11 Stories That Blow up: How to Anticipate When the Realisticness of Assumptions Will Matter[1]

As was suggested in the previous chapter, the approach that is here proposed for social economics will finally bring the issue of making bridges between the concrete and the abstract down to questions about the nature of reality. It is not, after all, just any 'concreteness' that is being made abstract: it is something real, something 'out there', beyond our minds. In most attempts to apprehend this reality it is reasonable to assume that there are many descriptions which are approximately equally accurate (arguably, none of them is perfectly accurate), but there are also some which are less accurate, and there are some descriptions which do not relate to reality well enough to serve usefully as inputs to a science.

As we return to subjects which were raised in Chapter 6, the thesis of this chapter will be developed with special care, because it will dispute a contention that has remained dominant in neoclassical economics for more than thirty years: namely, that

> ***the realisticness*** **(or lack thereof)** ***of the assumptions employed in economics*** **(by extension in the other social sciences too; as also, some economists have supposed, in the physical sciences)** ***does not matter.***

This idea, somewhat carelessly stated by Milton Friedman and even more carelessly used by myriad economists after him, has been extensively written upon and effectively disputed.[2] It nevertheless continues to prevail in economics and in other social sciences which are influenced by the methodology of economics.[3]

This chapter will attempt to provide a new framework for understanding when, and in what ways, the realisticness of certain assumptions does matter. It will begin by building upon the discussion of the three preceding chapters, where the relation between theory and reality was considered in terms of bridges between the concrete things of the real world on the one hand, and, on the other, the various methods of abstraction (neuronal, linguistic, theoretic) which symbolise these things in the process of 'communication' – still stretching the term 'communication', as we have done throughout Part II, to include writing or speaking; reading or listening; or using one's logical and intuitive faculties to work over an idea.

VARIOUS APPROACHES TO THE SELECTION AND USE OF ASSUMPTIONS

A news item in the summer of 1989 was a decision by voters in conservative Orange County, California, to legislate for expensive restrictions and requirements on businesses and consumers. The goal was to reduce the contributions made by Orange County to ozone depletion and global warming.

Economists might view these voters as acting irrationally. Orange County alone cannot save, and probably cannot destroy, the Earth's ozone layer. A more 'rational' kind of behaviour would be to let everyone else bear the costs of trying to do something about the problem; if Orange County could manage to be a free rider in this situation, it could reap handsome economic rewards. A prediction of how the vote on this issue would go, based only on neoclassical economic assumptions about the way rational individuals behave, would have been wrong. The distance between such a prediction and what actually happened in Orange County is a very mild case of what I will call 'stories that blow up'. The more extreme cases strike us more forcefully, as paradoxes or bizarre absurdities.

Economics, like most other social sciences, is almost entirely concerned with the way human beings behave. Understandings, descriptions and predictions of human behaviour are based upon beliefs about human nature. In economics, particularly, these beliefs have been axiomatised to a set of core assumptions which may be summed up by saying that '**Rational individuals maximise their perceived utility**'. Many other, secondary assumptions follow from this; particularly assumptions about markets, as collections of rational individuals. Another assumption may be discovered to be necessary to the development of this basic assumption into a system of theory which is supposed to apply to what happens in real economies: namely, that most people behave rationally most of the time. Given that most people behave rationally, and that the meaning of rationality is utility maximisation, the rest of neoclassical economics follows.

Some neoclassical economists define the words in the critical sentence '**Rational individuals maximise their perceived utility**' so that, tautologically, it must be true. (For example, defining rationality as the maximisation of perceived utility.) Others go to sociobiology to 'prove', by evolutionary history, that rational individuals are programmed to do nothing but maximise their perceived utility. That argument can be stated convincingly, to meet all the requirements of internal consistency; all the same, there are times, as in the case of the Orange County voters, when it creates a situation of external inconsistency, where the conclusions drawn from the assumption of rational maximising (non-tautologically defined) are contrary to the facts of the real world.

Other neoclassical economists take the position that the assumption of rational maximising is not *strictly* true, but that it's a 'good enough' approximation. And others, like Milton Friedman, have said we simply shouldn't be worrying about the realisticness of our assumptions: his position has most often been interpreted as saying that: **the goodness of the results we get shouldn't be judged by, and won't depend upon, the realisticness of the assumptions upon which we have built our theories.**

Beyond the core set of assumptions which a social science may hold, there are also provisional assumptions, taken on for the sake of a particular exercise. We might say, for example,

1. 'Let us assume perfect competition'; or
2. 'Let us assume that the interest of every actor in every long and short run is represented in a perfect market'; or
3. 'Let us assume that utility will be maximised when preferences are satisfied.'[4]

Even more commonly, we build into our models an assumption that

4. 'Equilibrium is where things are, or whither things are tending.'

The first of these assumptions is quite often made explicit: the other three are more often assumed than stated. What relation does 'assumption' bear to 'belief'? When closely questioned, most of the time we don't really believe that perfect competition *really* is a fact of life (1), or that perfect markets do exist which perfectly represent all interests (2). Many economists probably believe by default the third assumption, of the identity between utility-maximisation and preference-satisfaction simply because it has not occurred to them to question this, but when brought to think about it, many or most economists would be a little uneasy about committing their personal belief to this as a fact of the real world. They might be even more uneasy with a positive or teleological definition (4) of equilibrium as 'where things are or whither things are tending'. It is, however, much easier for us deal with certain economic problems, especially within the neoclassical framework of analysis, when we act as if all of these were our beliefs.

Such *as if* behaviour is not without implications. In the case of perfect competition, for example, as for any assumption which has to do with perfect markets, it requires us to go further: to assume a world where buyers and sellers are too small and powerless to dominate the market; where there is no collusion; there is perfect information; and all the other familiar conditions. Assumptions 3 and 4 have different kinds of implications, of a slightly less 'cognitive' nature. Assumption 3 has to do with an additional belief about human nature, which might or might not be considered a part of the rationality assumption: namely, the assumption that people know what is good for them – or at least, that every individual is the best judge of what is good for him or herself. This belief, like any alternative which may be offered, has strong political overtones. A gravitation toward assumption 4 may, indeed, have as much to do with temperament as with intellect.

As was the case with the *core* assumptions, individual social scientists respond to their, and their colleagues', *provisional* assumptions on a spectrum which goes from Friedman's position – that the realisticness of the assumptions doesn't matter – to the alternative, which holds that unrealistic assumptions will always produce inaccurate and unuseful theories. The latter extreme is, practically, an even more difficult position to maintain than is Friedman's, because any process of thinking is an abstraction – a *remove* from reality. It becomes extremely difficult to theorise at all if we try to ban all unrealistic assumptions.

What I hope to contribute to this debate is some common sense, and then some systematic thinking. The common sense is the observation that sometimes unrealistic assumptions do seem to lead inevitably to bad theory and sometimes they don't; sometimes the realisticness of the assumptions matters more and sometimes less.

The systematic thinking is where I hope something new and interesting may turn up.

> What if it were possible somehow to characterise the situations in which unrealistic assumptions have the greatest potential to lead us into trouble?
>
> I would like to work towards a system that could predict, for example: 'if you proceed with such and such an assumption, you are going to end up with results that won't be useful.'

