leviathans

In reality political borders are rarely determined democratically. Moreover, for most of history decisions over defense spending, types of governments, taxation, wars and conflict have not been taken by democratic "median voters" but by all sorts of nondemocratic rulers (kings, emperors, dictators, etc.). In this section we derive a model of conflict and size of countries in a world of rent-seeking "Leviathans" that generalize our previous results.

As before, individuals are uniformly distributed over the segment [0,1], and have utility equal to (1). As above, we assume that each individual pays the same tax t. That is, Leviathans are unable to tax individuals as a function of their preferences over the type of government.[20] Therefore, a Leviathan ruling a country of size s_h and spending d_h in defense obtains net rents equal to

$$R_h \equiv ts_h - K - d_h. \tag{21}$$

We assume that each Leviathan must guarantee at least a utility a level of u_0 to a fraction of δ of his citizens. The parameter δ captures the idea that no ruler can keep

[20] An interesting topic, which we do not develop here, is whether Leviathans would use compensation schemes, if available.

power without the support of at least a fraction of its subjects. A dictatorial ruler would need a low δ to maintain power. In other words, δ measures the degree to which subjects' preferences constrain the ruler's decisions (i.e., the degree of "democratic accountability" of the Leviathan). Without loss of generality, we will assume that a Leviathan faced with a $\delta > 0$ will locate the government in the middle of the country.[21] Therefore, a Leviathan who rules a country of size s_h faces the following constraint:

$$z_i - t - \frac{g\delta s_h}{2} \geqslant u_0, \tag{22}$$

where z_i is income per capita in country h. This means that the utility of an individual at a distance equal to $\delta s_h/2$ from the government must be at least as high as u_0. Clearly, if the above constraint is satisfied, at least δs_h individuals have a utility larger or equal to u_0 , and the Leviathan's constraint is satisfied.

Clearly the Leviathan will not choose to increase welfare of its citizens above the minimum level that satisfies the constraint (22). Any additional welfare would come at the expense of the Leviathan's rents. Thus Eq. (22) will hold with equality. Substituting this equation holding with equality into (21) one obtains:

$$R_h = z_i s_h - \frac{g\delta s_h^2}{2} - u_0 s_h - K - d_h. \tag{23}$$

As before, we assume that each government chooses defense spending after political borders have been formed, and taking other countries' defense spending as given.[22] However now, unlike in the previous model, defense is not chosen by voters but by Leviathans in order to maximize their net rents. Nonetheless, *for given borders* Leviathans will choose the same level of defense spending that would be chosen through direct democracy. In fact, since $z_i = y + e_i$ where e_i is given by Eq. (7), it is immediate to check that the first-order condition for the maximization of (23) given (7) is identical to (10). In particular, in a symmetric world in which all countries have equal size s, defense in each country is given as follows:

$$\frac{\widetilde{d}}{s} = \frac{(1-2)pe}{2}. \tag{24}$$

But what is the equilibrium number and size of countries in a world Leviathans? In what follows we will assume that borders are determined in order to maximize government's joint net revenues. In this respect we follow Friedman (1977) who argues that this is a reasonable assumption to predict long-run equilibrium borders in a world of rent-maximizing governments. Although explicit cooperative behavior by rulers may be rare (and mainly limited to those areas of the world run by related aristocracies or homogeneous nomenclatures), one may expect that even non-cooperative behavior may, under appropriate assumptions, lead to such a solution.

[21] Formally, if b denotes the location of the "Eastern border," the Leviathan will maximize his rents by locating the government in the interval $[b + \delta s_h/2, b + s_h - \delta s_h/2]$, which always includes the middle point $b + s_h 2$ for $0 \leqslant \delta \leqslant 1$, and includes *only* the middle point for $\delta = 1$.

[22] Therefore we maintain the assumption that conflict is not directly over borders.

The idea is that—through peaceful bargaining, war, dynastic alliances, etc.—each portion of land and population will be eventually allocated to the Leviathan who values it more—i.e., to the Leviathan who obtains the highest net revenues from it.

In a world of Leviathans in which all countries have equal size (a necessary condition for joint maximization) total rents are

$$R = y + e - u_0 - g\,\frac{\delta s}{2} - \frac{(1-s)pe}{2} - \frac{k}{s}. \tag{25}$$

Hence, the equilibrium size is[23]

$$s_\delta = \sqrt{\frac{2K}{g\delta - pe}} \tag{26}$$

and the number of countries is

$$N_\delta = \frac{1}{s_\delta}. \tag{27}$$

Note that as above the higher the probability of conflict (p) the larger the size of countries with Leviathans, as in the case of democracies. Also the higher δ (more "democratic" Leviathans) means smaller countries. It is important to notice that for $\delta = 1$ (maximum "democratic" accountability) *the Leviathan solution coincides with the democratic equilibrium in which borders themselves are determined via majority voting*. The fact that the same solution is obtained in two very different politico-economic frameworks highlights the robustness of our previous results.

As already noticed, the qualitative effects of p on the size of nations is the same as in the democratic model: all other things being equal, more conflict leads to larger countries. But how do different degrees of "democratization" (i.e., different δ's) affect the *impact* of p on the size of countries? In fact we have that

$$\frac{\partial s_\delta}{\partial \delta \partial p} < 0 \tag{28}$$

which means that, at higher levels of conflict, democratization has small effects on the size of countries and vice versa. In other words, in a world of high conflict democratization is "less important" in reducing the size of countries, and in a world of more widespread democracy, conflict is less important in determining the size of countries.

