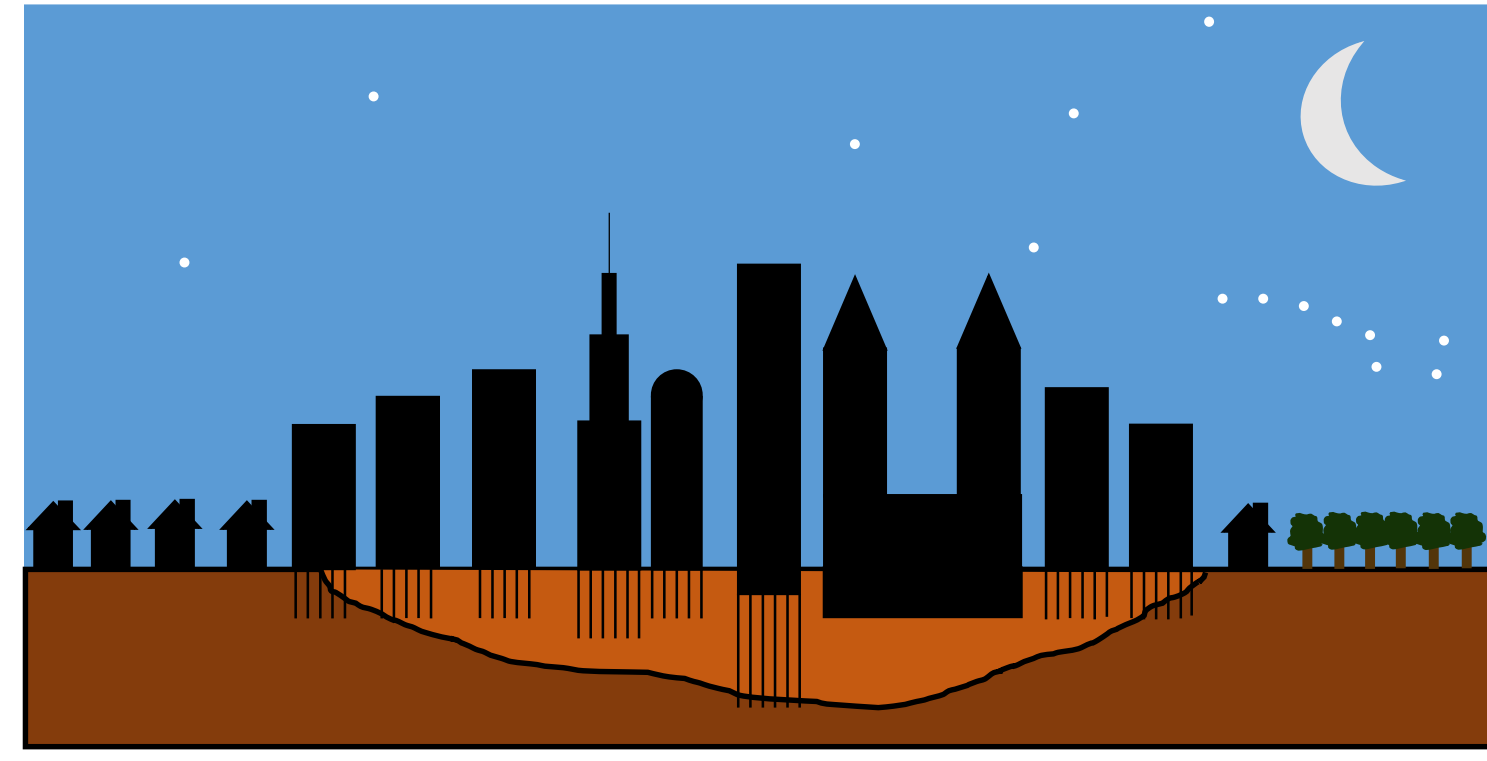


# Classifying Sedimentary Basin Resonance Using H/V Ratios: Mexico City Case Study



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## Abstract

Strong impedance contrasts are known to play a role in soil amplification both from the fundamental physics of conservation of energy across a strong impedance contrast and from observations with examples from around the world. From prior work by the author and others (Thompson et al., 2009; Thompson et al., 2012), we know that a strong peak in the weak motion surface/borehole transfer function is consistent with resonance due to vertical propagation of shear waves through horizontally layered soil systems. The shape of the transfer function can be altered by scattering through heterogeneous materials, significant attenuation, non-vertical incidence, and other complexities in the subsurface. In this work, we focus on evaluating site response complexity using only surface stations. Microtremor studies using Nakamura's technique (Nakamura, 1989), which is the ratio of the Fourier amplitude noise spectra of the horizontal component to that of the vertical component (H/V), have proven useful in strong impedance-contrast environments to estimate the fundamental site period, sediment thickness, and ultimately soil