In order to dramatise this, I will use as examples of what I mean by 'running into trouble' some stories that stray so far from reality that they blow up into paradoxes. Some of the classic paradoxes in philosophy, economics and other areas may turn out to be examples of what goes wrong when unrealistic assumptions are pushed to their limit. Before I take on some paradoxes, however, I would like to introduce a few more conceptual tools.

REALITY, UNICORNS AND A METAPHOR

In the last chapter I proposed the terms 'unicorn word' or 'unicorn phrase' to indicate a word or phrase which refers to something people can talk about in common – something we have all heard of, so that we each have a reasonable idea of what others mean by it – but where the thing itself does not exist. I suggested that some of the very common and useful concepts of mathematics (infinity, *i*, lines, points, instants, continuity) may be best categorised as unicorn terms. I stressed – and will want to stress this again, and yet again – that this does not mean we should not use these concepts, in economics or anywhere else: only that we should use them with rather more care and caution than has been common.

At this point I would like to introduce a deliberate use of a little piece of mathematics, emphasising that its use, throughout this chapter, will be *as a metaphor*. It will be developed as an aid in understanding what I will put forth as a resolution to the paradoxes that will be discussed in this chapter. It will be particularly important to recall the metaphorical character of this discussion when we come to Zeno's paradoxes, for some commentators have slipped into error by thinking that the mathematical example below was a literal, not a metaphorical solution to the problems of motion that Zeno posed.

The mathematical metaphor I wish to use may be illustrated by the unbounded integral where, as X approaches infinity, Y asymptotically approaches zero. Since Y never reaches zero, one would think that the area under such a curve would be infinite: this is, in fact, the case for 'the integral from one to infinity of one-over-X, dx' (see Figure 11.1).

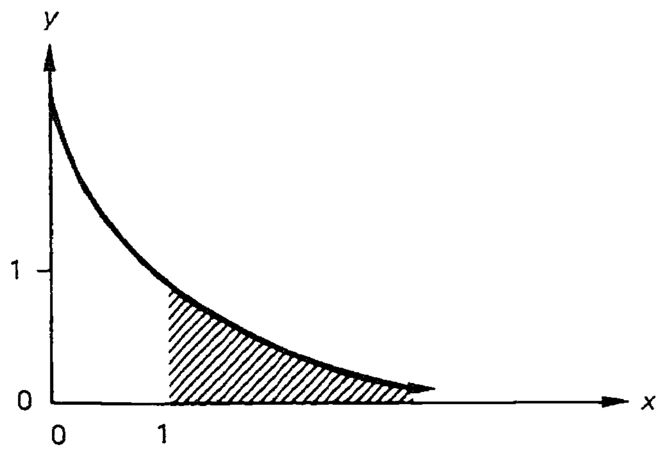

Figure 11.1 The graph of $\int_1^{\infty} \frac{1}{x}\,dx$

However, $\int_1^{\infty} \frac{1}{x^2}\,dx$ ('the integral from one to infinity of one-over-X-squared, dx' – the graph of which looks almost identical to Figure 11.1) *has a finite area* (equal, in fact, to 1). This gives us an illustration of an extra-natural limit [$X \rightarrow \infty$] which, in one case, pulls its consequences outside the real world with it; in the other, it returns a result which has a place within our experience. Note that either result – convergence or nonconvergence – can occur in a situation where the limit is infinity, even though 'infinity' is a unicorn word.

The first use I want to make of this metaphor of mathematical limits, and what happens to functions when we take them to their limits, is to establish a comparison with the set of *assumptions that could be characterised as idealisations*. 'Perfectly competitive markets', 'maximising behavior', and 'perfect precision' are all examples of this kind of assumption. They are also concepts which can be described on a spectrum, such as the spectrum from less to more competitive markets, or the spectrum from lesser to greater precision.

We may feel intuitively that, at least in their middle ranges, 'precision' and 'competition' lie on continuous spectra. What happens, however, when we approach the limit case, or idealisation, of perfect precision, perfect markets? Is there a discontinuity, such as we find in some mathematical functions, between the limit and the limit minus epsilon? **I would suggest that such a discontinuity, if it exists, is not to be looked for where we depart from the *actual*, but where we depart from the *possible*.**

Thought-experiments often ask us to explore conditions which do not actually exist in the real world. The thought-experiments which have the potential to end up as paradoxes, I am going to suggest, are those which contain some premise that is not only counter-factual, but is also counter-possible.[5]

The metaphor being developed here depends upon a comparison between, on the one hand, a mathematical function that goes toward some limit; and, on the other hand, an explanation that depends upon the idea of real events tending toward some 'ideal case'. When the mathematical limit is outside reality (e.g., infinity), or the ideal case is something not encountered in the real world (e.g., perfect markets), it will sometimes, but not always, happen that the integral will blow up, or the model will break down into paradox or absurdity. Without trying to define, on the mathematical side of the analogy, what are the contexts within which functions approaching certain limits do or do not converge, I will ask the question: what is the relationship between context and idealisation (or 'limit case assumption') which will result in 'nonconvergence' in a description of the real world?

My hypothesis is that nonconvergence – meaninglessness – is the fate of any argument, model, theory or system of theories where two conditions obtain:

1. first, when there is a discontinuity between the limit case idealisation and the real world;
2. and second, when the action in the argument, model, etc., is required to go continuously along the idealisation all the way to its limit.

Light is shed upon both of these conditions by a comparison between an idealisation commonly used in economics – perfect competition – and one from the physical sciences upon which economists have often drawn for analogy – the perfect vacuum. In such analogies it is often stressed, from the social science side, that the spectrum from less, to more, to a perfect vacuum, is continuous, with no phase-shift involved in the achievement of the limit. The analogy implies that the same may be said of all the social science idealisations which are employed as assumptions.

When, historically, this analogy first began to be employed, the perfect vacuum of the natural sciences was something of which no human being had had direct experience (indeed, Aristotle denied that such a thing *could* exist). During the formative period of the early 1900s, when the young science of economics was looking to physics for methodological guidence,[6] physicists were optomistic that a perfect vacuum could be achieved. The methaphor has remained influential, even though the prevailing beliefs in physics have changed.

If a perfect vacuum is not to be found in nature, is a perfect market? Not, I would suggest, strictly defined: there are anti-competitive characteristics in human nature, such that, as long as humans and their institutions continue to be the chief actors in markets, the counter-balance to perfect competition will always be effectively in play.[7] In particular, there may be a discontinuity (something like a 'phase shift') between the kinds of information actually found in the real world and the perfect information required for perfect markets. (More will be said on this in relation to Newcomb's paradox.)

If, indeed, perfect markets as strictly defined in economics not only do not but *cannot* occur in nature, this still does not mean that it will never be useful to think about them, or to assume them for certain modeling exercises. Their explosive potential for such use depends as well upon the second condition given above.

The second condition – **that the action in the argument, model, etc., is required to go continuously along the idealisation all the way to its limit** – applies to the context in which the idealisation is treated in a particular model or theory. If the context is carefully defined so that the action only depends upon the middle ranges (e.g., where behaviour is quite competitive, but never required to be completely so), then the potential for serious trouble need not be realised.

