GLOBAL DEVELOPMENT AND ENVIRONMENT INSTITUTE

WORKING PAPER NO. 00-01

Still Dead After All These Years: Interpreting the Failure of General Equilibrium Theory

Frank Ackerman

November 1999

Tufts University
Medford MA 02155, USA
http://ase.tufts.edu/gdae

Still Dead After All These Years: Interpreting the Failure of General Equilibrium Theory[1]

Frank Ackerman
fackerma@tufts.edu

For years after the Spanish dictator actually died, the mock television newscast on "Saturday Night Live" was periodically interrupted with a "news flash" informing viewers that "General Franco is still dead!" This served both to satirize the breathlessly urgent style of television news reporting, and to suggest that after many decades of taking an absolute ruler for granted, the world needed more than one reminder that he was no longer alive and well.

Much the same is true for general equilibrium theory. In the course of its long decades of rule over the discipline of economics, general equilibrium became established as the fundamental framework for theoretical discourse. Its influence continues to spread in policy applications, with the growing use of computable general equilibrium models. It has successfully colonized much of macroeconomics, with the insistence on the derivation of rigorous microfoundations for macro models and theories. General equilibrium theory is widely cited in a normative context, often in textbooks or semi-technical discussion, as providing the rigorous theoretical version of Adam Smith's invisible hand and demonstrating the desirable properties of a competitive economy.

Yet those who follow the news about microeconomic theory have known for some time that general equilibrium is not exactly alive and well any more. The equilibrium in a general equilibrium model is not necessarily either unique or stable, and there are apparently no grounds for dismissing such ill-behaved outcomes as implausible special cases. This conclusion is clearly at odds with established modes of thought about economics; several more "news flashes" will be required to assimilate and interpret the failure of earlier hopes for general equilibrium models, and to formulate new directions for economic theory.

The first section of this paper presents one such news flash, summarizing and explaining the evidence of fundamental flaws in general equilibrium theory. But simply hearing the news one more time is not enough. The goal of this paper is to develop a basic, intuitively comprehensible understanding of *why* it happened, as a guide to future theorizing. What features of the general equilibrium model led to its failure? What changes in economic theory are needed to avoid the problem in the future?

Section 2 examines contemporary interpretations of the findings of instability. Some attempts have been made to avoid the issue, without success. Despite occasional claims to the contrary, general equilibrium remains fundamental to the theory and practice of economics. Analysts who have faced the problem have identified two underlying causes: the inherent difficulties of the aggregation process, and the unpredictable nature of individual preferences.

Section 3 pursues the roots of the problem in the early history of general equilibrium theory: a mathematical framework transplanted from nineteenth-century physics was far less fruitful in economics, due to fundamental differences between the two fields. The provocative treatment of this topic by Philip Mirowski asks the right questions, but falls short of adequately answering them.

The final section briefly describes alternative approaches that might remedy the earlier flaws in neoclassical theory. Post-general-equilibrium economics will need a new model of consumer behavior, new mathematical models of social interaction, and an analysis of the exogenous institutional sources of stability.

1. The limits of general equilibrium

> *Now the reason for this sterility of the Walrasian system is largely, I believe, that he did not go on to work out the laws of change for his system of General Equilibrium.*
>
> -- John Hicks (1939, p.61)

The best-known results of general equilibrium theory are the two theorems proved by Kenneth Arrow and Gerard Debreu in the 1950s. First, under familiar assumptions defining an idealized competitive market economy, any market equilibrium is a Pareto optimum. Second, under somewhat more restrictive assumptions, any Pareto optimum is a market equilibrium for some set of initial conditions.

There is a longstanding debate about the interpretation of the Arrow-Debreu results, in light of the obvious lack of realism of some of their assumptions. For example, nonconvexities, such as increasing returns to scale in production, are common in reality. If they are allowed into the theory then the existence of an equilibrium is no longer certain, and a Pareto optimum need not be a market equilibrium (i.e., the second theorem no longer holds).

Yet despite awareness of this and other qualifications, economists frequently talk as if deductions from general equilibrium theory are applicable to reality. The most common and most important example involves the relationship between efficiency and equity. (For a critical review of the standard approach to the subject see Putterman et al. 1998.) The second fundamental theorem is often interpreted to mean that any efficient allocation of resources -- for instance, one based on a preferred distribution of income -- could be achieved by market competition, after an appropriate lump-sum redistribution of initial endowments.

This interpretation is a mistaken one. Even if the conditions assumed in the proofs applied in real life (which they clearly do not), meaningful application of the Arrow-Debreu theorems would require dynamic stability. Consider the process of redistributing initial resources and then letting the market achieve a new equilibrium. Implicitly, this image assumes that the desired new equilibrium is both unique and stable.

If the equilibrium is not unique, one of the possible equilibrium points might be more socially desirable than another, and the market might converge toward the wrong one. If the equilibrium is unstable, the market might never reach it, or might not stay there when shaken by small, random events.

Beyond stability

In the 1970s theorists reached quite strong, and almost entirely negative, conclusions about both the uniqueness and the stability of general equilibrium. There is no hope of proving uniqueness in general, since examples can be constructed of economies with multiple equilibria. The fundamental result about uniqueness, achieved by Debreu in 1970, is that the number of equilibria is virtually always finite (the set of parameters for which there are an infinite number of equilibria has measure zero). There are certain restrictions on the nature of aggregate demand that ensure uniqueness of equilibrium, but no compelling case has been made for the economic realism of these restrictions.

For stability the results are, if anything, even worse. There are examples of three-person, three-commodity economies with permanently unstable price dynamics (Scarf 1960), showing that there is no hope of proving stability of general equilibrium in all cases. The basic finding about instability, presented in a limited form by Sonnenschein (1972) and generalized by Mantel (1974) and Debreu (1974), is that almost any continuous pattern of price movements can occur in a general equilibrium model, so long as the number of consumers is at least as great as the number of commodities[2]. Cycles of any length, chaos, or anything else you can describe, will arise in a general equilibrium model for some set of consumer preferences and initial endowments. Not only does general equilibrium fail to be reliably stable; its dynamics can be as bad as you want them to be.