An interesting extension would make p a decreasing function of δ. This would capture rudimentarily the concept of "democratic peace." The larger is δ the closer a Leviathan is to a democracy. In this case an increase in δ (democratization) would reduce the size of countries for two reasons: a direct effect (democratization) and an indirect effect through a reduction of the probability of wars, p.

[23] Again, we abstract from the fact that $N_\delta = 1/s_\delta$ must be a positive integer.

7. Other extensions

Our basic model is highly stylized and simplified in many dimensions. In the previous section we have seen how our results—which we derived in a democratic framework—can be extended to allow for decisions by nondemocratic (or imperfectly democratic) Leviathans. In this section we will discuss two other extensions. First, we will relax the assumption that the probability of a match between individuals is not a function of their relative distance. In particular, we will present a simple extension in which the importance of distance is captured by one parameter (γ) and show how our results are robust to such extension. In the last part of this section we will briefly discuss the implications of relaxing another simplifying assumption, that is, uniform distribution of individuals on the geographical/ preference space.

7.1. Distance-dependent matching

So far we have abstracted from possible links between matching probabilities and relative geographical/ideological location. At one extreme, one may think that only individuals that are "close" to each other meet. At the other extreme, individuals may face substantial chances of interacting with people who are "far" from themselves, both geographically and ideologically. One can view our assumption of random matching as a stylized version of a world where "higher-distance" matchings occur with significant probabilities. This seems a useful first approximation since it allows us to focus, with a minimum of analytical complication, on the case for which conflict and defense are more likely to play an important role. However, it is useful to check whether the insights obtained under our extreme assumption of distance-dependent matchings carry on to more realistic settings in which individuals are more likely to meet people who are "close."

Specifically, assume that a match only occurs between individuals located within a distance $\gamma/2$ from each other. In other words, the parameter γ captures the importance of geographical/ideological distance in the matching progress. At one extreme ($\gamma = 0$) only identical individuals meet each other. In such world, conflict and defense would play no role. At the other extreme ($\gamma = 1$), individuals are as likely to meet close individuals and very distant "strangers." This is the world studied in the previous sections. In what follows we will consider the general case $0 < \gamma \leqslant 1$.[24]

In our previous analysis we showed how the Leviathan solution for $\delta = 1$ coincides with the voting equilibrium. In this section, to simplify algebra and

[24]To avoid excessive algebraical complications associated with asymmetries between the middle and the tails of the space distribution when relative distance matters, we will also assume that the interval [0,1] is mapped—both geographically and ideologically—on a circle of unit perimeter. In other words, without loss of generality, we will consider a circular version of our spatial model, therefore maintaining symmetry across individuals. For more details see Appendix.

notation, we will consider the Leviathan case with maximum democracy ($\delta = 1$).[25] The analysis can be easily extended to the general case $0 \leqslant \delta \leqslant 1$.[26]

When matching is a function of distance, our main results can now be summarized as follows.[27] Equilibrium defense per capita is now given by

$$\frac{d^*}{s} = \frac{p\gamma e}{4} \quad \text{for} \quad \frac{\gamma}{2} \leqslant s, \tag{29a}$$

$$\frac{d^*}{s} = \frac{p(\gamma - s)e}{2} \quad \text{for} \quad \frac{\gamma}{2} > s, \tag{29b}$$

which reduces to our previous result (12) for $\gamma = 1$. The intuition for the above Eq. (29a)–(29b) is straightforward: as the probability of having potential conflicts with distant individuals increase (higher γ), so does defense per capita.

As shown in the Appendix, the equilibrium size of countries is now given as follows:

$$s_\gamma = \sqrt{\frac{2K}{g}} \quad \text{for} \quad \gamma < 2\sqrt{\frac{2K}{g}}, \tag{30a}$$

$$s_\gamma = \frac{\gamma}{2} \quad \text{for} \quad 2\sqrt{\frac{2K}{g}} < \gamma < 2\sqrt{\frac{2K}{g - pe}}, \tag{30b}$$

$$s_\gamma = \sqrt{\frac{2K}{g - pe}} \quad \text{for} \quad 2\sqrt{\frac{2K}{g - pe}} < \gamma. \tag{30c}$$

When γ is low, distance is a major obstacle to interactions: people tend to meet mostly people within their own jurisdictions, and conflict plays on role in the determination of the size of countries. By contrast, at higher levels of γ, international conflict becomes more important, and directly affects the number and size of countries. Hence, the size of countries is (weakly) increasing in p—and strictly increasing for high levels of γ—qualifying and confirming our previous results. Moreover, since a higher γ means that conflict and defense are more important, the number of countries is also (weakly) increasing in γ. Of course, the functional form of the relationship between the number of countries and the "importance of distance" (inversely related to γ) depends on the specification we have chosen in order to obtain simple, closed-form solutions. But the two key messages of this extension seem to be pretty general and robust: (a) one should expect a positive relationship between p and defense—this relationship is likely to be *stronger* when distance is *less* important (high γ in our specification) and (b) one should also expect

[25]In our previous analysis we showed how the Leviathan solution for $\delta = 1$ coincided with the voting equilibrium.

[26]The details of the derivation are available upon request.

[27]Derivations are in Appendix A.

a positive relationship between size of countries and likelihood of interactions with (geographically and ideologically) distant individuals.