PASS-THROUGH LIMITS

I have suggested that the determination of whether an idealisation is to be understood as a 'continuous' or a 'discontinuous' limit is a first step toward discovering what circumstances will create paradoxes or absurdities (the analogy to which, in mathematics, is nonconvergence). Besides 'continuous' or a 'discontinuous' idealisations, there is another category of assumptions which I would call a 'pass-through limit'. A pass-through limit is an idealisation which you keep aiming at, until you find yourself on the other side of it, but there is no duration of real time when you can say, 'Here I am at the limit'. Put differently, it is a case where the coincidence of the conceptual limit and the real-world experience never exists in real time, but where, in the real world, we seem to go *through* the conceptual space which may be identified as a limit-case to reality. I am reminded of a grafitto in the women's lavatory at Boston University (the university where I took my economics doctorate): 'The long run will never come, and equilibrium is where you're never at.' In many instances equilibrium is set up in such a way as to be a pass-through limit. The concept of instantaneous time may also be usefully described in this way. Zeno's paradox will provide a more concrete example of this concept.

Mathematicians, when talking about a non-convergent integral, sometimes say that it 'blows up'. That is what happens, I will claim, to logic that tries to integrate real-world reasoning with an ideal case that stands outside of the possibility in the given context. A number of examples of logic 'blowing up' in this way have been preserved in the special form of paradoxes. The approach which is being suggested here, with the three categories of 'continuous', 'discontinuous' and 'pass-through' limits, may be able to slice through the Gordian knot of a number of logical paradoxes.

APPLICATION OF THE LIMIT METAPHOR

A Newcomb's paradox

As a start, let us consider Newcomb's paradox,[8] where a 'predictor' (possessing an extraordinary ability to predict, from knowledge of a person's character, what that person will do) has set up rewards based upon foreknowledge (or an extraordinarily good guess) of which of two boxes a subject will choose to open. Specifically,

> There are two boxes, (B1) and (B2). (B1) contains \$1000. (B2) contains either \$1000000 (\$M), or nothing .
>
> You have a choice between two actions:
> (1) taking what is in both boxes
> (2) taking only what is in the second box.[9]

The catch is that the 'predictor' (initially described as, possibly, an extra-terrestrial being, to account for its exceptional predictive powers) will decide what to put in box (B2) *depending upon* its assessment of what the subject will choose, in the future, to do. Thus,

> if it predicts that the subject will take only what is in the second box, then it will put \$M in the second box:
>
> but if it predicts that the subject will open both boxes, then it leaves the second box empty.

Importantly, the subject knows these rules; the subject also knows that *the predictor has almost never been wrong.*

The sequence of events appears in our familiar linear time to be as follows:

(t_1) predictor deposits rewards
(t_2) according to some decision rule, the subject chooses to open box (B2) only, or else both boxes
(t_3) the subject gets the reward that was already in the box(es) when s/he opens it (them).

Normal, rational rules of decision-making would assume that the result, at time (t_3) depends only upon what happened in prior times (t_1) and (t_2), *and not vice versa.* Given that the rewards are already in the boxes at time t_2, the dominance principle would say that, whatever decision the predictor has already made, at time t_2 it is the case that the subject has a better chance of getting a higher reward if s/he chooses to open both boxes. However, this story contains the appearance of backwards causality, linking the choice at (t_2) to the event of (t_1) through the predictor's extraordinary abilities. This is how Nozick argued it:

> If one believes, for this case, that there *is* backwards causality, that your choice causes the money to be there or not, that it causes [the predictor] to have made the prediction that he made, then there is no problem. One takes only what is in the second box. Or if one believes that the way the predictor works is *by looking into the future*; he, in some sense, sees what you are doing, and hence is no more likely to be wrong about what you do than someone else who is standing there at the time and watching you, and would normally see you, say, open only one box, then there is no problem. You only take what is in the second box. *But suppose we establish or take as given that there is no backwards causality*, that what you actually decide to do does *not* affect what he did in the past, that what you actually decide to do is not part of the explanation of why he made the prediction he made . . .' (ibid., p. 134; italics added).

The paradox, as the story is presented here, is that there are two plausible, mutually exclusive, courses of action; on the face of it, there exist reasons both necessary and sufficient to support each one, but only one course of action can be taken. It is, in (Nozick's) actual experience of discussing this paradox with a variety of people, very difficult to persuade the proponents of either course of action that the other is right; worse yet is the situation of the person who can feel the force of both sets of arguments – and yet who knows that only one decision can be made.

My answer is that the problem was set up in an 'impossible' way – a way that was *bound* to 'blow up', rather than converging upon a single solution that would accord with all parts of the story. The problem is that

(a) we are asked to believe that there is no backwards causality (conditions which seem to abide by the laws of our known world); and, simultaneously,
(b) we are asked to believe in a very, very high degree of predictive ability on the part of the predictor (conditions which go beyond the limits of what is possible in this world).

The individuals who come down strongly in favour of one decision *or* the other are those who choose to place their credence according to (a) *or* (b) – effectively managing to ignore the other request for belief. The individuals who suffer from simultaneously feeling the force of both arguments are those who somehow did succeed in accepting, with approximately equal weight, both (a) *and* (b).

Belief (a) is consonant with what we know of this world. On the face of it, belief (b) is described so as to make it, also, plausibly part of the world we live in (especially because we are *told* that there is no backwards causality); but this is where something illegitimate is slipped in. Under the mask of the insistence that backwards causality is not involved, we are asked to believe in a degree and kind of predictive power that simply does not exist in this world.

We (human beings) can make very good predictions about the way certain chemicals will react together under a set of stated conditions. We have learned to make pretty good predictions about how pigeons will respond to a variety of stimuli, and we are learning, all the time, additional quite reliable predictions about more pigeon responses to more diverse stimuli. We can make some predictions about large patterns of human behaviour (e.g., marriage, or voting) that seem to hold for a while after they are first made, but that disintegrate slowly over time. As individuals, we may be quite good at predicting some of the choices that will be made by other individuals to whom we are close (which of two recipes my husband will prefer; what my son or daughter will do on the first night at the end of the school year; etc.) But the skill we are asked to believe in, of the extra-terrestrial predictor, does not fall into any of these real-world categories. Essentially, we are asked to believe in a predictor so accurate that it can know things about the subject's character which the subject him/herself does not know until the moment of irrevocable choice – *general truths about character which will absolutely (or almost absolutely) determine a particular choice.* In effect, we are asked to accept actions or states being codetermined by 'personality' or 'genes' in a way that does not fit our known world.[10] If the predictor does not have supernatural powers of prediction, it can only make its extraordinarily good guesses about the future by deduction from what it knows about the subject's state at t_1 – knowledge, one can only presume, which must be about either personality or genetic make-up.

Such deduction, I claim, is outside of possibility. Many actual people, actually thinking about this story, have found themselves so divided between the two choices that they might well come down one way on one day, the other on the next, depending upon random outside events that might have affected their moods, etc. Experience suggests that few if any individuals are entirely consistent: the best possible knowledge of an individual is likely to yield, not an absolute prediction of behaviour, but a probabilistic one, such as: 'Faced with such and such a situation, the subject will take choice (A) 75 per cent of the time'.

The first discontinuity in Newcomb's paradox is its assumption of a consistency in human behaviour, arising from 'genes' and/or 'personality', which permits (nearly) perfect prediction. The next discontinuity is the assumption of the combination of knowledge and understanding (of 'genes' and/or 'personality') which must be present in order to make the (nearly) perfect prediction of the story.