A common reaction to this Sonnenschein-Mantel-Debreu (SMD) theorem is to guess that instability is an artifact of the model, perhaps caused by uncommon or unrealistic initial conditions, or by the nature of the assumed market mechanisms. Investigations along these lines have failed to revive general equilibrium, but instead have driven more nails into its coffin.

The SMD result cannot be attributed to a specific, rigid choice of individuals' preferences, nor to a particular distribution of income. In a sweeping generalization of the SMD theorem, Kirman and Koch (1986) proved that the full range of instability can result -- i.e., virtually any continuous price dynamics can occur -- even if all consumers have identical preferences, and any arbitrarily chosen income distribution is used, as long as the number of different income levels is at least as great as the number of commodities. This means that the SMD theorem can be established even for a population of nearly identical consumers -- with identical preferences and almost, but not quite, equal incomes (Kirman 1992).

Another important generalization shows that "SMD instability" may be a property of an economy as a whole even if it is not present in any part, or subset, of the economy (Saari 1992). Suppose that there are n commodities; even if every subset of the economy with n-1 or fewer commodities satisfies conditions that guarantee stability of equilibrium, it is still possible to have "arbitrarily bad" dynamics in the full n-commodity economy. This means, among other things, that the addition of one more commodity could be sufficient to destabilize a formerly stable general equilibrium model. More generally speaking, dynamic results that are proven for small general equilibrium models need not apply to bigger ones.

Might instability be just a result of the unrealistic method of price adjustment assumed in general equilibrium models? Again, the answer is no. In Walrasian general equilibrium, prices are adjusted through a *tâtonnement* ("groping") process: the rate of change for any commodity's price is proportional to the excess demand for the commodity, and no trades take place until equilibrium prices have been reached. This may not be realistic, but it is mathematically tractable: it makes price movements for each commodity depend only on information about that commodity[3]. Unfortunately, as the SMD theorem shows, tâtonnement does not reliably lead to convergence to equilibrium.

On the other hand, any price adjustment process that does reliably converge to equilibrium must be even less realistic, and far more complex, than tâtonnement. There is an iterative procedure that always leads to a market equilibrium, starting from any set of initial conditions (Smale 1976). However, there is no apparent economic justification for this procedure, and it requires overwhelming amounts of information about the effects of prices of some goods on the demand for other goods.

A final, negative result has been achieved on this question, showing that any price adjustment process that always converges to an equilibrium has essentially infinite information requirements (Saari 1985). Consider any iterative price adjustment mechanism, in which current prices are a smooth function of past excess demand and its partial derivatives. If there is an upper bound on the amount of information used in the adjustment process, i.e., if it relies solely on information about any fixed number of past periods and any fixed number of derivatives of the excess demand function, then there are cases in which the process fails to converge. These cases of nonconvergence are mathematically robust; that is, they occur on open sets of initial conditions, not just at isolated points.

Safety in numbers?

Not much is left, therefore, of the original hopes for general equilibrium. One direction in which theoretical work continued in the 1990s is the attempt to deduce regularities in aggregate economic behavior from the dispersion of individual characteristics. This approach abandons efforts to prove that market economies are generically stable, and instead suggests that conditions that lead to stability are statistically very likely to occur, even if not quite guaranteed.

In particular, Hildenbrand (1994) and Grandmont (1992) have explored the hypothesis that the dispersion of individual preferences is a source of aggregate stability. That is, predictable, smoothly distributed differences in individuals' demand functions and consumption patterns, of the sort that are observed in reality, could lead to a definite structure of aggregate demand that might imply stability of equilibrium. (For reviews of this line of work, see Kirman 1998, Lewbel 1994, and Rizvi 1997.)

There are two problems with the statistical approach to economic stability. First, it has not yet succeeded. The assumption of a smooth distribution of consumer characteristics seems to help, but has not entirely freed the proof of market stability from arbitrary restrictions on individual preferences or aggregate demand functions.

Second, even if the statistical approach were to succeed in explaining past and present market stability, it would remain vulnerable to future changes in preferences. Suppose that it is eventually demonstrated that the empirically observed dispersion of consumer preferences is sufficient to ensure stability in a general equilibrium model. This finding might not be reliable for the future since, in the real world, fads and fashions episodically reorganize and homogenize individual preferences. That is, coordinated preference changes involving the media, fashions, celebrities, brand names, and advertising could, in the future, reduce the dispersion of consumer preferences to a level that no longer guaranteed stability.

2. Explanations of the fall

In the aggregate, the hypothesis of rational behavior has in general no implications.

-- Kenneth Arrow (1986)

The mathematical failure of general equilibrium is such a shock to established theory that it is hard for many economists to absorb its full impact. Useful interpretations of its causes and significance have been slow to appear. This section begins with a presentation and critique of three views that suggest that the SMD theorem is not as important as it looks. It then turns to other interpretations offered by two of the theorists whose work appeared in Section 1.

Three styles of denial

Is the SMD result only a mathematical curiosity, of limited importance for economics? At least three major arguments make that claim, on the basis of disinterest in dynamics, disinterest in abstraction, and disinterest in the particular theories of the past. As we will see, none of the three is persuasive.

First, some essentially say that we're just not a dynamic profession. A recent graduate text in microeconomic theory presents a detailed explanation and proof of the SMD theorem and then, a few pages later, tells students that

> A characteristic feature that distinguishes economics from other scientific fields is that, for us, the equations of equilibrium constitute the center of our discipline. Other sciences, such as physics or even ecology, put comparatively more emphasis on the determination of dynamic laws of change. (Mas-Colell, Whinston and Green 1995, p. 620.)