amplification. H/V ratios have also been used to evaluate site response using earthquake ground motions. While fundamental site period is reliably estimated with H/V ratios, the amplification is sensitive to the noise environment and the signal processing and therefore, is not generally trusted. In this work, we focus on the shape of the peak in H/V ratios as an indicator of site response complexity. Using earthquake recordings from Mexico City, we demonstrate that H/V ratios can be used as an indicator of complexity in site response, in a similar manner to that demonstrated by Thompson et al. (2012) using surface/borehole transfer functions. In Mexico City, non-resonant H/V ratios are consistent with the transition zone at the edge of the Lake Bed sediments.

Site response: describing equations 1, 2 and 3 as a cartoon

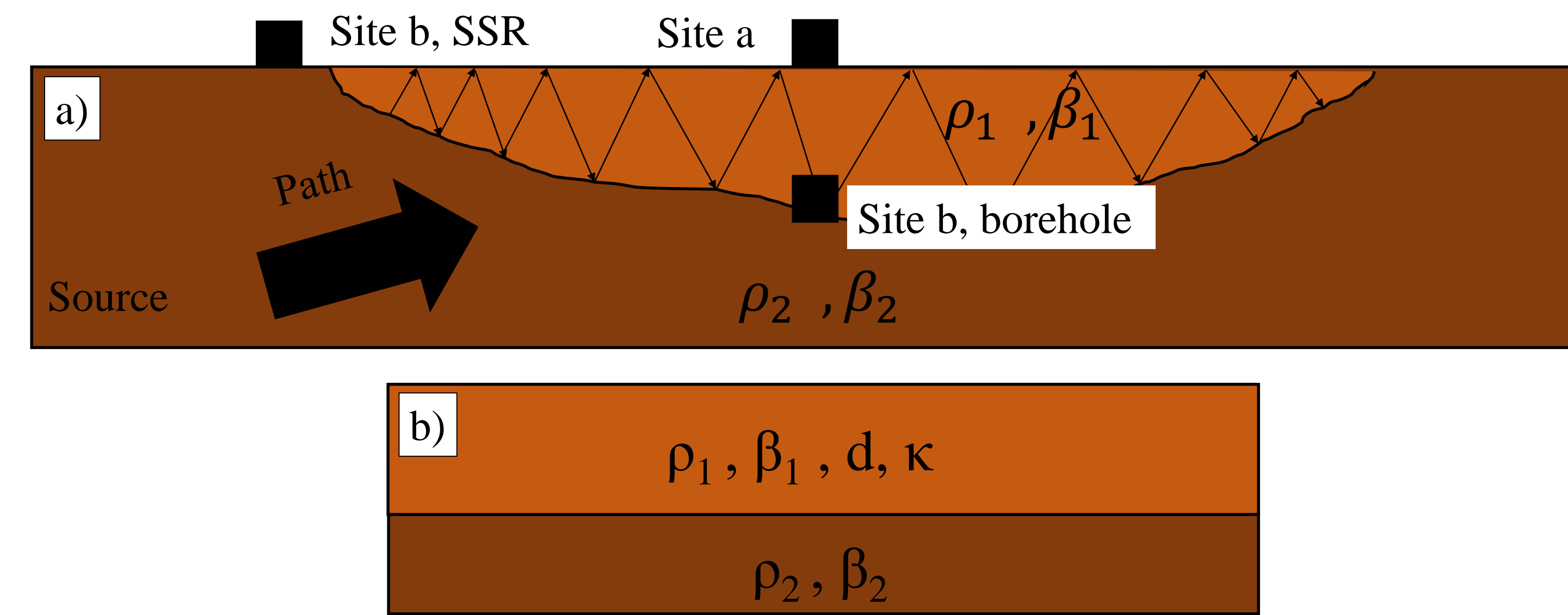


Figure 1. a) Cartoon of a sedimentary basin with three seismic stations, one at the site of interest, one at the base of the overburden and one on a reference site. The source, path and site are on the right side of equation 1. The recordings at these sites can be used for computing the borehole, standard spectral and H/V transfer function using the ratio in equation 5. b) The variables used for computing the theoretical transfer function