The story is told as not necessarily reaching the limiting cases (of *perfect* predictive ability, or *perfect* consistency in behaviour), but it ignores what I perceive as a discontinuity between the limit and the limit-minus-epsilon (the best possible prediction, the highest possible degree of consistency). It is doubtful that we would ever be able to describe conclusively what constitutes 'the best possible prediction'; but I suggest that the supposed (perhaps extra-terrestrial) predictor and its predictable subject have been described in a way that depends upon assumptions that go beyond the limits of any possible reality of this world. The attempt to apply rational decision rules to the problem is paradoxical because of an absurdity in trying to apply this-world behaviour to what is, in effect, another world.

B The paradox of the 'selfishness gene'

It will be useful to compare this conclusion, briefly, with a plausible resolution of what some have seen as a paradox in the theory of evolution.

By some interpretations, a particular strand of sociobiology[11] shows that reality cannot be as we, in fact, know it to be. The puzzle is posed thus:

> Suppose a species to be genetically programmed for altruism, so that individual members will risk their lives for the good of other members or of the whole (e.g., in rescuing an endangered individual).
>
> Now suppose that a mutation arises such that individuals carrying the mutated gene are 'selfish': they will be 'free riders' on the altruism of the rest of the group, but will not put themselves at risk to enhance the welfare of others.
>
> Such selfish individuals will therefore, on average, live longer than the altruistic members, have more children, and pass on their selfishness to a disproportionate fraction of the next generation. This will be repeated until most of the population carries the 'selfishness' genes instead of the altruistic ones. By similar reasoning, we see that, in a generally selfish population, it would be impossible for a self-sacrificing gene to arise.
>
> Yet (here is the paradox) we know that there actually are a number of species in which individuals will perform acts that reduce their own 'inclusive fitness' (at the extreme, sacrificing their lives) for their children, for the children of other individuals, or in defense of a larger group. How can this be?

The answers proposed by the philosopher, Mary Midgley, depend upon finding the limit case assumptions implicit in the statement of 'the selfishness gene paradox', and showing these assumptions to be discontinuous with reality in the given context. She starts with the 'atomising approach to impulses' which, projecting from what is known about genetic determinism of physical characteristics (certain chunks of DNA can be isolated as critically affecting e.g., eye colour in humans, maturation speed in fruit flies, etc.) assumes that there are also isolable DNA groupings which can be – and someday will be – recognised as determining any psychological characteristic one cares to mention. Midgley says:

> If you believe that the tendency to each specific sort of action is inherited separately, then all tendencies carrying personal danger are

fairly well at the level of Gregor Mendel's experiments with peas. However, it contains a hidden bomb that will explode '**when the action in the argument, model, etc., is required to go continuously along the idealisation all the way to its limit**': e.g., when, as we just saw, it is carried into a context where complexity is more critical. To generalise Midgeley's point, when we have to include 'patterns of behavior' in our analysis of survival schemes (as is the case with virtually all animals, as well as at least some plants), then we are dealing with contexts where the discontinuous idealisation, 'one gene for each characteristic', will disable the story.

C Zeno's 'dichotomy' paradox

Newcomb's paradox and the paradox of the selfishness gene are particularly interesting for the social sciences because they deal with human choices. At the same time, it is instructive to see the same approach applied to something that may appear, on the face of it, to be a paradox about the physical world, but where the problem in fact hinges on generalised assumptions of precision, divisibility and continuity which are commonly made in social sciences (such as economics) where mathematics are much employed. Zeno's paradoxes of motion may also be seen to have a 'solution' (if it is a solution to point out that the problem, as posed, simply does not belong to our known world) through application of what I have been calling the limits metaphor.

We will look particularly at the one of Zeno's paradoxes which is known as 'the Dichotomy'. We will find that it is like the paradox of the selfishness gene (and unlike Newcomb's paradox), in that it imagines a situation which is inconsistent with our experience of reality (rather than creating an internal inconsistency). The paradoxical conclusion of the Dichotomy is that **motion is impossible** – a conclusion which, of course, everyone knows, by experience, to be false. And yet, the argument is very compelling.

The argument, most simply stated, proceeds thus:

> If you set out to run a mile, you first have to run a half mile; but before you can get to the half-mile mark, you have to get to the quarter-mile mark; and so on. The image is one of paralysis in the face of infinite regress: since infinite divisibility (e.g., of a distance) is assumed, you can never reach point B, because you are always required, *first*, to get to the point halfway between A and B. That halfway point is redefined as the new point B, and the problem is repeated, *ad infinititum.*

A more rigorous statement of the problem, connecting it with modern mathematics and physics, is the following:

> Among the four paradoxes with which Zeno sought to discredit the possibility of physical motion, two are of primary relevance to contemporary mathematical physics, because of that discipline's affirmation that the time variable ranges over the real numbers just as the space variable does. More particularly, it is the denseness of the ordering of the points of space and of the instants of time that provides the point of application for Zeno's polemic. The claim that for any point on the path of a moving object, there is *no* next point, anymore than there is an immediately following or preceding instant for any instant during the motion, enables Zeno to ask incisively: In what sense can the events composing the motion be significantly said to succeed one another temporally, if they succeed one another densely rather than in the consecutive manner of a discrete sequence? This question takes the form of asking . . how can a temporal process even begin, if, in order to survive the lapse of a positive time interval *T*, a body must first have endured through the passage of an infinite regression of overlapping subsidiary time intervals $\frac{T}{2^n}$ *($n = \ldots, 3, 2, 1$) which has no first term* because the denseness postulate entails infinite divisibility . . .?[13]

A logical trick which allows us to deal with one aspect of the Dichotomy is to perform (theoretically) the exact same operation upon *time* as that which is being performed (theoretically) upon *space*: one hour, infinitely divided, would surely suffice to walk one leisurely mile, infinitely divided. Here an infinity (whose size is 'alph naught' in Cantor's terms) of infinitesimal increments of time is paired up with the same type of infinity of infinitesimal increments of space. However, a logical problem still remains after this operation: there is no reason to think that an infinite addition of infinitesimals (if they are true infinitesimals, with zero dimension) would add up to a finite distance or time.