Second, perhaps it was always silly to care so much about empty abstractions. According to Deirdre McCloskey, the whole category of general equilibrium theorizing is merely "blackboard economics," exhibiting the "rhetoric of mathematical formalism":

> None of the theorems and countertheorems of general equilibrium theory has been surprising in a qualitative sense... *But the qualitative sense is the only sense they have...* The problem is that the general theorem of Arrow and Debreu or any of the other qualitative theorems do not, strictly speaking, relate to anything an economist would actually want to know. (McCloskey 1994, p.135)

Things an economist would actually want to know, for McCloskey, inevitably involve information about how big something really is compared to something else.

Finally, it could be that those on the inside track already have learned to avoid the theoretical dead ends of the past. This view is common in conversation with economists, if not in writing. "No one," it is claimed, believes in general equilibrium theory any more; the profession has moved on to game theory, complexity theory, evolutionary frameworks, and other techniques, allowing the creation of sophisticated new models that do not fit into the old Arrow-Debreu mold.

Each of these arguments is narrowly true and broadly false. In a narrow sense, they describe the behavior of numerous economists: many do focus on static rather than dynamic theoretical problems; many others are predominantly engaged in empirical work; and there are theorists who no longer use a general equilibrium framework. Yet in a broad sense, each of these observations misses the point.

The dismissal of dynamics fails because all significant applications of theory are inherently dynamic. In an ever-changing world (or a model of that world), static properties of equilibrium have no practical meaning unless they persist in the face of small disturbances. Advocacy of policies based on their static optimality in a general equilibrium framework -- a very common form of applied economics -- implicitly assumes some level of dynamic stability, since otherwise the optimum might not last for long enough to matter. Yet the dynamic stability of the general equilibrium framework is precisely what is called into question by the SMD theorem.

For those who are overwhelmed by the ongoing mathematical escalation in economic theory, McCloskey's call to turn away from empty formalism toward real empirical work has a certain refreshing charm. But even applied researchers often present their work in terms of the abstractions of what she calls "blackboard economics." Some theory is present, explicitly or implicitly, in any empirical study. In applied economics today, it is common to find explicit reliance on a general equilibrium framework, and hence implicit reliance on the dynamic stability of its equilibrium points.

The notion that "no one" believes in general equilibrium any more is true only of small circles of avant-garde theorists. Look anywhere except at the most abstractly theoretical journals, and general equilibrium still characterizes the actual practice of economics. General equilibrium models have become ubiquitous in such important areas as trade theory and environmental economics, and are continuing to spread. Macroeconomics, a field that once developed its own very different theories, has been led into the pursuit of rigorous microfoundations – in effect seeking to deduce aggregate behavior from general equilibrium theory. An on-line search for publications on "general equilibrium" turns up hundreds of citations each year, with no evidence of declining interest in the subject.[4]

General equilibrium is fundamental to economics on a more normative level as well. A story about Adam Smith, the invisible hand, and the merits of markets pervades introductory textbooks, classroom teaching, and contemporary political discourse. The intellectual foundation of this story rests on general equilibrium, not on the latest mathematical excursions. If the foundation of everyone's favorite economics story is now known to be unsound – and according to some, uninteresting as well – then the profession owes the world a bit of an explanation.

Individualism and aggregation

There are some theorists who have recognized the importance of the failure of general equilibrium theory. Two of the authors cited above, Alan Kirman and Donald Saari, have published thoughtful reflections on the subject. Kirman, in a dramatically titled article ("The Intrinsic Limits of Modern Economic Theory: The Emperor Has No Clothes"), argues that

> The problem seems to be embodied in what is an essential feature of a centuries-long tradition in economics, that of treating individuals as acting independently of each other... This independence of individuals' behavior plays an essential role in the construction of economies generating arbitrary excess demand functions [the source of instability in the SMD theorem]. (Kirman 1989, pp.137-38)

Saari considers the problem from a mathematician's perspective. Examining the "Mathematical Complexity of Simple Economics" (Saari's title), he explores the SMD theorem and related results, and concludes that

> [T]he source of the difficulty -- which is common across the social sciences -- is that the social sciences are based on aggregation procedures... One way to envision the aggregation difficulties is to recognize that even a simple mapping can admit a complex image should its domain have a larger dimension than its image space... [T]he complexity of the social sciences derives from the unlimited variety in individual preferences; preferences that define a sufficiently large dimensional domain that, when aggregated, can generate all imaginable forms of pathological behavior. (Saari 1995, pp.228-229)

There are two separate points here: one involves the methodology of aggregation, and the other concerns the behavioral model of the individual. Both are basic causes of the instability of general equilibrium.

Instability arises in part because aggregate demand is not as well-behaved as individual demand. If the aggregate demand function looked like an individual demand function -- that is, if the popular theoretical fiction of a "representative individual" could be used to represent market behavior -- then there would be no problem. Unfortunately, though, the aggregation problem is intrinsic and inescapable. There is no representative individual whose demand function generates the instability found in the SMD theorem (Kirman 1992). Groups of people display patterns and structures of behavior that are not present in the behavior of the individual members; this is a mathematical truth with obvious importance throughout the social sciences.

For contemporary economics, this suggests that the pursuit of microfoundations for macroeconomics is futile. Even if individual behavior were perfectly understood, it would be impossible to draw useful conclusions about macroeconomics directly from that understanding, due to the aggregation problem (Rizvi 1994, Martel 1996). This fact is reflected in Arrow's one-sentence summary of the SMD result, quoted at the beginning of this section.

The microeconomic model of behavior contributes to instability because it says too little about what individuals want or do. From a mathematical standpoint, as Saari suggests, there are too many dimensions of possible variation, too many degrees of freedom, to allow results at a useful level of specificity. The consumer is free to roam over the vast expanse of available commodities, subject only to a budget constraint and the thinnest possible conception of rationality: anything you can afford is acceptable, so long as you avoid blatant inconsistency in your preferences.