Fundamental site frequency is a function of shear wave velocity and depth

Equation 1.1:  $|v_0/2v_2| = \rho_2\beta_2/\rho_1\beta_1$   
Equation 1.2:  $f_m = m\beta_1/4d_1$

Equation 1.1.1) Normal incidence shear waves propagating vertically through low velocity overburden overlying high velocity basement amplify as a function of the impedance contrast where  $v_0$  is the displacement at the free surface,  $v_2$  is the displacement amplitude of the 2<sup>nd</sup> layer and  $\beta_1$  and  $\beta_2$  are the shear wave velocities of the overburden and basement layers respectively (figure 1). 1.2) This amplification occurs at fundamental frequencies as a function of the shear wave velocity ( $\beta_1$ ) and depth ( $d$ ) of the overburden layer (figure 1) (Haskell 1960).

We leverage the assumption that the Earth is a linear, time invariant system (eq. 2) to compute site transfer functions using spectral ratios (eq. 3)

Equation 2.1:  $s(t) = i(t) * h_e(t) * h_g(t) * h_r(t)$

Equation 2.2:  $S(f) = I(f) \times H_e(f) \times H_g(f) \times H_r(f)$

Equation 2.2.1) Assuming linear soil properties, ie. strain independent shear modulus, and vertically propagating shear waves, we can solve for the surface response to a forcing function in the frequency domain using the Fast Fourier Transform (FFT). If we assume that the source input, earth's crust, site geology, and instrument response are linear time invariant systems, then the seismogram recording  $s(t)$  is a linear combination of these systems given by the convolution: where  $i(t)$  = impulse response (source),  $h_e(t)$  = earth's crust (path),  $h_g(t)$  = site geology (site), and  $h_r(t)$  = instrument response (instrument) and "\*" is the convolution operator (figure 1a) (Borcherdt, 1970). 2.2) The Fourier response spectra of the seismogram is equal to the product of the Fourier transforms of the respective systems:

Equation 3:  $a(f) = \frac{S_{ha}(f)}{S_{hb}(f)}$

Equation 3. Assuming an earthquake at a large hypocentral distance from two stations with the same recording instrument, a and b, the source, path and instrument response of these stations are equal, thus the transfer function,  $a(f)$ , from station b to station a is represented by the ratio of the amplitude response spectra of the horizontal component of station a and station b (figure 1). Spectral ratios allow us to compare the frequency content of the ground motions at each station (Thompson et al. 2012). Since we are interested in the magnitude of the acceleration at different frequencies, we compute equation 1.2.3 using the amplitude spectra of  $S_{ha}(f)$  and  $S_{hb}(f)$ . There are three methods of computing the ETF using spectral ratios, in order of decreasing cost and accuracy: borehole, simple spectral and H/V ratio (see figure).

We also think about soil response to a vibration in the time domain as a function of the damping ratio  $\zeta$  and the ratio of fundamental frequency of soil column and forcing function (earthquake ground motion in bedrock).

Equation 4:  $\frac{u_0}{(u_{st})_0} = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2}}$

Equation 4. Seismic waves propagating through the bedrock come into contact with the basin overburden and cause it to resonate. In this instance, the seismic waves are the "forcing function" and they excite the overburden based on the relationship between the frequency of the seismic waves ( $\omega_n$ ) and the fundamental frequency of the overburden ( $\omega$ ). The closer the fundamental frequency of the overburden is to the frequency of the forcing function, the greater the ratio of the maximum amplification of the overburden to the initial displacement imposed on it by the forcing function.  $u_0$  is the steady state amplitude and  $(u_{st})_0$  is the maximum value of the static deformation and their ratio is  $R_d$  or the amplification factor. As the ratio of the excitation frequency to that of the fundamental frequency approaches one, the overburden amplifies. The smaller the damping ratio, the greater the amplification factor (Chopra, 2007) (figure 6).