Alfred Marshall has not been the only person to find that time and change are stumbling blocks on which the scientific method must repeatedly bark its shins. In the introductory essay to the book just quoted, the editor, Wesley Salmon, comments that Henry Bergson

> takes the paradoxes of Zeno to prove that the intellect is incapable of understanding motion and change. In his celebrated 'cinemato-

> graphic' characterisation of ordinary knowledge he maintains that the usual approach to a physical process consists in accumulating a series of static descriptions of its successive states, much as a motion picture consists of a large number of still pictures. By stringing these static representations together, Bergson argues, we can never come to grips with movement and change themselves . . . . It is only by entering into the process and perceiving it directly that we can genuinely understand physical becoming. Such insight cannot be achieved by mathematical analysis or by logical reasoning; metaphysical intuition is the only way (Wesley Salmon, 'Introduction' to *Zeno's Paradoxes*, p. 19).[14]

Continuing the discussion in Chapter 6, a first cut at the Dichotomy paradox might be to say that the problem is that space and time are continuous, while matter is discrete (it is discrete *as matter*: it's less clear what one can say about it on the level at which it resolves to energy). Moreover, 'discrete' and 'continuous' have different meanings when discussed in the context of (1) 'matter', (2) 'space', (3) 'time', and (4) 'energy'. If we suppose that the very first use of the words *discrete* and *continuous* was presumably with respect to material objects, then we should treat the later application of these words to space, time, and energy as *metaphorical extensions*, not as identical usages. Bergson (who is imprecise but, I think, accurate, as far as he goes) says that Zeno's paradoxes 'all consist in applying the *movement* to the *line* traversed, and supposing that what is true of the line is true of the movement.'[14] What one needs to add is that what is assumed in Zeno's story of the immobilised runner is true only of a theoretical line, in theory, not of any real line, in reality. This point has been made in a variety of commentaries, such as the following: (this is from a discussion of another paradox of Zeno, 'Achilles and the Tortoise', but it is equally applicable to the Dichotomy):

> In a physical race, what can we do in the way of marking points on Achilles' distance corresponding to the terms of the infinite geometric series? We may mark many such points. But they are physical points and are therefore unlike mathematical points that have no size. Physical points always have some size. Hence arises the difficulty of packing an infinite number of them into a finite distance. Even if we make the points extremely small, this cannot be done. Even though we make them as small as we please, they still, so long as they are physical and thus greater than zero, cannot be

> packed into a finite distance. And, if they are reduced to zero, they are no longer physical, but mathematical and no longer relevant. Nor can any device of 'infinitesimals' enable us to pack in an infinite number of them: 'vanishing quantities', 'ghosts of departing quantities' of whatever minuteness greater than zero can always by amassed in too great numbers to be packed into a finite distance.
>
> This, I think, is the easiest way of seeing that Zeno's premise cannot characterise a physical race: the 'and so on' is inapplicable because somewhere two neighboring physical points will touch each other and it will be impossible to subdivide the distance between them without altering the assigned size of the points (J. O. Wisdom, 'Achilles on a Physical Racecourse', in Salmon, 1970, pp. 86–7).

As with Newcomb's paradox and the paradox of the selfishness gene, there is something in the way Zeno's problems are set up that brings an other-world assumption into a real world situation. A salient problem is the assumption of infinite divisibility of matter, space, time and motion: the same word, *continuity*, is used with all four, and the *theoretical possibility of infinite divisibility* which accompanies 'continuity' is assumed to follow.

It is difficult to accept that an idea like infinite divisibility, which has become so familiar to us, has no real world meaning.[15] Another way to think about problems of measurement is to focus upon the fact that *perfectly precise* physical measurement requires the identification of *points*; e.g., if you wish to measure the distance between A and B, you have to define precisely the points where A and B are located.

Suppose we try to get around this by deciding that, rather than dealing with pre-existing, real world points, we will create points which, *a priori*, are twelve inches apart. In conception, this seems reasonable; what happens when we try to put it into practice? As we take out stronger and stronger microscopes with which to focus on the two ends of our real-world, twelve-inch-ruler, we will find the ruler to be more and more bumpy. We will have to make judgments on whether to measure from the end of *this* molecule or *that* (for a while we are back in the terrain of fractals); then, as we get down to the atomic, and finally the subatomic, levels (moving into the field of physics), we will find that there is much more 'space' than there is 'matter'; that the matter, such as it is, refuses to stay still; and that there is no way of locating '*the* point indicated by the end of the ruler'.[16]

A similar argument will show that 'the point halfway between' the two ends of the ruler – or between points A and B – *has no real world* *existence.* Zeno supposed that before we go *to* the end point we must go *to* the halfway point. The Gordian-knot-cutting fact is that, in this real world, where location is as uncertain as fractals shows it to be (see n. 15, above), *we never go 'to' any points.* The best we can do, in trying to use language to bridge the gaps between metaphors and molecules, is to say that the only relation *real objects* can have with the *ideal concept* of 'points' (in time or in space) is that they go *through* them; hence the notion of the pass-through limit.

In the world of mathematics, we may have dimensionless points; in the real world, dimensionless entities (such as photons?) play no role in dividing up space; dimensioned space is only partitioned (and never precisely so, even at the molecular level) by other dimensioned entities. Nor do the possible interactions of real entities include the idea of precise juxtaposition which Zeno evidently had in mind when he spoke of going *to* a point.

My foot may go *through* a point (when the latter is defined as a dimensioned area), or else it may come to rest *on* it (if the 'point' is imagined on the ground). There is no way that my foot can come to rest 'right next to' a dimensioned point B, because both objects, being material, are composed of a combination of matter and space, where, at the subatomic level, (we are told by physicists), the relations between matter/energy and space are such that the space around each subatomic particle is proportionally like the space around a few dust-motes in a great cathedral. There is only probability to tell us about the relative positions of a particle in my foot and a particle in the graphite pencil mark signifying point B; the probability of their actually colliding (being right 'next to' one another – does that phrase still have a meaning at this level?) is vanishingly small.

Summarising this discussion we might say: 'You can't get there (to the world of mathematics) from here (the physical world)'. A very simple example of this fact emerges from Zeno's Dichotomy: you cannot subdivide a real distance so as to arrive at a point. In the real-world game, points are out. Similarly, 'However many moments you can mention you are still only specifying the limits of the periods that separate them, and at any stage of the division you like it is these periods that make up the overall period.'[17]

A note is in order here on the metaphor which introduced the approach of this chapter to certain paradoxes. I am far from the first to have noticed a similarity between the idea of a convergent limit and Zeno's paradoxes of motion. However, I have been careful to keep my use of the limit idea a metaphorical one, while others (e.g., Alfred

North Whitehead and Charles Sanders Pierce[18]) have claimed that Zeno's story really *is about* an infinite series which, in the limit, is convergent. That is what the *mathematical* version of the story is about, but this is precisely why it creates a paradox, for there is another, *real* version of the story, which is about real motion through real space in real time; and this version has nothing to do with infinity.

EMPIRICISM (AGAIN) What may be found particularly alarming about the preceding discussion is that it is so empirical. I have depended upon what I believe about the real world – including beliefs that stem from a lay reading of modern physics, similarly from experimental psychology, as well as from personal experience with decision-making and prediction in myself and other people – to suggest that what have seemed to be paradoxes are, in fact, situations where unreal assumptions (a degree of predictability in human behaviour; a one-to-one relation between definable genes and predictable outcomes; the possibility of accurate measurement to an infinite degree of precision; the idea that material things in the real world may be positioned 'right next to' one another; or the translation, from mathematics, of the concept of a 'point' as something that has meaningful existence in the real world) have been introduced into an otherwise plausible story. Each story blows up – it produces conclusions that are mutually contradictory, or else that contradict our experience of the world – *when such an unrealistic assumption is pushed too close to its limit.*

Past attempts to 'solve' such paradoxes have depended, for the most part, upon logic operating *within* each story. The assumptions of the story were accepted as written, and the game was to try to use the rules of logic to operate on the given elements so powerfully as to escape from their paradoxical traps.[19] I have proposed an empirical approach which, in addition to logical analysis of internal consistency, inspects the story to see how it relates to our beliefs about this world.

If an integral is calculated, and found not to converge to any real number, we do not try to use logic to find ways around the answer: we accept that a given function, integrated up to a given limit, is one that simply does not converge.