The assumed independence of individuals from each other, emphasized by Kirman, is an important part but not the whole of the problem. A reasonable model of social behavior should recognize the manner in which individuals are interdependent; the standard economic theory of consumption fails to acknowledge any forms of interdependence, except through market transactions. However, merely amending the theory to allow more varied social interactions will not produce a simpler or more stable model. Indeed, if individuals are modeled as following or conforming to the behavior of others, the interactions will create positive feedback loops in the model, increasing the opportunity for unstable responses to small fluctuations (see Section 4).

3. The limits of analogy

> *There is a fairly close analogy between the earlier stages of economic reasoning and the devices of physical statics. But is there an equally serviceable analogy between the later stages of economic reasoning and the methods of physical dynamics? I think not.*
>
> – Alfred Marshall (1898)

How did economists come to spend so much time and effort on general equilibrium, only to arrive at a mathematical dead end? What was the source of such longstanding devotion to an ultimately unworkable theory? The problems identified in the last section -- the inherent difficulties of aggregation, and the underspecified model of individual behavior -- are not new, and cannot be blamed on the latest mathematical wrinkles in the formulation of general equilibrium. In particular, the microeconomic behavioral model was an intentional feature of the theory, and has been present in something like its current form ever since neoclassical economics was born. A look at the history of economic thought may help to identify what went wrong in the beginning.

In their history of the idea of economic equilibrium, Ingrao and Israel (1990) argue that mathematical modeling of economic systems was a continuation of a major current in eighteenth- and nineteenth-century European social thought, seeking to identify lawlike regularities in social life and organization. Once the idea of equilibrium was given a mathematical form, though, the mathematics itself became the predominant influence on the further development of the theory.

The outlines of general equilibrium theory first appeared in the work of Leon Walras, as part of the "marginalist revolution" of the 1870s. The sudden interest in marginalism in economics in the 1870s is commonly attributed to the influence of mid-nineteenth-century advances in mathematics and the physical sciences. Thus the original structure of general equilibrium theory reflects the manner in which economists applied the new mathematical techniques of the era.

Breaking the conservation law

The influence of physics on the origins of neoclassical economics is analyzed in depth in a controversial work by Philip Mirowski (1989). The pioneers of marginalism in economics, including Walras, tried to develop analogies to mechanics in some detail. According to Mirowski, several of the early neoclassical economists adopted very similar mathematical models, and incorporated similar flaws.

The physics of the day, much admired by economists, assigned a central role to the conservation of energy. Potential energy could be represented as a vector field, indicating the direction in which particles would move unless constrained by other forces. The economic analogy treated individuals as particles moving in commodity space, where the spatial coordinates are quantities of different commodities[5]. Utility was the vector field indicating the direction in which individuals would move, to the extent allowed by budget constraints.

The problem with this economic analogy, for Mirowski, was the failure to take, or to understand, the logical next step. In physics, the model of potential energy as a vector field induces predictable movements of particles, and leads to a related concept of kinetic energy, measured in the same units as potential energy. The law of energy conservation applies to the sum of potential plus kinetic energy, not to either one alone. Much of the power of the physical theory and the effectiveness of its use of mathematics derives from the energy conservation law.

In the economic analogy, if utility as a potential field induces predictable movements of individuals in commodity space, the "kinetic energy" of that movement should be consumer expenditure. The exact analogue of the law of energy conservation would thus be the conservation of the sum of utility plus expenditure, an economically meaningless concept. At this point the analogy breaks down. The resulting economic theory remained fragmented, using bits and pieces of the mathematical apparatus related to energy conservation, but unable to draw on the full strength and coherence of the original physical theory. In particular, the economists adapted some of the relationships of static equilibrium, but failed to incorporate the more complicated dynamic relationships from physics.

The debate surrounding Mirowski's argument (see, for example, Walker 1991, Varian 1991, Hands 1992, Cohen 1992, de Marchi 1993, and Carlson 1997) raises many other issues, as does his original work. On the central point about the close relationship between physics and early neoclassical economics, Mirowski poses the right question, but his answer is at best incomplete. Mirowski certainly demonstrates that Walras, Jevons, Pareto, Fisher, and other neoclassical pioneers discussed analogies to physics in great detail, without always understanding the mathematics that was involved. Moreover, he is persuasive in suggesting that this episode of intellectual history had a formative impact on modern economic theory. Yet Mirowski's version of what went wrong with neoclassical theory is frustrating on two accounts.

First, why should economics need an exact analogue to energy conservation? The failure to create a precisely analogous principle might be taken as a recognition that economics and physics are not identical in structure. While the particular mathematical methods that work in physics are therefore not available, others, more appropriate to economics, could be created. In a sense, it is true that *something* must be conserved in any economic theory that allows quantification and causal analysis; otherwise, there would be no way to compare magnitudes and events at different times (Mirowski 1990). However, this does not imply that the same thing, or the analogous thing, must be conserved in two different theories.

Second, while Mirowski makes a remarkably strong case for the idea that the earliest neoclassicals were mediocre mathematicians, that early history does not explain the persistence of mistakes through successive generations of economists. In the thorough reworking of neoclassical theory in the 1930s and 1940s by Hicks, Samuelson, von Neumann, and others, it seems unlikely that past mathematical errors and oversights would have survived unnoticed.

Nonetheless, the problem remains as Mirowski describes it: the original formulation of general equilibrium by Walras and others relied heavily on analogies to physics, often using the same mathematical structures, i.e. the same mathematical metaphor for reality. Why did this metaphor prove so much more fruitful in physics than in economics? Answers must be sought in features of the economic model that are *intentionally* different from the physical analogue, and have therefore persisted through more than a century of development of economic theory. Two such answers are suggested in the following subsections, involving the number of dimensions in the model, and the individual, asocial nature of preferences.

Lost in commodity space

The analogy between mechanics and economics makes the spatial coordinates of a particle correspond to the quantities of commodities held by an individual. Once this step is taken there is already a significant difference between the two theories, involving the number of dimensions.