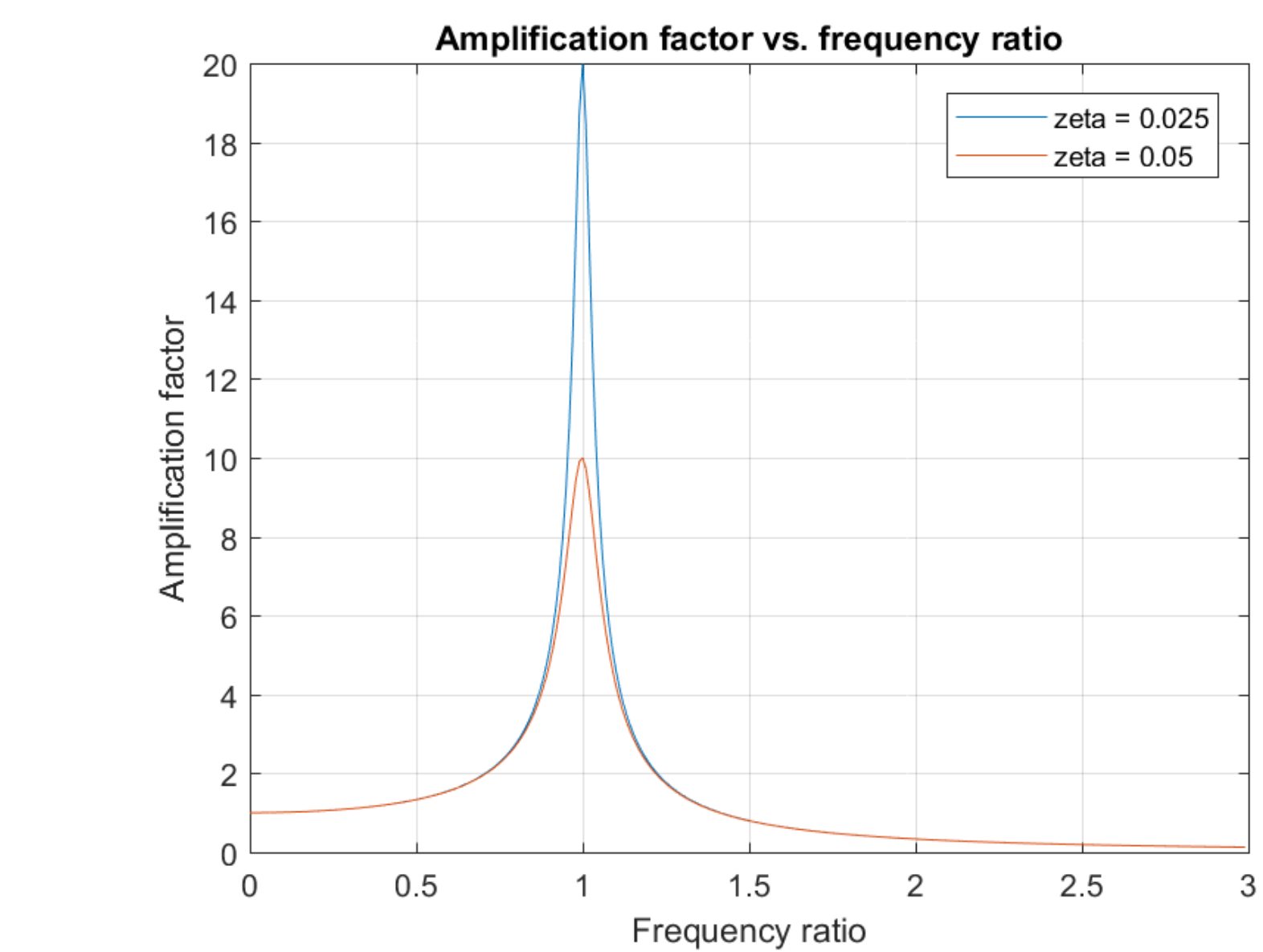


Figure 6. Amplification factor versus frequency factor using two different values of the damping ratio  $\zeta$ . From equation 4, the "amplification factor" is  $u_0/(u_{st})_0$ . The "frequency ratio" is  $\omega/\omega_n$ .

## Conclusion

The H/V ratios at sites presented in this study exhibited behavior expected of a soil column subject to an earthquake forcing function assuming a single laterally homogenous soil layer ( $V_s = 70$  m/s) overlying basement rock ( $V_s = 475$  m/s), vertically propagating shear waves (horizontal component), frequency independent damping, and a strain independent shear modulus. The harmonic modes and amplitude are well modeled in the lake zone. Although the fundamental peak amplitudes in the transition zone match well, the observed peak width is wider than the model, and the higher harmonics are not observed, perhaps suggesting attenuation due to wave scattering. The H/V ratio, therefore, may serve as a proxy for the site transfer function providing an inexpensive site characterization method for site response studies.