When encountering a story that contains a paradox we may, similarly, choose to examine, for plausibility, the way the 'plot' of the story (Achilles tries to overtake the Tortoise; a human subject tries to decide which box to open) combines with the built-in assumptions (space is infinitely divisible; genes or personality 'determine' choices).

Some, at least, of the persistent paradoxes kicking around in the literature of philosophy, mathematics, etc., represent cases where the flaws are not to be found in the internal logic. What then remains for examination is the possibility that it is 'external inconsistency' which has made the story fail to converge. I have suggested that what we should be on the look-out for is an element which has been allowed to go unchallenged because it appears reasonable in the 'middle ranges' (some degrees of probability in some kinds of prediction fit within the real world, as do measurements and locational activities down to some degree of precision; there is a middle range of influence – rarely, absolute determinism – from genes upon behaviour; etc.), but which has slipped too far along toward its limit, or ideal case (e.g., perfect prediction, or perfect precision).

There may be something alarming in such a suggestion. For many of the people who are good at what it takes to be successful in academia it is easier, more pleasant, cleaner, somehow, to work on the logic of internal consistency than to get mixed up with the 'external consistency' issue of how theories, models, etc. relate to empirical facts. Then, too, there is so much known today; as soon as one steps outside of a very narrowly defined area of expertise one will encounter others who know much more than one does oneself. One risks being wrong. If one is *shown* to be wrong, this is embarrassing and uncomfortable, and may be professionally damaging. As academics, we have gotten where we are because we have demonstrated a fairly good ability to protect ourselves against being logically wrong. If, however, we are to lay ourselves on the line in terms of facts about the real world, and, worse yet, if we cannot always choose the factual area to be discussed – if we are expected to address any old kind of reality whose nature happens to be at issue – then there is absolutely no surety. There is no one of us who could not make mistakes in some – many – areas.

All that being said, it is hard to believe that there is much future in an academic attitude that does not try, ultimately, to test theories, hypotheses, models, and other constructs against all that is known about reality. We don't have to be experts on everything to be able to make a first cut at what is reasonable.

The greatest difficulty we are likely to encounter in such an endeavour is with fields that are currently in the process of shaking up old, 'common sense' notions – replacing them with what will probably be the common sense of the future. There, what seems 'reasonable' may now be under attack by the experts, and may cease to seem reasonable a few years hence. For example, at the time of the classical Greek philosophers, many of the discoveries which were most surprising and upsetting to existing beliefs were coming from mathematics; indeed, perhaps it was his effort to accommodate some of the new concepts in mathematics that led Zeno to postulate the impossible situations that have come down the centuries to us. In the nineteenth century such unsettlement came dramatically from evolutionary biology. At our own time in history – indeed, throughout much of the twentieth century – physics has been the area of understanding which most often seems to shake us on matters which we might have thought empirically obvious.

Recall the 'unicorn word' idea which encourages us to draw distinctions between what we believe to be real and what we believe does not belong to this world. We may find it useful to refer to a gnomic formulation:

That we haven't experienced it doesn't mean it isn't.
That we can imagine it doesn't mean it is.

It becomes more difficult to keep these distinctions between reality and conception in mind when we wander into abstruse subjects such as theoretical physics. Modern physics obliges us, for example, to consider it an open question whether such familiar concepts as 'space' and 'time' may be unicorn words. Some of the contentions of modern physics appear to us as paradoxes, in creating conclusions which we intuitively feel to be inconsistent with reality. A prime example is the story of Schrödinger's cat, which extends (to what appears, in commonsense terms, to be an absurdity) the contention that the context of the experiment of the observation will determine, in a deeper way than is familiar to us in ordinary life, the nature of reality at a given moment.

Context, again, is critical. It seems likely that Heisenberg's or Shrödinger's conclusions on the interaction between reality and observation are more relevant to the wave versus particle nature of electrons than to our more commonly experienced reality. What causes an idealisation (such as 'pure observation') to create a 'nonconvergence' is not whether it is, alone, a discontinuity from reality; but whether it is discontinuous from reality in the particular context wherein we are considering it. For example, Schrödinger proposed very special conditions where the state of a cat would depend upon the probabilistic behaviour of subatomic particles. The cat may be simultaneously both dead and alive (just as the particle is, in some probabilistic sense, in more than one position) *as long as it is not observed*; its state will resolve to one or the other only as a result of observation.[20] However, in most situations it is reasonable to assume that the state of a cat in a box is in more determinate than this, even while the cat is not being observed.

THE USEFULNESS AND THE DANGERS OF THE IDEA OF IMPOSSIBILITY

Consider the modern physicist's beliefs (e.g., Heisenberg's or Schrödinger's conclusions on the interaction between reality and observation) as tools. Consider human nature, with its nearly irresistible urge to use, on every object at hand, any new tool that it has in hand. A great deal that is of value – much, indeed, of what we regard as human civilisation – has resulted from this tendency. *Ex ante* we would usually not wish to restrain such exuberance (at least, in the case of *abstract* tools; when the tool is something physical, like a hammer, we may wish to exercise some restraint over the child who has just discovered it!). Nevertheless, *ex post* we can say that some uses of such and such a tool are better than others; and some are positively misleading. Zeno had applied a set of tools – the abstract idea of infinity, and its corollary idea, infinite divisibility -- to a piece of reality where it had no place. This inappropriate application is hard to spot, because the ideas are entirely appropriate with respect to some things (e.g., the real-number line, or set theory) whose dissimilarities from the stuff (real time, real space) to which Zeno had applied them are not immediately obvious.

The concept of 'inapplicability' to which I have just appealed depends upon the idea, brought out earlier, of 'impossibility'. It is important to post some warnings on this subject. History is littered with impossibilities disproven, and of successes which could only be achieved by ignoring the very idea I have been promulgating, of skirting impossibilities.

A good example is a story which is told of Albert Einstein: when praised for his mathematical prowess, he replied that there were many better mathematicians than he; his successes stemmed from the fact that, when his mathematics led him to what seemed an impossible conclusion, he went ahead with the mathematics instead of being blocked by the common sense notion of what is possible. It appears that this was an appropriate and useful frame of mind for a physicist in Einstein's time; it is likely that there will be other times in other disciplines where it is also the essential precondition for progress.[21]

In this, as in so many things, there is a pendulum of fashion. In the Victorian era, imaginative writers railed against the prevailing fashion of thinking, which required that people only believe in 'hard facts' – in what they could see in front of their noses. The influence of Einstein and his followers in physics has doubtless played a part in turning this fashion around; the field of physics has had such an impact in this century that what works there is bound to be tried out almost everywhere else. In any case, intellectuals now seem eager to follow the advice given to Alice by the Red Queen – to practice believing a few impossible things every day, before breakfast.

I can propose no algorithm for testing when the 'Einsteinian' frame of mind is, and when it is not, appropriate. The ability to believe the seemingly impossible is obviously of great importance in some cases: in other cases, dogged insistence on remaining within the bounds of what seems to 'make sense', though less romantic, may be the strategy that will stay closest to the truth. The 'Einsteinian' frame of mind may have been, and may continue to be, the doorway to knowledge in physics (though perhaps it has somewhat overused by now). In the field of economics, which is, after all, largely about subjects that are on the human scale of magnitude (because the subjects are human beings), I would suggest that, in the majority of cases, we should only as a last resort abandon (though we should always be willing to reexamine) our common-sense beliefs about what is possible in this world. Ploughing the fields of the counter-intuitive has been practised so much in recent economic work that what was to be harvested therefrom may almost all have been gathered in.