Physics is, in this respect, the more modest of the two fields. Physical particles have three spatial coordinates; they travel in the familiar world of three-dimensional space. A paradigm-changing innovation, the theory of relativity, adds just one more dimension to create a four-dimensional space-time continuum. Abstract higher-dimensional constructions are common in physics (e.g., the phase spaces of elaborate theoretical systems), but the resulting theories have observable, testable implications for events in the low-dimensional space of our physical experience.

The analogous space of our economic experience is a commodity space that, in a modern industrial economy, may have hundreds of thousands of dimensions. Even in Walras' day, there must have been thousands of distinct commodities, and hence thousands of dimensions in a general equilibrium model of the economy as a whole. This is no phase space used to explain a simpler experiential space; the thousands of independent dimensions (commodities) are fundamental to the characterization of economic experience in neoclassical theory.

Intuition is a poor guide to the vast commodity spaces of economics. The ability to visualize shapes and motion drops off rapidly as the number of dimensions increases beyond three. The dynamic possibilities for a model are far more complex in three dimension than in two; how much greater complexity is introduced by going to thousands of dimensions? No system of actual equations in such high-dimensional spaces can be comprehended or manipulated. All that can be done is to prove completely generic results, or to talk about low-dimensional -- usually visualizable, two- or three-dimensional -- examples and special cases. Yet as noted in Section 1, these special cases may be misleading: dynamic results that can be proved for smaller general equilibrium models need not apply to bigger ones.

The huge number of dimensions of commodity space is not only a mathematical problem. It also affects the plausibility of the economic model of the consumer. The consumer must be able to reveal her preferences about any of the commodities on the market; this may require knowledge of hundreds of thousands of different items. Whenever a new commodity appears on the market, she must be able to revise her preference ordering at once to reflect the change. Clearly, no real person can come close to fulfilling this role. Responses to this problem are discussed in Section 4.

A different drummer

The original neoclassical analogy to physics made utility comparable to energy. Yet there are crucial ways in which utility and energy differ. In physics, the same potential energy fields affect all particles in the same manner, allowing structure and predictability in the movements of large groups of particles. In the analogous economic theories, a different utility function motivates each individual, giving a group of people a structureless, unpredictable pattern of change. No common forces move them all in parallel; no interactions with each other, save through market exchange, coordinate their motions.

This is no mathematical accident, but rather a result of the microeconomic behavioral model, which concentrates on one aspect of human activity and assumes away everything else. In fact, human behavior involves a complex combination of relatively predictable responses to social forces on the one hand, and unpredictable individual preferences and choices on the other hand. The former aspect is where an analogy to physical laws of motion might have proved most valuable – but the latter is the focus of neoclassical economics.

At first glance, physics and economics both appear to rely on an unobservable force: potential energy, like utility, is not directly observable. But potential energy is indirectly deducible from, and commensurable with, observable data. The potential energy of any one particle is readily compared to any other. In economics, in contrast, the lack of an observational measure of utility and the absence of interpersonal comparability meant that Walrasian general equilibrium was devoid of empirical content.[6]

Not all the early neoclassical economists saw utility as a completely individual and unmeasurable matter. Alfred Marshall and Arthur Pigou maintained that interpersonal comparison of utility was at least sometimes possible, for group averages if not for individuals. If this "material welfare" school of economics (Cooter and Rappoport 1984) had remained dominant, a different analysis of utility and preferences, and hence a different model of equilibrium, might have emerged. However, any hints of a distinct Marshallian paradigm were swept away by the "ordinalist revolution" of the 1930s, which resolved the conceptual problems with utility by banishing it altogether in favor of revealed preference.

The revealed preference account of consumer choice does not escape all the philosophical problems surrounding the subject (Sen 1973, Sagoff 1994). Nor does it eliminate the asocial individualism of the model, the feature which subverts structure and prediction of group behavior. Each individual still marches to a different drummer, even if the drums are now labeled "revealed preference relation" instead of "utility function." Naturally, this leaves no way of telling where the parade is headed.

4. Alternatives for the future

> *How disappointing are the fruits, now that we have them, of the bright idea of reducing Economics to a mathematical application of the hedonistic calculus...*
>
> --John Maynard Keynes (1972), p.184n

General equilibrium is still dead. Exactly 100 years after the 1874 publication of Walras' most important work, the SMD theorem proved that there was no hope of showing that stability is a generic property of market systems. Another quarter-century of additional research has found no way to sneak around this result, no reason to declare instability an improbable event. These negative findings should challenge the foundations of economic theory. They contradict the common belief that there is a rigorous mathematical basis for the "invisible hand" metaphor; in the original story, the hand did not wobble.

While the SMD theorem itself appears mathematically esoteric, we have seen that its roots are traceable to simple, intentional features of the neoclassical model, which have been present since the beginning. What happens to economic theory when those features are changed? Where should we look for new alternatives for the future? This concluding section examines three areas where new theoretical approaches are needed in response to the failure of general equilibrium: the commodity-based model of consumer choice; the analysis of social interactions; and the role of institutional sources of stability.

So many commodities, so little time

The discussion, in Section 3, of the high-dimensional nature of the traditional model of consumer choice led to criticism of the implausibly large information processing requirements which the theory imposed on consumers. This criticism of the neoclassical model has been raised before, perhaps initially in Herbert Simon's arguments for bounded (rather than global) rationality.

It is also reminiscent of a classic series of attempts to reconceptualize consumer choice. In roughly simultaneous, independent work, Kelvin Lancaster (1966), Richard

Muth (1966), and Gary Becker (1965) each proposed that what consumers actually want is not goods per se, but rather characteristics of the goods, or experiences produced by consuming or using the goods. This is consistent with the manner in which psychologists, sociologists, and anthropologists generally understand the process of consumption. Yet surprisingly little has come of this approach in economics. Of the two major versions of the theory, Lancaster's more rigidly – probably too rigidly – structured model was never developed much beyond its provocative initial presentation. Meanwhile, Becker's more amorphous "household production function" model is, like the neoclassical theory of consumption, capable of being stretched to fit virtually all possible situations, and hence ends up explaining very little. (See Ackerman 1997 and Goodwin et al. 1997 for further discussion.)