7.2. Nonuniform distributions

The assumption of a uniform distribution of individuals can also be relaxed, but at the price of major algebraical complications. A key advantage of maintaining the uniform-distribution assumption is that it allows the derivation of simple closed-form solutions for the equilibrium number and size of countries. In a world where countries are symmetric and all have equal size, comparative statics is highly simplified: one can talk about *the* size of countries, rather than having to trace complex changes in *the size distribution* of countries. By contrast, nonuniform distributions usually imply that different countries will have different sizes, different levels of aggregate defense and/or defense per capita, etc. Hence obtaining closed-formed solutions will be pretty elusive, and one will have to resort to analytical simulations in order to obtain comparative statics results. However, the main message of our paper (a positive relationship between probability of conflict and size of countries, and its "general equilibrium" implications) will tend to carry on to more general settings. In what follows we will illustrate this point by briefly discussing an example of nonuniform distribution.

Assume that individuals are *not* distributed uniformly on the interval [0,1]. Without much loss of generality, assume that the density function $f(x)$ is increasing over [0,1]. In particular, assume

$$f(x) = 2x \tag{31}$$

which implies a cumulative distribution function $F(x) = \int_0^x f(z)\,\mathrm{d}z = x^2$. Hence, the world population, as before, has mass 1, but half of it is (nonuniformly) located between 0 and $1/\sqrt{2} \simeq 0.71$ while the other half is (nonuniformly) located between 0.71 and 1.

Hence, one can see that countries of equal size would not satisfy requirement 1, even in the absence of conflict. For example, consider the case $p = 0$ and $s = \frac{1}{2}$ (that is, two equal-size country, one "on the left" and one "on the right"). In a democracy in which the type of government is chosen through majority voting, the left country's government would be located at $F(m_{\mathrm{L}}) = m_{\mathrm{L}}^2 = \frac{1}{4}$—that is, $m_{\mathrm{L}} = \frac{1}{2}$, while the right country's government would be located at $F(m_{\mathrm{R}}) = m_{\mathrm{R}}^2 = 1 - \frac{1}{4}$—that is, $m_{\mathrm{R}} = \sqrt{3}/2 \simeq 0.87$. Since taxes per capita would be the same in the two countries, the individual at the border (i.e., at location $1/\sqrt{2} \simeq 0.71$) would have higher utility in the "right" country, whose government is closer, therefore violating Requirement 1. The conditions to satisfy Requirement 1 in a two-country world would imply countries of different size. Specifically, in the absence of conflict, Requirement 1 is satisfied, if and only if the following holds:

$$g(b - m_{\mathrm{L}}) + \frac{K}{s_{\mathrm{L}}} = g(m_{\mathrm{R}} - b) + \frac{K}{s_{\mathrm{R}}}, \tag{32}$$

where

$$F(m_L) = m_L^2 = s_L/2, \tag{33}$$

$$F(m_R) = m_R^2 = 1 - s_R/2, \tag{34}$$

$$F(b) = b^2 = s_L \tag{35}$$

and

$$s_L + s_R = 1. \tag{36}$$

Eqs. (32)–(36) simultaneously determine the four variables s_L, s_R, m_L, m_R and b, where s_L (s_R) is the size of the left (right) country, m_L (m_R) is the location of the left (right) government, and b is the location of the border. Analogous equations can be obtained for the case of three countries, four countries, etc.

What if conflict is introduced? Then defense spending can be obtained for countries of different sizes. For example, in the case of two countries of size $s_L = s$ and $s_R = 1 - s$, respectively and returning to the case of random matching ($\gamma = 1$), we have the following first-order conditions for, respectively, d_L and d_R:

$$\frac{d_R}{(d_L + d_R)^2}\, ep(1 - s) = \frac{1}{s} \tag{37}$$

and

$$\frac{d_L}{(d_L + d_R)^2}\, eps = \frac{1}{1 - s} \tag{38}$$

which implies

$$\frac{d_L}{s_L} = \frac{eps_R}{2} \tag{39}$$

and

$$\frac{d_R}{s_R} = \frac{eps_L}{2}. \tag{40}$$

Hence, the smaller country will have a larger defense per capita, and the larger country will have smaller defense per capita, confirming the existence of economies of scale in defense for this asymmetric case.[28] The equilibrium condition (32) extend to the case with conflict as follows:

$$g(b - m_L) + \frac{eps_R}{2} + \frac{K}{s_L} = g(m_R - b) + \frac{eps_L}{2} + \frac{K}{s_R}. \tag{41}$$

Analogous conditions can be obtained for $N = 3, 4$, etc.

By the same token, the analysis can be extended to derive the equilibrium conditions that would satisfy Requirement 2. Of course, in general one cannot be obtained in this extended setting, but analytical simulations could be used to

[28] On the other hand, *aggregate* defense spending is the same in both countries.

compare different configurations of borders. All other things being equal, voters would prefer smaller countries at lower levels of p.[29] This example illustrate how the key insights from the analysis carry on to more general distributions of individuals over the geographical/idelogical space. Analogously, the analysis could be extended to deal with the case of Leviathans and/or with the case of distance-dependent matchings.

8. Concluding remarks

This paper provides a model consistent with three observations. First, secessions and, more generally, breakup of countries should go hand in hand with a reduction of international conflict. Second, the number of conflicts among small countries may go up, as a result of the breakup of previous larger political units. Third, the size of the 'peace dividend' is influenced by the process of country breakup which follows the reduction in the likelihood of international conflict.