At the same time we must be warned that, while the set of 'the possible' may (or may not) be finite, it seems most likely that it is, in any case, unbounded. If we rule certain topics, approaches, packages of ideas, etc., 'out of bounds' because they seem to us to make an 'impossible' use of some idealisation, we could be running the risk of letting common sense blind us to some important counter-intuitive reality. We are in a situation like that of statistics, where every action that decreases the alpha risk increases the beta risk.

There is, and always will be, a tug-of-war between common-sense empiricism and theoretical speculation. These two aspects of human nature often pull in opposite directions. At the same time, each is essential as a complement to the other. Perhaps the chief effect of this chapter's extended use of a metaphor from the mathematical concept of limits, with their properties of convergence, divergence, etc., is to give aid and comfort to the common sense side of this 'dichotomy' in human nature. We all knew, all along, that altruism exists, even though some users (or misusers) of sociobiology claim that it cannot; and we all know that we are capable of moving, whatever Zeno may have 'proved'. What I have tried to show is that the common-sense response to these paradoxes of external inconsistency – 'I don't care what you say you've proved; that's not the way the world is!' – is soundly based: the paradoxes arose because the stories incorporated assumptions which are, in fact, contrary to the way the world is.

The particular counter-factual assumptions which we encountered in the paradoxes of this chapter fell into two groups:

A Some had to do with **human nature**: how and to what degree it is determined by personality; and how and to what degree personality is determined, in turn, by genetic composition. From this there arise questions about how accurate and precise predictions about human behaviour can be made to be; and on what such predictions would be based.

B The other group had to do with **the nature of the physical universe**. Some aspects of this physical reality are unlikely to be the basis for a paradox because they are sufficiently obvious that we could not easily be fooled into accepting their denial as part of a realistic story. (For instance, if Zeno's stories began with the statement, 'In order to travel any distance it is necessary to be able to move at infinite speed', we would reject it outright. By contrast, the statement 'Before you can go the whole distance, you must first traverse half the distance' raises no problems; only later is the corollary slipped in that, to traverse half the distance you must *come to the half-way point*; that is a subtle enough issue that it, too, fails to raise our intuitive hackles.)

Those aspects which were teased out in the discussion as containing hidden problems had to do with the realisticness of applying the ideas of e.g., 'points' and 'lines' and 'infinity' to the natural world; if these things are called into question, we then have to consider that an idea such as 'precise measurement' may also be without real-world meaning (although it has plenty of abstract meaning.)

To extend our knowledge of the physical world as far as possible, it was necessary to appeal to some of what is becoming the subject of common belief in the field of modern physics. Some of the conflicts which arose here could be seen as between, on the one hand, pure mathematics (with its zoological garden of unicorn concepts such as infinity, the square root of minus one, and other far fancier impossibilities), and, on the other hand, applied physics (which reveals more than is evident to the unaided senses about the nature of matter, space and time – although all of these concepts tend to suffer identity-shifts when examined at great removes from ordinary sensory experience). Economics, which has for so long suffered from physics-envy, can take comfort at least in this: that, to the extent that physics is 'about' real things in the real world, and mathematics is not, economics is more similar to the former than to the latter.

Notes

. This chapter was first presented as a seminar at the World Institute for Development Economic Research in Helsinki. I am extremely grateful to WIDER for the opportunity there afforded me to receive much helpful commentary.

The title originally referred to 'realism', but a participant in the seminar, Uskali Mäki, suggested the change on the grounds that 'While realism is a philosophical doctrine (or divides into many such doctrines), realisticness is a property (or many such properties) of representations, including economic theories and their assumptions.' (Uskali Mäki, 'On the Problem of Realism in Economics', in *Ricerche Economiche*, special issue on 'Epistemology and Economic Theory', March 1989 Abstract). See further Mäki's definition of the philosophical meaning of realism in economics in Chapter 8, above, under 'the Standards Required for "Scientific Belief"'.

2. Some of the debate is summarised in the 1979 essay by Lawrence Boland, 'A Critique of Friedman's Critics', *Journal of Economic Literature*, 17, pp. 503–22.). Some later commentaries include:

Bruce Caldwell, 'A Critique of Friedman's Methodological Instrumentalism', *Southern Economic Journal*, 47 (1980), pp. 366–74;

William Frazer Jr and Lawrence Boland, 'An Essay on the Foundations of Friedman's Methodology', *American Economic Review*, 73 (1983) pp. 129–44;

Abraham Hirsch and Neil de Marchi, 'Making a Case when Theory is Unfalsifiable; Friedman's Monetary History', *Economics and Philosophy*, 2 (1986) pp. 1–22;

Uskali Mäki, 'Rhetoric at the Expense of Coherence: A Reinterpretation of Milton Friedman's Methodology', *Research in the History of Economic Thought and Methodology*, vol. 4 (1986) pp. 127–43.

Uskali Mäki, 'Friedman and Realism', forthcoming in *Research in the History of Economic Thought and Methodology*, vol. 8 (1990) (preliminary draft, March 1989).

I have found Mäki's work especially helpful, and have further benefited by discussion with him. His formulation of Friedman's view is the statement that

> economic theories should be accepted as good predictors (but not believed to be true) and rejected as bad predictors (but not believed to be false). According to this conception, nothing follows from acceptance of a theory about its truth and about the existence of its objects. Beliefs about these questions (i.e., the truth value of a theory and the existence of its objects) are formed on grounds independent of accepting or rejecting a scientific theory (Mäki, 'On the Problem of Realism in Economics', p. 25).

3. The sociology-of-knowledge kind of reasons why Friedman's approach has continued to have such force are well laid out in Mäki, 'Friedman and Realism'.
4. I am grateful to Thomas Schelling for mentioning this to me as a particularly questionable frequently made assumption.
5. This line of reasoning, if it proves fruitful, may ultimately require definition of the implications of different kinds of impossibility; but it will not be possible to pursue that here.
6. See Philip Mirowski, 'The Probabilistic Counter-revolution, or How Stochastic Concepts Came to Neoclassical Economic Theory', *Oxford Economic Papers*, 41 (1989) pp. 217–35). Also, Mirowski's history of the influence of physics upon economics, *More Heat Than Light* (Cambridge University Press, 1989).
7. This subject will be discussed in *Social Economics*, volume 2, where the counter-balance to competition just cited will be given the name of 'the pan-human conspiracy'.
8. This paradox was first constructed by the physicist, William Newcomb, of the Livermore Radiation Laboratories. It was first published by the philosopher, Robert Nozick, in 'Newcomb's Problem and Two Principles of Choice', in *Essays in Honor of Carl G. Hempel*, ed. by N. Rescher *et al.* (Reidel, Dordrecht, 1969). Among the many articles that have been written about it since, two that are particularly relevant to the discussion that will follow are J.L.Mackie, 'Newcomb's Paradox and the Direction of Causation', *Canadian Journal of Philosophy*, 7 (1977); and M.Dummett, 'Causal Loops', in *The Nature of Time* (R. Flood and M. Lockwood, (eds) (Basil Blackwell, 1986).
9. R. Nozick, 'Newcomb's Problem and Two Principles of Choice', p. 114.
10. Cf. Nozick's footnote:

> But it also seems relevant that in Newcomb's example not only is the action referred to in the explanation of which state obtains . . . but there is also another explanatory tie between the action and the state; namely, that both the state's obtaining and your actually performing the action are both partly explained in terms of some third thing (your being in a certain initial state earlier) (ibid., p. 146).