Whether an alternative is based on these foundations or other approaches, it remains important to create a mathematically manageable behavioral model with information requirements on a human scale. Such a model cannot be expressed primarily in terms of ownership, knowledge of, and response to individual commodities, simply because there are so many of them. Human needs and behavior must be described in terms of other categories, more limited in number.

Among other changes, this makes it virtually impossible to demonstrate the optimality of consumer choices and market outcomes. Optimization would require global rationality with its unrealistically high information requirements. Any realistic behavioral theory will, in contrast, embody some form of bounded rationality, defined over a far smaller set of choices – because that is all that real people have time for.

Blowing bubbles

The weaknesses in neoclassical theory that ultimately led to the SMD theorem, as described in Section 3, included not only the high-dimensional model of consumer choice, but also the asocial, individualistic nature of preferences. The absence of social forces that influence individuals (with the sole exception of market exchange) makes the results of the theory underdetermined and unpredictable. In reality, of course, there are numerous nonmarket social interactions which impart structure to group behavior. The importance of these social interactions has been recognized in some recent work in economics, leading to models that add a touch of realism to the theory of economic behavior.

However, models of interaction do not necessarily contribute to an explanation of market stability. On the contrary, social conformity or emulation, such as wanting a consumer good because other people already have it, creates positive feedback in the market, with obvious potential for destabilization. This can be seen most directly in models of fads and market bubbles. For example, Bikhchandani et al. (1998) show that erratic and fragile market "cascades" can occur if individuals consider the behavior of others to be a better source of information than their own knowledge or preferences.

The same logical sequence – social interaction leads to positive feedback and instability – can be seen in other, more elaborate models, of which two varieties deserve particular mention. Deterministic nonlinear models can give rise to chaotic dynamics, while agent-based models, involving simulation of the actions of individuals under hypothesized behavioral rules, often lead to nearly chaotic outcomes that have been dubbed "complexity." In such models, positive interactions – situations in which one person's action makes it more likely that another will act in the same way – are the source of either chaos or complexity. These interactions, and hence these outcomes, would be commonplace in a realistic, comprehensive theory of individual economic activity. As Saari puts it, "Economics so effortlessly offers the needed ingredients for chaos that, rather than being surprised about exotic dynamics, we should be suspicious about models which always are stable." (Saari 1996, p. 2268)

Chaos and complexity models are characterized by sensitive dependence on initial conditions, the antithesis of stability. Therefore, quantitative replication or prediction of empirical data is impossible; at best it is possible to qualitatively reproduce the types of fluctuations and instability that arise in some real-world markets. For example, a "chaos" model of stock trading among sophisticated investors, crowd-following investors, and brokers generates patterns of volatility very similar to those of real stock markets (Day 1994, chapter 11). So does a "complexity" model with endogenous, adaptive expectations, in which market participants continuously revise their expectations of future prices based on observations of each other's behavior and of market outcomes (Arthur et al. 1997). In each of these models, positive feedback is created by investors who follow and imitate other investors' decisions, thereby amplifying small fluctuations in the market.

In short, the leading styles of modeling social interactions threaten to compound the problem of instability in economic theory. While the underspecified social structure assumed by neoclassical economics may have contributed to the indeterminacy of its outcomes, the most obvious cure seems to make the disease worse.

Macrofoundations of microeconomics

Where, then, does stability come from? Theoretical analysis to date, which has been impressive in its depth and breadth, has shown that stability is simply not an endogenous mathematical property of market economies under all initial conditions. This provides an elegant theoretical justification for a return to traditional styles of macroeconomics, in which cyclical fluctuations and potential instability of aggregate incomes are central topics of concern. Yet the demonstration of the robustness of "SMD instability," combined with recent research on nonlinear dynamics, chaos, and complexity, appears to prove too much. Market economies are only episodically unstable or chaotic; it is certainly the norm, not the exception, for markets to clear and for prices to change smoothly and gradually.

In short, it is more obvious in practice than in theory that large, complicated market economies are usually stable. If it is so difficult to demonstrate that stability is endogenous to a market economy, perhaps it is exogenous. That is, exogenous factors such as institutional contexts, cultural habits, and political constraints may provide the basis for stability, usually damping the erratic endogenous fluctuations that could otherwise arise in a laissez-faire economy. Variations on this theme can be found in several alternative schools of thought, such as Marxist, feminist, and institutionalist economics.

There are other approaches, closer to conventional theory, that also make institutions central to economic analysis. The need for exogenous sources of stability is one of the tenets of what David Colander calls "post-Walrasian macroeconomics." Colander (1996) identifies three distinguishing characteristics of the post-Walrasian perspective. First, the equations necessary to describe the economy have multiple equilibria and complex dynamics. Second, individuals act on the basis of local, bounded rationality, since global rationality is beyond anyone's information processing capabilities. Finally, institutions and non-price coordinating mechanisms are the source of systemic stability in a market economy. Colander refers to the last of these characteristics as establishing the macrofoundations of microeconomics. The "post-Walrasian" initiative is an encouraging one, but much more remains to be done to create a comprehensive alternative theory, building on what is now known about the limitations of established models.

Colander's approach, like any of the alternative schools of thought, would lead economists to take a humbler stance than they often do in public debate. The guaranteed optimality of market outcomes and laissez-faire policies died with general equilibrium. If economic stability rests on exogenous social and political forces, then it is surely appropriate to debate the desirable extent of intervention in the market – in part, in order to rescue the market from its own instability.