While these implications of the model appear quite consistent with recent events, we should emphasize several necessary extensions of our approach. First and foremost, we have ignored the role of alliances and the related problem of free riding in defense spending by smaller numbers. To some extent, one can reinterpret the 'country' of our model as a group of allied countries, and view our model of country formation as a model of alliance formation. However, the analogy can be pushed only to a point, because critical issues of bargaining (and free riding) amongst allied countries cannot be addressed in our model. Second, in our basic model of conflict we assumed that the probability of conflict amongst citizens of different countries is the same around the world. We have also considered extensions in which the probability of conflict between groups of individuals may depend on their geographical/ideological distance. Further extensions linking the probability of conflict between groups to their political, economic and social characteristics are left for further research. Third, our assumption concerning the identity between the geographic and preference dimension excludes the consideration of ethnic minorities. In the context of our model, in fact, an ethnic minority could be viewed as a group of individuals with preferences very different from those of individuals on their right and left geographically. In reality, the existence of ethnic minorities is a critical determinant of both country formation and secessions and of regional conflicts. More generally, focusing on continuous distributions of individual characteristics allows us to study some important and robust relationships between conflict, defense and country size with the minimum amount of notational and analytical complexity. However, additional insights can probably be obtained by extending and modifying

[29]For example, this can be illustrated by comparing the two-country case, studied above, with the one-country case. For all individuals, the net benefits from a breakup are decreasing in p. Hence, assuming that a majority of voters prefer the one-country solution to the two-country solution for $p = 0$, for an appropriate choice of parameters g and K there must exist a p low enough to ensure that a majority of voters will be in favor of country breakup.

the basic framework in order to allow for asymmetries and discontinuities. These extensions are left for future research.

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Appendix A

Proof of Proposition 2. Let $l_{ih}\{N\}$ denote the distance of individual I, belonging to country h, from his country's government when the world is divided in N countries of equal size $s = 1/N$, and let $t_{ih}\{N\}$ denote the taxes paid by i. Each individual i in country h will prefer N to $N+1$ countries, if and only if:

$$t_{ih}\{N\} + gl_{ih}\{N\} \leqslant t_{ih'}\{N+1\} + gl_{ih'}\{N+1\}. \tag{A.1}$$

That is

$$t_{ih'}\{N\} + t_{ih}\{N\} \geqslant g[l_{ih'}\{N\} - l_{ih'}\{N+1\}]. \tag{A.2}$$

Since

$$t_{ih}\{N\} = NK + \left(1 - \frac{1}{n}\right)\frac{pe}{2}, \tag{A.3}$$

$$t_{ih}\{N+1\} = (N+1)K + \left(1 - \frac{1}{N+1}\right)\frac{pe}{2} \tag{A.4}$$

we have that, for every individual i in each country h

$$t_{ih'}\{N+1\} - t_{ih}\{N\} = K + \frac{pe}{2N(N+1)}. \tag{A.5}$$

Denote with $l_{mh}\{N\} - l_{mn'}\{N+1\}$ the median distance change in country h. It can be shown[30] that the median distance change is the same in all countries, and equal to

$$l_m\{N\} - l_m\{N+1\} = \frac{1}{2N(N+1)}. \tag{A.6}$$

[30]A proof by construction is provided in Alesina and Spolaore (1997, Appendix, Lemma 1).

Hence, N is always preferred to $N+1$ in all countries, if and only if

$$K+\frac{pe}{2N(N+1)}\geqslant\frac{g}{2N(N+1)}, \tag{A.7}$$

which is satisfied if and only if

$$N(N+1)\geqslant\frac{g-pe}{2K}. \tag{A.8}$$

Analogously, each individual i is country h will prefer N to $N-1$ if and only if

$$t_{ih}\{N\}+gl_{ih'}\{N-1\}\leqslant t_{ih}\{N-1\}+g[l_{ih'}\{N-1\}-l_{ih}\{N\}]. \tag{A.9}$$

That is

$$t_{ih}\{N\}-t_{ih'}\{N-1\}\leqslant g[l_{ih'}\{N-1\}-l_{ih}\{N\}]. \tag{A.10}$$

It is immediate to verify that, for every i in each country h, we have

$$t_{ih}\{N\}-t_{ih'}\{N-1\}\leqslant K+\frac{pe}{2N(N-1)}. \tag{A.11}$$

N will satisfy Requirement 2 only if there exists at least one country h in which

$$K+\frac{pe}{2N(N+1)}\leqslant g[l_{mih'}\{N-1\}-l_{mh}\{N\}]. \tag{A.12}$$

It can be shown[31] that, for any N, the maximum median distance change $l_{mih'}\{N-1\}-l_{mh}\{N\}$ is given by

$$l_{mih'}\{N-1\}-l_{mh}\{N\}=\frac{1}{2N(N-1)}. \tag{A.13}$$

Therefore, there exists no country h in which the median voter prefers $N-1$ to N is and only if

$$K+\frac{pe}{2N(N+1)}\leqslant\frac{g}{2N(N-1)}, \tag{A.14}$$

which is satisfied if and only if

$$N(N-1)\leqslant\frac{g-pe}{2K}. \tag{A.15}$$

Hence, N satisfied Requirement 2 if and only if it satisfies both (A.8) and (A.15). The integer that satisfies both equations is the integer closest to

$$\sqrt{\frac{g-pe}{2K}}. \qquad \square$$

Proof of Proposition 5. In order to prove Proposition 5 we first need to prove a Lemma. Define as d_1' and d_2' the defense spending in countries 1 and 2 after the

[31] A proof by construction is provided in Alesina and Spolaore (1997, Appendix, Lemma 2).