## References

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## Hill Zone and Compact sites, FLAT

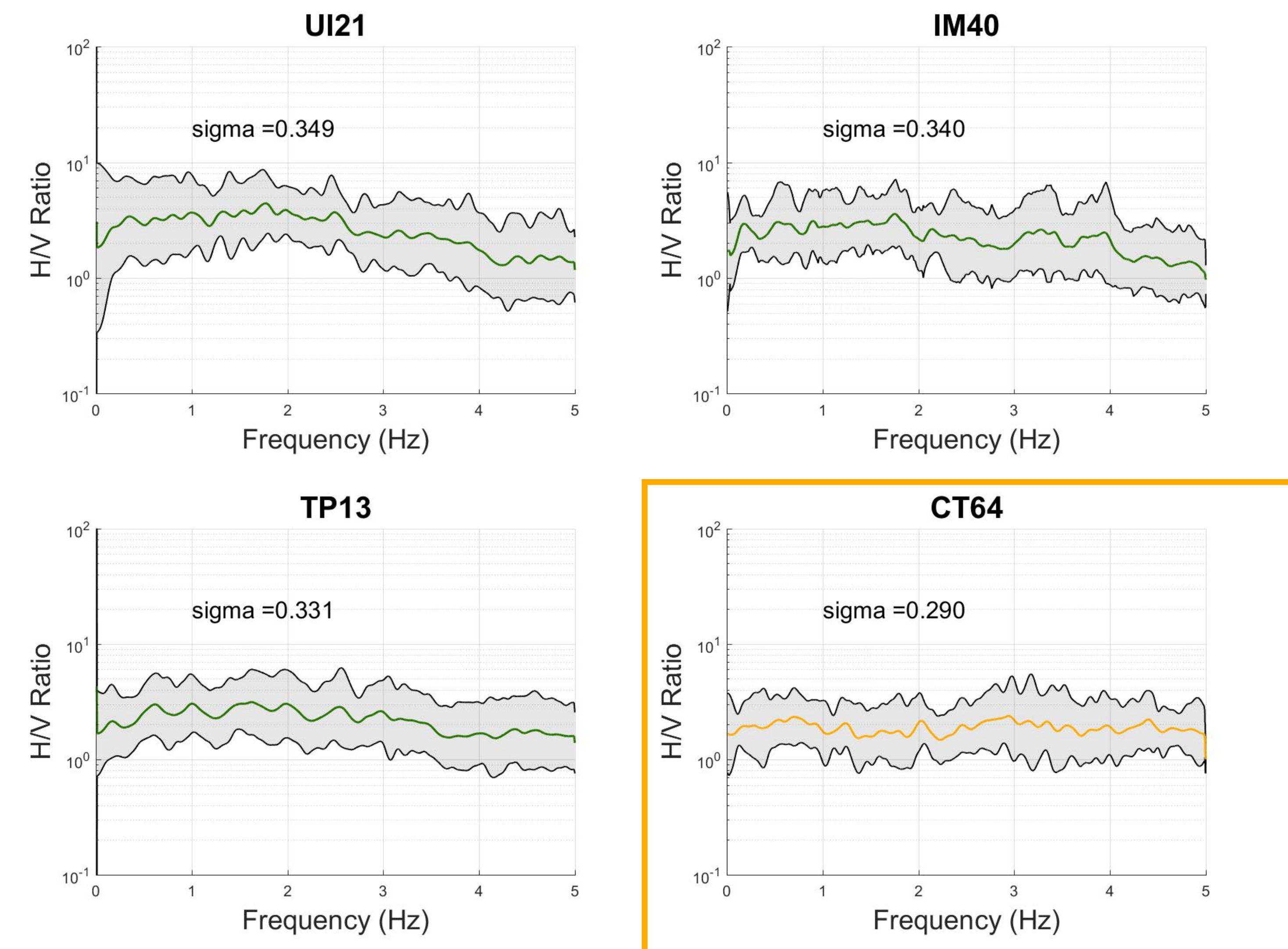


Figure 3. Three examples of hill zone sites plotted in green and one compact site plotted in orange. Their interevent variabilities are indicated and calculated from the median standard deviation between 1 and 5 Hz. These sites are flat because they don't exhibit resonance, ie. the frequency content of the horizontal component is roughly equal to that of the vertical component. In these examples, however, the H/V ratio is slightly above one implying that the horizontal energy content is actually slightly larger than that of the vertical. This is being explored by comparing the HVSRs to the SSRs.