To recapitulate the main points in closing: Section 1 explained the fact that general equilibrium is, indeed, still dead after all these years. There are two principal causes of the death, as seen in Section 2. The instability of the neoclassical model can be attributed to the inescapable difficulties of the aggregation process, and the highly individual, asocial nature of consumer preferences. These are not recent innovations, but design flaws that have been present since the origins of the theory in the late nineteenth century. Section 3 argued that two intentional features of the theory, present in the original neoclassical analogy to physics, led economics astray: the huge number of dimensions and information requirements of the "commodity space" framework, and the individualistic behavioral model.

Repairing these flaws, as described in Section 4, will require a model of human needs and behavior that is not defined in terms of individual commodities. It will involve mathematically complex analyses of social interaction. And it will have to recognize the central role of social and institutional constraints. These new departures will make economics more realistic, but will not demonstrate the inherent stability or optimality of market outcomes, as general equilibrium theory once seemed to do.

*****

The death of General Franco was not a panacea for the problems of Spain. Yet it did open many democratic, pluralist options, no longer requiring the whole country to follow one authoritarian leader. Spain after Franco looks a lot more like neighboring countries in the freedom of expression that it offers its citizens, and the diversity of opinions that can be expressed in public debate. The same might yet be true of economics after general equilibrium.

Frank Ackerman is Director of the Research and Policy Program of the Global Development And Environment Institute and Research Associate Professor at Tufts University's Urban and Environmental Policy Program.

NOTES

[1] Thanks to Herb Gintis, Neva Goodwin, Deirdre McCloskey, Philip Mirowski, Irene Peters, Donald Saari, and participants in the University of Massachusetts/Amherst political economy seminar for comments on earlier drafts.

[2] Recent work in general equilibrium theory has typically assumed a pure exchange economy, without production. The obstacles to proving uniqueness or stability seem to arise on the consumer side of the market; including production would make the mathematics more complicated, but would not change the results discussed here. Real-world applications of the theory, of course, require modeling of production as well as consumption.

[3] Other relatively simple adjustment mechanisms have been proposed, such as quantity adjustment in a fixed-price environment. Rizvi (1994) argues that analyses of such mechanisms have often relied on specific, ad hoc forms for aggregate excess demand, making them vulnerable to the SMD critique.

[4] Publications with the subject "general equilibrium" listed in the EconLit database of the American Economics Association increased from about 100 per year in the early 1980s to about 800 per year by the mid-1990s. Although the number of all EconLit citations grew rapidly in those years, the number of "general equilibrium" citations grew even faster.

[5] In comments on an earlier draft of this paper, Mirowski has objected that the original neoclassical model, as described in his work, makes points in physical space analogous to commodity bundles, but says nothing about physical analogues to individuals. The distinction seems to me a rather thin one: commodity bundles are meaningful as bundles only because they are, or could be, held by individuals; conversely, individuals are located, in an exchange economy, solely by the commodity bundles they possess.

[6] More recently it has been argued that, if people respond rationally to lotteries, it is possible to deduce their utility functions, which are unique up to a linear transformation

(solving the problem of measurement for an individual, though still not allowing interpersonal comparison). However, this view, introduced by von Neumann and Morgenstern, emerged only after the "ordinalist revolution," and has always been a minority perspective among neoclassical economists. Thus it has played very little part in the historical developments described in the text. Moreover, empirical evidence suggests that people often do not respond rationally to lotteries, undermining this approach to measurement of utility.

REFERENCES

Ackerman, Frank, 1997. "Consumed in Theory: Alternative Perspectives on the Economics of Consumption," *Journal of Economic Issues*, vol. 31, 651-64.

Arrow, Kenneth, 1986. "Rationality of Self and Others in an Economic System," *Journal of Business*, vol. 59, no. 4, pt. 2, S385-S399.

Arthur, Brian, and John Holland, Blake LeBaron, Richard Palmer, and Paul Tayler, 1997. "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market," *Economic Notes* vol. 47, 297-329.

Becker, Gary, 1965. "A Theory of the Allocation of Time," *Economic Journal* vol. 75, 493-517.

Bikhchandani, Sushil, and David Hirshleifer and Ivo Welch, 1998. "Learning From the Behavior of Others: Conformity, Fads, and Informational Cascades," *Journal of Economic Perspectives*, vol. 12, 151-170.

Carlson, Mathieu J., 1997. "Mirowski's Thesis and the 'Integrability Problem' in Neoclassical Economics," *Journal of Economic Issues* vol. 31 no. 3, 741-760.

Cohen, Avi, 1992. "Seeing the Light Despite the Heat: Post-Mirowski History of Economic Thought," *Philosophy of the Social Sciences* vol. 22 no. 1, 83-96.

Colander, David, editor, 1996. *Beyond Microfoundations: Post-Walrasian Macroeconomics* (New York: Cambridge University Press).

Cooter, Robert, and Peter Rappoport, 1984. "Were the Ordinalists Wrong About Welfare Economics?", *Journal of Economic Literature*, June, 507-530.

Day, Richard, 1994. *Complex Economic Dynamics, Volume I: An Introduction to Dynamical Systems and Market Mechanisms* (Cambridge MA: MIT Press).

Debreu, Gerard, 1974. "Excess Demand Functions, *Journal of Mathematical Economics* vol. 1, 15-21.

de Marchi, Neil, editor, 1993. *Non-Natural Social Science: Reflecting on the Enterprise of 'More Heat than Light'*, annual supplement to *History of Political Economy* (Durham, NC: Duke University Press).

Goodwin, Neva, and Frank Ackerman and David Kiron, editors, 1997. *The Consumer Society* (Washington: Island Press).

Grandmont, Jean-Michel, 1992. "Transformations of the Commodity Space, Behavioral Heterogeneity, and the Aggregation Problem," *Journal of Economic Theory* vol. 57, 1-35.

Hands, Wade, 1992. "More Light and Less Heat: Mirowski on Economics and the Energy Metaphor," *Philosophy of the Social Sciences* vol. 22 no. 1, 97-111.

Hicks, John, 1939. *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory* (Oxford: Clarendon Press).