ε-perturbation. Remember that $s_1' = s - \varepsilon, s_2' = s + \varepsilon$, and that we are assuming that $d_i,\ i = 3, \ldots, N$, remain unchanged, so that $d_i = s(1-s)pe/2 \equiv d$.

Lemma.

$$d_1' d_2'; \quad \frac{d_1'}{s_1'} > \frac{d_{21}'}{s_2'} > 0.$$

Proof. Using the first order conditions (10), after some algebra, one obtains the following system of equations in d_1' and d_2':

$$(s+\varepsilon)\left[(1-2s)\frac{d}{(d_2'+d)^2} + (s-\varepsilon)\frac{d_1'}{(d_1'+d_2')^2}\right] = \frac{1}{2pe}, \tag{A.16}$$

$$(s-\varepsilon)\left[(1-2s)\frac{d}{(d_2'+d)^2} + (s+\varepsilon)\frac{d_1'}{(d_1'+d_2')^2}\right] = \frac{1}{2pe}. \tag{A.17}$$

Differentiate both sides with respect to ε, noting that the right-hand size of (A.16) and (A.17) are a constant. Then evaluate the two expressions at $\varepsilon = 0$, noting that $\varepsilon = 0$, $d_1' = d_2' = d$. We obtain

$$\left[\frac{1-2s}{4d} + \frac{s}{4d}\right] + s\left[-\frac{(1-2s)}{4d^2}\frac{\partial d_2'}{\partial \varepsilon} - \frac{1}{4d} - s\,\frac{\frac{\partial d_2'}{\partial \varepsilon}}{4d^2}\right] = 0, \tag{A.18}$$

$$\left[-\frac{1-2s}{4d} + \frac{s}{4d}\right] + s\left[-\frac{(1-2s)}{4d^2}\frac{\partial d_1'}{\partial \varepsilon} - \frac{1}{4d} - s\,\frac{\frac{\partial d_1'}{\partial \varepsilon}}{4d^2}\right] = 0. \tag{A.19}$$

Solving, one obtains:

$$\frac{\partial d_2'}{\partial \varepsilon} = \frac{1-2s}{1-s}\frac{d}{s}, \tag{A.20}$$

$$\frac{\partial d_1'}{\partial \varepsilon} = \frac{1-2s}{1-s}\frac{d}{s}. \tag{A.21}$$

For $s < 1/2$ (A.20) is positive and (A.21) is negative. Using (A.20) and (A.21), and evaluating at $\varepsilon = 0$ one obtains:

$$\frac{\partial d_2'/s_2'}{\partial \varepsilon} = \frac{1}{s+\varepsilon}\left[\frac{\partial d_2'}{\partial \varepsilon} - \frac{d_2'}{s+\varepsilon}\right] = -\frac{d}{s(1-s)} = -\frac{pe}{2} < 0, \tag{A.22}$$

$$\frac{\partial d_1'/s_1'}{\partial \varepsilon} = \frac{1}{s-\varepsilon}\left[\frac{\partial d_1'}{\partial \varepsilon} - \frac{d_1'}{s+\varepsilon}\right] = \frac{d}{s(1-s)} = -\frac{pe}{2} < 0. \tag{A.23}$$

Eqs. (A.20), (A.21), (A.22) and (A.23) imply the Lemma. □

In words, for a small perturbation of the border between two countries, total defense is larger in the larger country, but defense per capita, thus taxes per capita are higher in the smaller country. We are now ready to prove the Proposition.

Define $\widetilde{s}(0)$ as the minimum stable size, for $p = 0$. From the results of Alesina and Spolaore (1997) we know that for $p = 0$, $\widetilde{s}(0) = 1/\widetilde{N}$, where $\widetilde{N}$ is the largest integer smaller than $\sqrt{g/2K}$. First, we want to prove that $\widetilde{s}(p)\widetilde{s}(0)$ for any $p>0$. If $p = 0$ the condition for stability can be written as follows:

$$g\left(\frac{s-\varepsilon}{2}\right) - \frac{K}{s-\varepsilon} < g\left(\frac{s+\varepsilon}{2}\right) + \frac{K}{s+\varepsilon}. \tag{A.24}$$

Suppose now that $\hat{s}(p) < \hat{s}(0)$. For some $p>0$, choose s' such that $\hat{s}(p) < s' < \hat{s}(0)$. Consider the ε perturbation when $s = s'$. Since $s' < \hat{s}(0)$ if $p = 0$ the individual at the new border prefers the bigger country. Since $s' > \hat{s}(p)$ for $p>0$ the border individual prefers the smaller country. This is a *contradiction*, since if $p>0$ defense per capita, thus taxes per capita, are higher in the smaller country and total defense, thus the expected revenue from conflict, are lower in the smaller country. Thus if the individual at the border between countries of size $s' + \varepsilon$ and $s' - \varepsilon$ prefers the bigger country for $p = 0$ he cannot prefer the smaller country for $p>0$. Thus $\hat{s}(p) \geqslant \hat{s}(0)$ for $p>0$. Consider now $p' > p$. A similar argument based on contradiction establishes that $\hat{s}(p') \geqslant \hat{s}(p)$;thus it follows that the function $\hat{s}(p)$ is weakly increasing in p. □