## Mexico City RACM Network

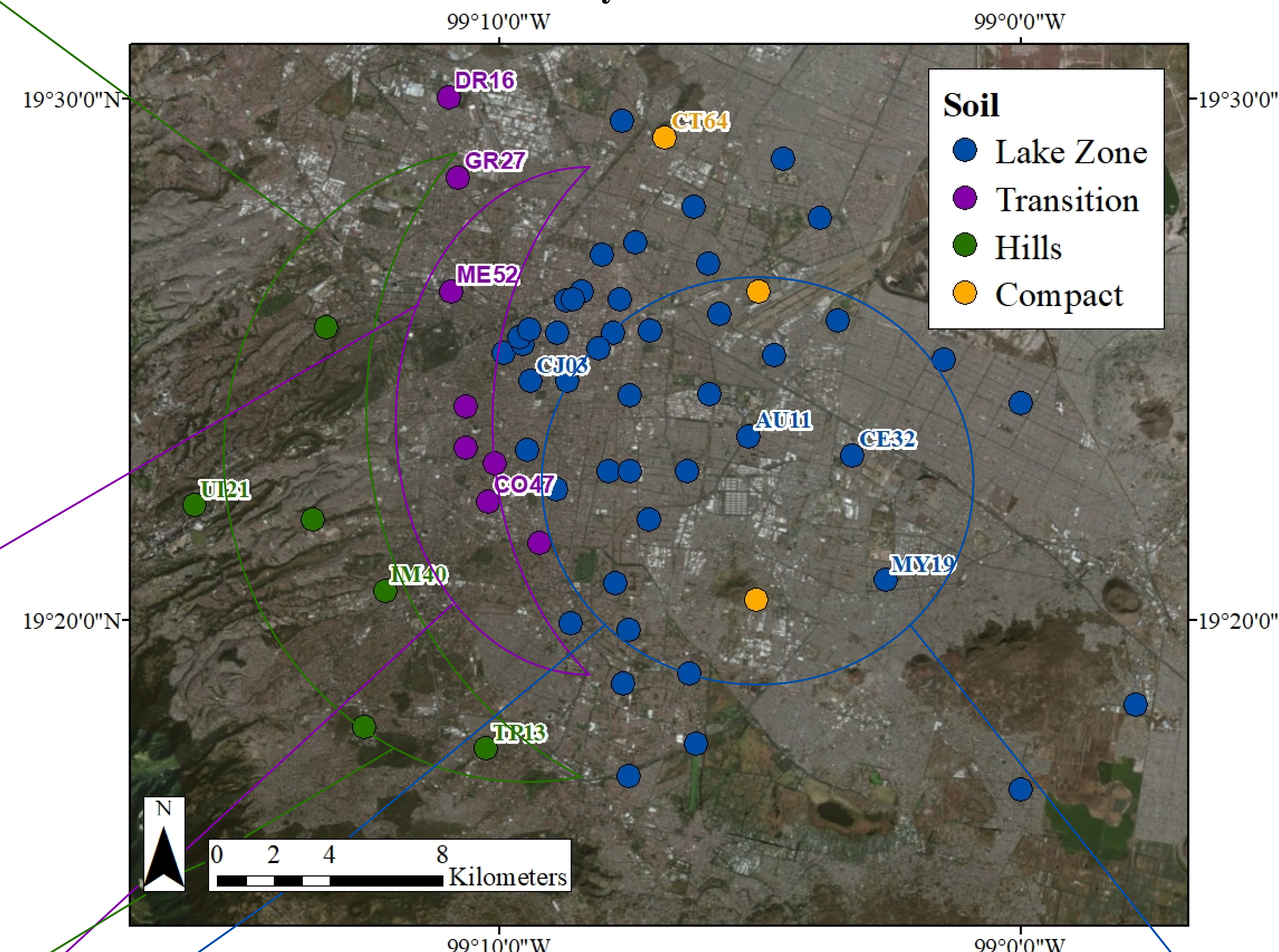


Figure 2. Map of the Mexico RACM seismic network with soil classification indicated in color. The four hill sites excluding UI21 and TP13 were reclassified from "compact" to adhere to their official geotechnical zone.