Hildenbrand, Werner, 1994. *Market Demand: Theory and Empirical Evidence* (Princeton: Princeton University Press).

Ingrao, Bruno, and Giorgio Israel, 1990. *The Invisible Hand* (Cambridge MA: MIT Press).

Keynes, John Maynard, 1972. *Essays in Biography, Collected Writings X*.

Kirman, Alan, 1998. Review of Werner Hildenbrand, "Market Demand: Theory and Empirical Evidence," *Economic Journal* vol. 108, 868-870.

–, 1992. "Whom or What Does the Representative Individual Represent?", *Journal of Economic Perspectives* vol. 6 no. 2, 117-136.

–, 1989. "The Intrinsic Limits of Modern Economic Theory: The Emperor Has No Clothes," *Economic Journal* vol 99, 126-139.

–, and K. J. Koch, 1986. "Market Excess Demand in Exchange Economies with Identical Preferences and Collinear Endowments", *Review of Economic Studies* vol. 53, 457-463.

Lancaster, Kelvin, 1966. "A New Approach to Consumer Theory," *Journal of Political Economy* vol. 74, 132-157.

Lewbel, Arthur, 1994. "An Examination of Werner Hildenbrand's 'Market Demand'", *Journal of Economic Literature* vol. 32 no. 4, 1832-1841.

Mantel, R, 1974. "On the Characterization of Aggregate Excess Demand," *Journal of Economic Theory* vol. 7.

Marshall, Alfred, 1898. "Distribution and Exchange," *Economic Journal* vol. 8, 37-59.

Martel, Robert, 1996. "Heterogeneity, Aggregation, and a Meaningful Macroeconomics," in Colander 1996.

Mas-Colell, Andreu, and Michael Whinston and Jerry Green, 1995. *Microeconomic Theory* (New York: Oxford University Press).

McCloskey, D. N., 1994. *Knowledge and Persuasion in Economics* (New York: Cambridge University Press).

Mirowski, Philip, 1990. "Learning the Meaning of a Dollar: Conservation Principles and the Social Theory of Value in Economic Theory," *Social Research* vol. 57 no. 3, 689-717.

–, 1989. *More Heat than Light: Economics as Social Physics, Physics as Nature's Economics* (New York: Cambridge University Press).

Muth, Richard, 1966. "Household Production and Consumer Demand Functions," *Econometrica* vol. 34, 281-302.

Putterman, Louis, and John E. Roemer and Joaquim Silvestre, 1998. "Does Egalitarianism Have a Future?", *Journal of Economic Literature* vol. 36, 861-902.

Rizvi, S. Abu Turab, 1997. "Responses to Arbitrariness in Contemporary Economics," *History of Political Economy* vol. 29 Supplement, 273-88.

–, 1994. "The Microfoundations Project in General Equilibrium Theory," *Cambridge Journal of Economics* vol. 18, 357-377.

Saari, Donald G., 1996. "The Ease of Generating Chaotic Behavior in Economics," *Chaos, Solitons & Fractals* vol. 7, no. 12, 2267-2278,

–, 1995. "Mathematical Complexity of Simple Economics," *Notices of the American Mathematical Society* vol. 42, no. 2, 222-230.

–, 1992. "The Aggregated Excess Demand Function and Other Aggregation Procedures," *Economic Theory,* vol. 2, no.3, 359-88.

–, 1985. "Iterative Price Mechanisms," *Econometrica* vol. 53, 1117-1131.

Sagoff, Mark, 1994. "Should Preferences Count?," *Land Economics* vol. 70, 127-145.

Scarf, Herbert, 1960. "Some Examples of the Global Instability of the Competitive Equilibrium," *International Economic Review* vol. 1 no.3, 157-172.

Sen, Amartya, 1973. "Behavior and the Concept of Preference," *Economica* vol. 40, 241-259.

Smale, Steven, 1976. "A Convergent Process of Price Adjustment and Global Newton Methods," *Journal of Mathematical Economics* vol. 3, 107-120.

Sonnenschein, H, 1972. "Market Excess Demand Functions," *Econometrica* vol. 40, 549-563.

Varian, Hal, 1991. Review of *More Heat than Light,* by Philip Mirowski, *Journal of Economic Literature* vol. 29, 595-596.

Walker, Donald, 1991. "Economics as Social Physics," *Economic Journal* vol. 101, 615-631.

The Global Development And Environment Institute (G-DAE) is a research institute at Tufts University dedicated to promoting a better understanding of how societies can pursue their economic goals in an environmentally and socially sustainable manner. G-DAE pursues its mission through original research, policy work, publication projects, curriculum development, conferences, and other activities. The "G-DAE Working Papers" series presents substantive work-in-progress by G-DAE-affiliated researchers. We welcome your comments, either by e-mail directly to the author or to G-DAE, Cabot Center, Fletcher School, Tufts University, Medford, MA 02155 USA; tel: 617-627-3530; fax: 617-627-2409; e-mail: gdae@tufts.edu; web: http://ase.tufts.edu/gdae.

Papers in this Series:

00-01 Still Dead After All These Years: Interpreting the Failure of General Equilibrium Theory (Frank Ackerman, November 1999)

00-02 Economics in Context: The Need for a New Textbook (Neva R. Goodwin, Oleg I. Ananyin, Frank Ackerman and Thomas E. Weisskopf, February 1997)

00-03 Trade Liberalization and Pollution Intensive Industries in Developing Countries: A Partial Equilibrium Approach (Kevin Gallagher and Frank Ackerman, January 2000)

00-04 Basic Principles of Sustainable Development (Jonathan M. Harris, June 2000)

00-05 Getting the Prices Wrong: The Limits of Market-Based Environmental Policy (Frank Ackerman and Kevin Gallagher, September 2000)

00-06 Telling Other Stories: Heterodox Critiques of Neoclassical Micro Principles Texts (Steve Cohn, August 2000)