A.1. Numerical simulations

ε-Stability: Consider N countries of equal size $s = 1/N$. Suppose that a small perturbation takes place at the border between country 1 and country 2, so that $s'_1 = s - \varepsilon$ and $s'_2 = s + \varepsilon$. The other $N - 2$ countries remain of size s. All countries adjust their defense spending after the perturbation. New defense spending in country 1 (2) will be denoted by d'_1 (d'_2). Defense spending in the remaining $N - 2$ countries is denoted by d'. Then, d'_1, d'_2 and d' are given as follows:

$$d'_1 = \arg\max\left\{2pe(s+\varepsilon)\,\frac{d'_1}{d'_1+d'_2} + 2pe(N-2)s\,\frac{d'_1}{d'_1+d'} - \frac{d'_1}{s-\varepsilon}\right\},$$

$$d'_2 = \arg\max\left\{2pe(s-\varepsilon)\,\frac{d'_2}{d'_1+d'_2} + 2pe(N-2)s\,\frac{d'_2}{d'_2+d'} - \frac{d'_2}{s+\varepsilon}\right\},$$

$$d' = \arg\max\left\{2pe(s-\varepsilon)\frac{d'}{d'_1+d'} + 2pe(s+\varepsilon)s\,\frac{d'}{d'_2+d'}2pe(N-3)\,\frac{d'}{d'+\bar{d}'} - \frac{d'}{s}\right\}.$$

The first-order conditions are:

$$2pe(s+\varepsilon)\frac{d'_2}{(d'_1+d'_2)^2} + 2pe(1-2s)\frac{d'}{(d'_1+d')^2} = \frac{1}{s-\varepsilon}, \tag{A.25}$$

$$2pe(s-\varepsilon)\,\frac{d_1'}{(d_1'+d_2')^2}+2pe(1-2s)\,\frac{d'}{(d_1'+d')^2}=\frac{1}{s+\varepsilon}, \tag{A.26}$$

$$2pe(s-\varepsilon)\,\frac{d_1'}{(d_1'+d')^2}+2pe(s+\varepsilon)\frac{d_2'}{(d_2'+d')^2}+2pe(1-3s)=\frac{1}{4d'}+\frac{1}{s}, \tag{A.27}$$

$$\hat{d}_1'=d_1'/(s-\varepsilon),\quad \hat{d}_2'=d_2'(s+\varepsilon),\quad \hat{d}_3'=d'/s. \tag{A.28}$$

For any given vector of parameters (pe, g, K) and for any given configuration of N countries of size $s=1/N$, it is possible to calculate the amount of defense per capita, that would be chosen, respectively, in country 1, country 2, and in the remaining $N-2$ countries when the border between countries 1 and 2 is perturbated, so that a fraction ε of the population of country 1 joins country 2, where ε is a number of much smaller than 1. These values can be obtained by solving the system (A.25)–(A.27) numerically. For example, we have calculated the values of $\hat{d}_1', \hat{d}_2'$, and $\hat{d}_3'$ for $pe=500$.[32] By using such defense values, we can then calculate the utility $u_1'(u_2')$ of the individual at the border between countries 1 and 2 if she belongs to country 1 (country 2). In general, these utilities are given as follows:

$$\begin{aligned}u_1' &= [1-p(1-(s-\varepsilon))]\,e+2pe\left[(s+\varepsilon)\frac{d_1'}{d_1'+d_2'}+(1-2s)\,\frac{d_1'}{d_1'+d'}\right]\\ &\quad -\frac{K}{s-\varepsilon}-\frac{d_1'}{s-\varepsilon}-g\frac{s-\varepsilon}{2}u_2'\\ &= \left[(s-\varepsilon)\frac{d_2'}{d_1'+d_2'}+(1-2s)\frac{d_2'}{d_2'+d'}\right]\\ &\quad -\frac{K}{s+\varepsilon}-\frac{d_2'}{s+\varepsilon}-g\frac{s+\varepsilon}{2}.\end{aligned} \tag{A.29}$$

If $u_1'>u_2'$, the individual at the border would like to join the smaller country. In this case, the original configuration would be stable. By contrast, if $u_1'<u_2'$, the individual at the border would like to join the larger country, henceforth amplifying the perturbation. In the latter case, the original configuration is not stable (i.e., it does not satisfy Requirement 3). Values of u_1' and u_2' can be calculated for different values of g and K. In particular, we have calculated u_1' and u_2' for different values of g, when $pe=500$ and $K=0.5$ and 4. For any given value of the vector $(pe,\ g,\ K)$, we have calculated the maximum N that satisfies Requirement 3, i.e., that is stable (note that if N is stable, so is $N-1$). Let N^* denote the maximum number of equally-sized countries that satisfies Requirement 3, i.e., that is stable (or, more specifically, ε-stable). Denote with N' the largest integer smaller than $\sqrt{(g-pe)/2K}$, and with N'' the integer that is closest to $\sqrt{(g-pe)/2K}$. In all our simulations we have

(1) N^* is decreasing in pe.

[32] The details of all simulations described in this Appendix are available from the authors upon request.