## Transition Zone sites, ONE BROAD PEAK

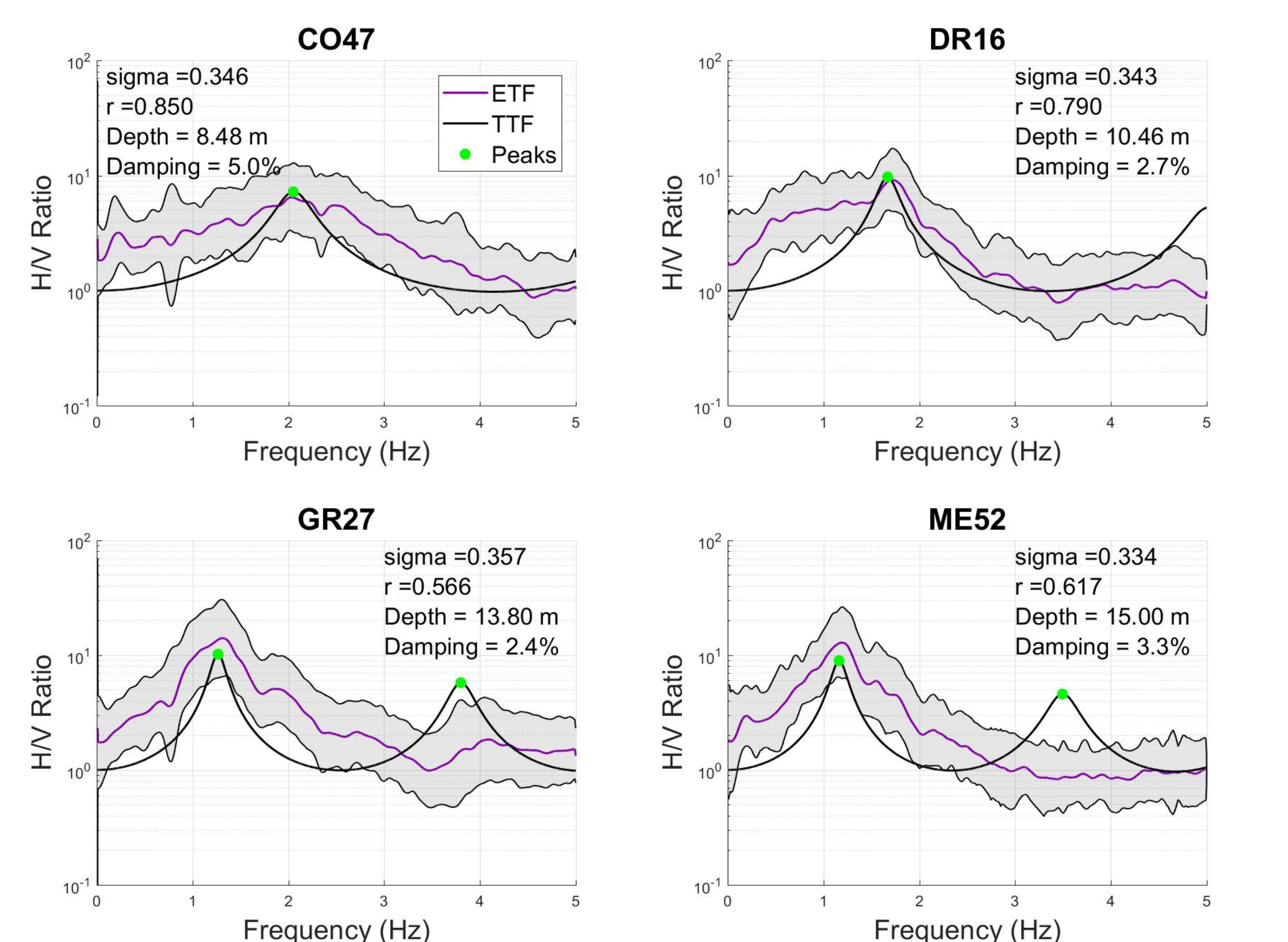


Figure 4. Four examples of transition zone sites plotted in violet. Interevent variabilities, Pearson's correlation coefficients are computed between the first peak of the TTF and 5 Hz. Depth and damping values were computed from the TTF inversion.