11. The strand of sociobiology here referred to is often taken to derive from Richard Dawkin's book, *The Selfish Gene*. He was not, however, talking about a *gene for selfishness*, which is what is at issue here. Martha Nussbaum kindly suggested to me the term, 'the selfishness gene'.
12. Mary Midgley, *Beast and Man: The Roots of Human Nature* (Cornell University Press, New York, 1979) pp. 128 and 134–5. It is regrettable that this excellent book is now out of print.
13. Adolf Grünbaum, 'Modern Science and Refutation of the Paradoxes of Zeno', in Wesley C. Salmon (ed) 1970. (With respect to the references to 'modern physics', note that this essay was written in 1955.)

    For further discussion of continuity and the denseness postulate, refer to Chapter 6, under 'the Use of Mathematics to Address Problems with Time and Change'.
14. Henry Bergson, 'The Cinematographic View of Becoming', in Salmon, 1970, p.64; italics added).
15. If we require persuasion that we have been much too optimistic in combining the notions of *precision*, *measurement* and *divisibility*, we may appeal to the field of fractals, which has brought into mathematical consciousness the imprecision of measurement and of location. The fractal emphasis upon level of focus shows that a different level will produce different measurements, e.g., the famous example of attempting to measure a shoreline, which runs as follows:

    > If you take a thread and lay it carefully on a map's depiction of a piece of shoreline – let us say, the shoreline of Cape Cod – you may then stretch out the thread and, by translating back into the measurement of reality from the key of the map (e.g., 'one inch equals five miles'), you will have a statement of the length of that shoreline. However, if you do the same with a map made to a much finer level of detail (e.g., one inch equals 1/2 mile), wherein the gross outline of promentories and bays is resolved to more detailed ins and outs, you will come up with a measurement considerably longer than the first.
    >
    > Now if you drive to Cape Cod and start laying a tape measure along the shore, curling it around each rock that meets the water, the measurement will become much longer again. It will increase once more when you begin using a thinner, more malleable tape, so as to account for all the roughnesses and barnacles on the rocks, and for the individuals granules of sand. When you get out your magnifying glass, and then your microscope, using appropriately more refined measuring instruments, the 'shoreline' will continue to have a greater measured length, as smaller and smaller bumps and indents are accounted for.
    >
    > I will not propose imagining that you continue measuring down to an infinitely fine level of detail (!); the picture is quite complicated enough without that. Which measurement, of those within the realm of conceivable, this-world possibility, represents the 'true' length of the shoreline of Cape Cod?

    For a trenchant illustration of how too glib a transition from the mathematical idea of a continuum (with infinite divisibility) to the spacial reality of a continuum can suggest that a finite space can contain an infinite amount (of copper, in the example given), see Daly and Cobb, p. 40.

16. Another way of understanding this was suggested to me in a comment by a physician, Richard Rockefeller: at the atomic level of reality it is meaningless to discuss matter in terms of *location*; matter resolves to energy, which never stands still, so that it can be said to 'come to' or to 'be at' any place.

If we try to locate our 'points' by means of triangulation – e.g., 'Point A is three metres south of the exterior corner of this building, and five metres west of that path' – we will be right back to where we began: how do we find the 'edge' of the path from which to measure five metres? Moreover, any way we devise of bringing out the measurements from the corner and from the path will have a thickness: the two real world representations of abstract 'lines' (even if they are laser beams) will intersect in an area, not a point – unless we do it only in our heads. The point, once again, is the difference between the idea and the reality.

17. G. E. L. Owen, 'Zeno and the Mathematicians', in Salmon, 1970, p. 158.

Putting this idea into more mathematical terms, 'the division of an interval effects *no* reduction in the cardinality of the resulting subintervals as compared to that of the original interval.' (Adolf Grünbaum, 'Zeno's Metrical Paradox of Extension', in Salmon, 1970, p. 191). Grünbaum discusses the idea of 'a line' within Cantorean set theory, where it represents the kind of infinity called 'non-denumerable'; any subdivision of such a set is also a non-denumerable infinity. Since points do not exist in the world of material things, and the distance between any two objects is a real distance (even if it cannot be represented, in the real world, by a line, because lines, like points, do not exist in this sphere; and even if its length and end points cannot, in actual fact, be measured and located with absolute precision) – this should make us wonder about the meaning of Grünbaum's introduction of a word to arithmetic from set theory, when he explains a finite interval as 'the union of a continuum of degenerate intervals' (ibid., p. 193). These 'degenerate intervals' (i.e., points) do not actually exist; they cannot be arrayed 'in a continuum' because you never move away from the original dimensionless point when you put other dimensionless points 'next' to it: 'next to' a point is right there, in the same place.

'Union' has a perfectly good meaning in set theory. The attempt to apply it to a spatial description must be recognised as, again, a metaphor, and not a very successful one, for it does not suggest any new or real answers to the original paradox. Grünbaum indeed concludes that 'We are here confronted with an instance in which set-theoretic addition (i.e., forming the union of degenerate subintervals) is meaningful while arithmetic addition (of their lengths) is not (*ibid.*).

18. 'Writing early in the twentieth century, Pierce remarked of "The Achilles" that " . . . this ridiculous little catch presents no difficulty at all to a mind adequately trained in mathematics and in logic". I presume his low opinion reflected a belief that the entire source of the paradox was Zeno's inability to realise that an infinite series could have a finite sum' (Salmon, 'Introduction' to *Zeno's Paradoxes*, pp. 25–6).

19. E.g., for an analysis for Newcomb's paradox which attacks the story from many angles, but in each case focussing only upon the *internal* logic, see Isaac Levi, 'Newcomb's Many Problems'; *Theory and Decision*, 6 (1975).
20. I would hazard the guess that this story may, indeed, be a true paradox, not merely a situation that appears paradoxical because of our naiveté about physical reality. The limit case that creates the problem could be any one of a number of things. One candidate may be the simplistic pairing of the on/off states of life/death with two possible sets of behaviours of the particle, so that either it *will* trip the hammer that breaks the flask that empties the poison that kills the cat – or it *won't*; the probabilistic behaviour of particles may not resolve so neatly. Examples of other places to look for a discontinuity between the limits assumed in the story and what is possible in reality are: the assumption that what is true for particles is true for the things they affect; or the assumption that all of the things which we think of within the category, 'observation', are continuous in their nature and their effects.
21. There are physicists who would say that Einstein could have made contributions to the emerging field of quantum mechanics, but that he drew back because he ran up against the limits of his credulity: a flexible space/time concept was acceptable, but a world stochastically determined was not – in his famous comment, 'God does not play at dice'. The 'Einsteinian position' cited in this chapter refers to his earlier willingness to give credence to the apparently incredible.