(2) N' is *always* ε-stable; N'' is ε-stable for values of pe larger than the critical value $\bar{p}\bar{e}$ definedby the following equation:[33]

$$N(\bar{p}\bar{e}) = \sqrt{\frac{g-\bar{p}\bar{e}}{2K}}. \tag{A.30}$$

Hence, if N'' is ε-stable, the number N which satisfies Requirement 2′ is equal to N^*. Otherwise, it is equal to N', which is always stable.

A.2. Secession-free equilibria

Consider N countries of equal size s. Consider a secession of size z taking place in country 1. Then, we have a new country of size z spending d_z in defense. The rest of country 1, being now of size $s-z$, spending d_{s-z} in defense, and the remaining $N-1$ countries, of size $s=1/N$, spending d' in defense. d_z, d_{s-z} and d' are given by the following first order conditions (as long as the system has strictly positive solutions):[34]

$$\begin{aligned}
&2pe(s-z)\,\frac{d_{s-z}}{(d_z+d_{s-z})^2}+2pe(1-s)\,\frac{d'}{(d_z+d')^2}=\frac{1}{z},\\
&2pez(s-z)\,\frac{d_z}{(d_z+d_{s-z})^2}+2pe(1-s)\,\frac{d'}{(d_{s-z}+d')^2}=\frac{1}{s-z},\\
&2pe(s-z)\,\frac{d_z}{(d_z+d')^2}+2pe(s-z)\,\frac{d_{s-z}}{(d_{s-z}+d')^2}=\frac{2pe(1-s)}{4d'}=\frac{1}{s}.
\end{aligned} \tag{A.31}$$

For any $z \leqslant s/2$, let u_n denote the status-quo utility of an individual located at a distance $s/2-z$ from the center, that is

$$u_n = e + g\left(1\left(\frac{s}{2}-z\right)\right) - \frac{K}{s} - \frac{d^*}{s}. \tag{A.32}$$

Let u_s denote the utility of that same individual should a secession of size z occur, so that he would be located at the border between the new country of size z and the rest of his old country, now of size $s-z$:

$$\begin{aligned}
u_s = {} & p(s-z)(2e) + p(N-1)s\frac{d_z}{d_z+d'}2e \\
& + 1(1-p(1-z))e + g\left(1-\frac{z}{2}\right) - \frac{K}{z} - \frac{d_z}{z}.
\end{aligned} \tag{A.33}$$

This individual would be in favor of (against) a unilateral secession of size z as long as u_n is smaller (larger) than u_s, and be indifferent in the case $u_n = u_s$. For a given configuration of N countries of size $s = 1/N$, we can calculate the values of u_n and u_s associated with different possible secessions of size $z = s$, where z takes different values between 0 and 0.5. If u_n is smaller than u_s for some values of z, that

[33] For instance, when $K = 0.5$, for $g = 800$, the critical $\bar{p}\bar{e}$ is 400; for $g = 1600$, the critical $\bar{p}\bar{e}$ is 700, etc.

[34] The conditions that characterize the corner solutions are available upon request.

configuration will not be secession free. On the other hand, if for every $u_n \geqslant u_s$ we can say that $s = 1/N$ is secession-free.

We have calculated the values of defense per capita and of u_n and u_s for $g = 800$, $K = 0.5$ and $pe = 50$. The z's take values between 0.05 and 0.5.[35] In this example, $s = \frac{1}{2}$ and $\frac{1}{3}$ ($N = 2$ and 3) are not secession free. Up to 25% of the citizens of each country of size s would be happier if they could form a smaller country on their own. Secessions of size $\frac{1}{40}$, $\frac{1}{20}$, $\frac{15}{200}$, $\frac{1}{10}$ and $\frac{1}{8}$ would all be approved unanimously by the relevant subset of citizens. $S = \frac{1}{4}(N = 4)$ is not secession-free either, because up to 20% of the citizen of each country of size s would like to form a smaller country. $N = 5$ is not secession free because secessions of size $z = s/10 = \frac{1}{50}$ and $z = 0.15s = \frac{3}{100}$ would be unanimously preferred by the relevant fractions of the population. By contrast, values of N larger than 5 aresecession free.

In general, if N is secession-free, so is $N + 1$. For each value of the parameters (g, K, pe), we can calculate the minimum N that is secession-free. We calculate the minimum N that is secession-free for different values of the parameters and find that

- N is decreasing in pe.
- The largest integer smaller than $\sqrt{(g - pe)/2K}$ is always secession free.

A.3. Derivations of results in Section 7.1

The Leviathan in country h, faced with $\delta = 1$, will choose defense in order to maximize his net rents R_h, given by

$$R_h = \left[z_b - g\frac{s_h}{2} - u_0 - \frac{d_h + K}{s_h} \right] s_h, \tag{A.34}$$

where $z_b = y + e_b$ is the income of the individual at the border. In a symmetric equilibrium all countries have the same size (s)[36] and e_b is given as follows:

$$e_b = \frac{d_h}{d_h + d_{h'}} p \frac{\gamma}{2} 2e \quad \text{for} \quad \frac{\gamma}{2} \leqslant s \tag{A.35a}$$

and

$$e_b = \frac{d_h}{d_h + d_{h'}} p(\gamma - s) 2e \quad \text{for} \quad \frac{\gamma}{2} > s, \tag{A.35b}$$

where d_h is defense in country h and $d_{h'}$ is defense in each of the other countries. By maximizing (A.34) subject to (A.35a)–(A.35b) and solving for the symmetric

[35] For this specific example, no secession would ever occur for $\lambda > 0.5$. This turns out to be true in all our calculations for $N > 2$. In some cases, when $N = 2$, a majority of the original population may want to secede and form a smaller nation.