## Lake Zone sites, SEVERAL THIN PEAKS

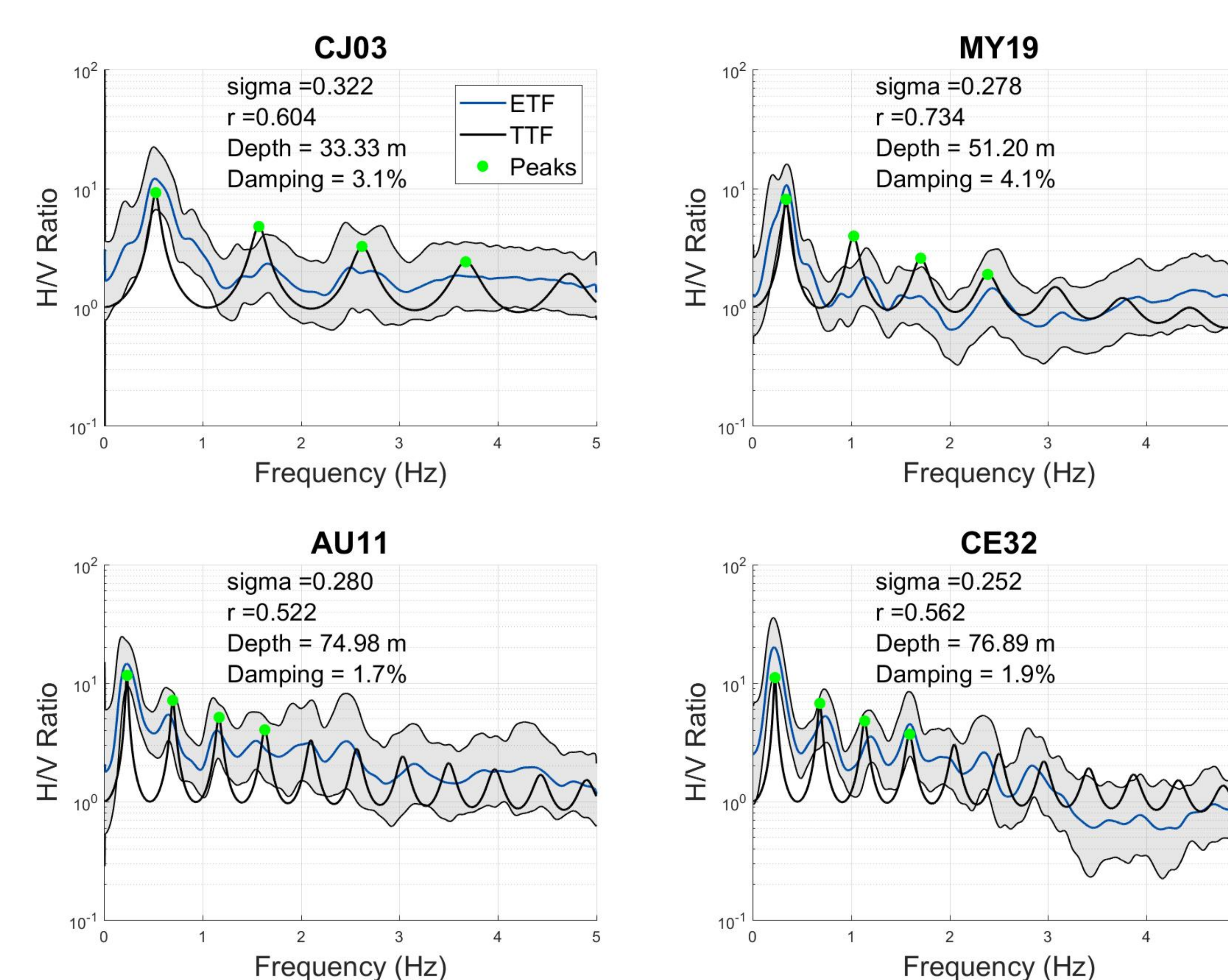


Figure 5. Four examples of lake zone sites plotted in blue. Interevent variabilities and Pearson's correlation coefficients are computed between the first peak and fourth peak of the TTF. Depth and damping values were computed from the TTF inversion.