[36] We have maintained symmetry by assuming that the interval [0,1] is mapped on a circle. That is, the individuals located at 0 or 1 face the same matching probabilities that the individuals located at any other point—for example, if $\gamma = 0.2$, the individual located at 0 faces equal chances of matching with each individual located between 0.9 and 1 and between 0 and 0.1. In other terms, the point 0 and 1 coincide (they are like "twelve o'clock" in an analogue watch). Given this assumption, the Leviathan solution is symmetric, and implies countries of equal size.

equilibrium, we obtain (29a)–(29b). Leviathans' aggregate rents are given as

$$R = y + e - g\frac{s}{2} - u_0 - \frac{d^*}{s} - \frac{K}{s}, \tag{A.36}$$

where d^*/s is given by (29a)–(29b). Straightforward optimization implies that the size of countries s that maximizes (A.36) is given by (30a)–(30c).

References

Alesina, A., Spolaore, E., 1997. On the number and size of nations. Quarterly Journal of Economics 112, 1027–1056.

Alesina, A., Spolaore, E., 2003. The Size of Nations. MIT Press, Cambridge, MA.

Alesina, A., Spolaore, E., 2004. War, peace and the size of countries. Journal of Public Economics, forthcoming.

Alesina, A., Spolaore, E., Wacziarg, R., 2000. Economic integration and political disintegration. American Economic Review 90, 1276–1296.

Alesina, A., Baqir, R., Hoxby, C., 2004a. Political jurisdictions in heterogeneous communities. Journal of Political Economy 112, 348–396.

Alesina, A., Spolaore, E., Wacziarg, R., 2004b. Trade, growth and the size of countries. In: Aghion, P., Durlauf, S. (Eds.), Handbook of Economic Growth. North-Holland, Amsterdam (forthcoming).

Bolton, P., Roland, G., 1997. The breakup of nations: A political economy analysis. Quarterly Journal of Economics 112, 1057–1090.

Boulding, K.E., 1962. Conflict and Defense: A General Theory. Harper, New York.

Casella, A., Feinstein, J., 2002. Public goods in trade: On the formation of markets and political jurisdictions. International Economic Review 43 (2), 437–462.

Clemens, B., Gupta, S., Schiff, J., 1997. What happened to the peace dividend? Finance and Development, March.

Findlay, R., 1996. Towards a model of territorial expansion and the limits of empire. In: Garfinkel, M.R., Skaperdas, S. (Eds.), The Political Economy of Conflict and Appropriation. Cambridge University Press, Cambridge.

Friedman, D., 1977. A theory of the size and shape of nations. Journal of Political Economy 85, 59–77.

Garfinkel, M.R., 1994. Domestic politics and international conflict. American Economic Review 84, 1294–1309.

Grossman, H.I., 1991. A general equilibrium model of insurrections. American Economic Review 81, 912–921.

Grossman, H.I., Kim, M., 1995. Swords of plowshares? A theory of the security of claims to property. Journal of Political Economy, December, 103.

Hartley, K., Sandler, T. (Eds.), 1995. Handbook of Defense Economics. North-Holland, Amsterdam.

Hess, G.D., Orphanides, A., 1995. War politics: An economic, rational voter framework. American Economic Review 85, 828–846.

Hess, G.D., Orphanides, A., 2001. War and democracy. Journal of Political Economy 109, 776–810.

Hirshleifer, J., 1989. Conflict and rent-seeking success functions: Ratio versus difference models of relative success. Public Choice 46, 247–258.

Hirshleifer, J., 1995. Anarchy and its breakdown. Journal of Political Economy 103, 26–52.

Hobsbawn, E.J., 1994. The Age of Extremes. Vintage Books, New York.

Le Breton, M., Weber, S., 2003. The art of making everybody happy: How to prevent a secession. IMF Staff Papers 50, 403–435.

Olson, M., Zeckhauser, R., 1966. An economic theory of alliances. Review of Economics and Statistics 48, 266–279.

Powell, R., 1999. In the Shadow of Power: States and Strategies in International Politics. Princeton University Press, Princeton, NJ.

Sachs, J., Sala-i-Martin, X., 1992. Fiscal federalism and optimum currency areas: Evidence for Europe from the United States. In: Canzoneri, M., Masson, P., Griles, V. (Eds.), Establishing a Central Bank: Issues in Europe and Lessons from the U.S. Cambridge University Press, Cambridge.

Sandler, T., Hartley, K., 1995. The Economics of Defense. Cambridge University Press, Cambridge.

Schelling, T.C., 1960. The Strategy of Conflict. Harvard University Press, Cambridge, MA.

Spolaore, E., 2002. Trade, conflict, and political borders. Brown University, Mimeo.

Tullock, G., 1974. The Social Dilemma: The Economic of War and Revolution. University Pub., Blacksburg, VI.

Tullock, G., 1980. Efficient rent seeking. In: Buchanan, J.M., Tollison, R.D., Tullock, G. (Eds.), Toward a Theory of the Rent-Seeking Society. Texas A&M University Press, College Station, TX.