CEE-245 Final Project: Modeling Mexico City with vertically propagating shear waves through a uniform, damped soil layer on elastic basement rock

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In this study, we solve the site response problem of vertically propagating shear waves through an infinite halfspace of one uniform damped layer bounded by a free surface at the top and an infinite basement layer with definite physical properties at the bottom. We find that the model is a relatively good approximation of the Mexico City soil profile.

Problem introduction and important equations

We begin our derivation with a description of the soil system and an overview of the important equations necessary for the derivation. The soil system in question is an infinite half space with a single uniform damped layer over a basement layer. Both layers have a modulus, G, density, ρ, shear wave velocity, v_s, and damping ζ. Each layer's respective properties are labeled in figure 1.

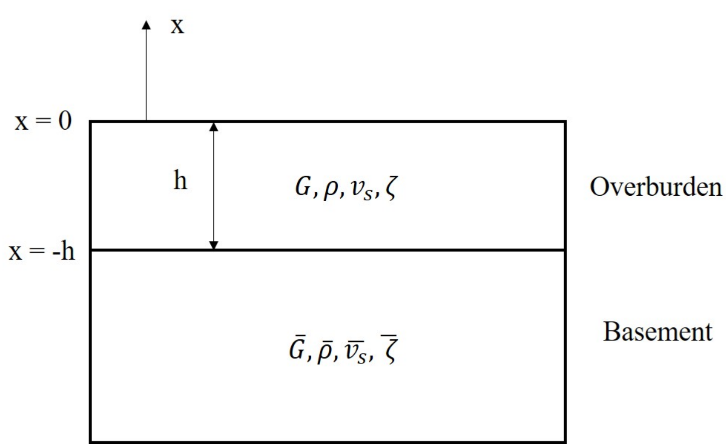

Figure 1: Cartoon representation of 1D shear wave propagation through uniform, damped soil over elastic rock soil system. The symbols for each layer's properties are labeled.

The system's shear modulus, density and shear wave velocity are interrelated by:

$$v_s = \sqrt{\frac{G}{\rho}} \tag{1}$$

and k, the wavenumber and ω, the angular frequency are related to v_s by

$$v_s = \frac{\omega}{k} \tag{2}$$

and the damped shear wave velocity (from Kramer 1996 page 260) is

$$v_{sdamped} = v_s(1 + \zeta) \tag{3}$$

To reduce terms of the form e^{ix}, we use

$$e^{ix} = cos(x) + isin(x) \tag{4}$$

and we use

$$cos(x) = cos(-x)\; and\; sin(x) = -sin(-x) \tag{5}$$

Mexico City subsurface properties

In this study, we will test our model using a simplified approximation of the Mexico City subsurface. Mexico City is built at the location of three historic shallow lakes: Texcoco, where most of the urban sprawl is now located, Xochimilco, to the southwest of Texcoco and Chalco, to the southeast of Texcoco. They were filled with windblown volcanic ash during the Wisconsin glacial period and are now characterized by compressible, high plasticity, high water content clays interspersed with horizontal lenses of sand and soil layers (Romo, 1988). The lakes were shallow and therefore never formed any significant deltas while the ash was settling, leaving the lake sediments mostly laterally homogenous (Stephenson and Lomnitz, 2005). The general stratigraphy of the lakes is a 1-2 meter crust underlain by 25-30 meters Upper Clay Formation (UCF) underlain by the roughly 3 meter thick First Hard Layer (FHL) underlain by around 20 meters of Lower Clay Formation (LCF) until the Deep Deposits (DD) (Fig 2, Romo, 1988; Stephenson and Lomnitz, 2005).

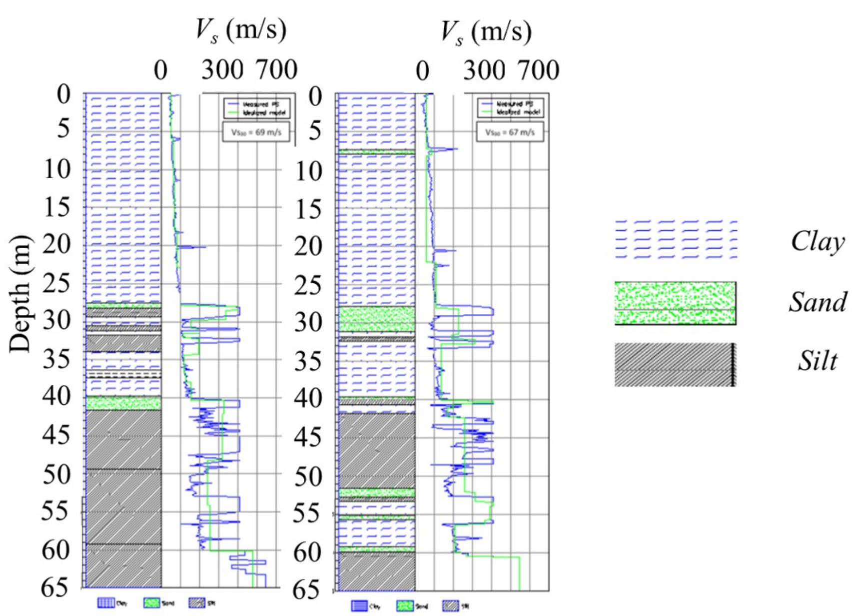

Figure 2: Typical lakebed soil shear wave velocity profiles in Mexico City (modified from Mayoral et al. 2016)

We simplify this stratigraphy as a single layer halfspace with density, shear wave velocity and damping of the overburden $1.5g/cm^3$, $70m/s$ and $\zeta = 7\%$ respectively and density, shear wave velocity and damping of the bedrock $2.7g/cm^3$, $475m/s$ and $\zeta = 5\%$ respectively, values provided in Stephenson (2005) and the shear wave velocity profile in Mayoral et al. (2016) (Fig. 3).

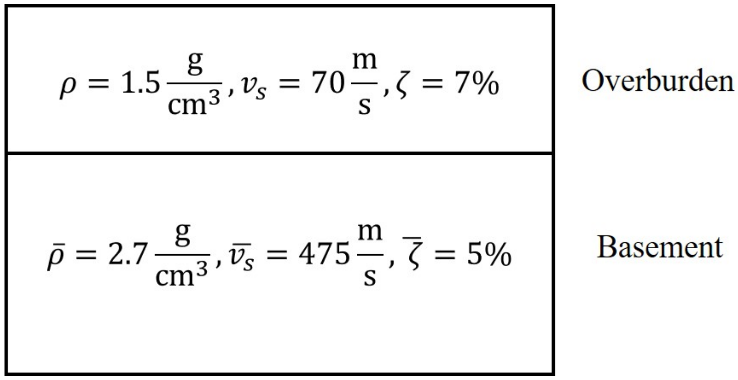

Figure 3: Simplified soil profile used in this study with parameters of overburden and basement

Applying problem's boundary conditions to the wave equation

From CEE-245 HW 4, we know that the most general solution for the displacement in the upper layer will have the form:

$$u(x,t) = A\ exp[i(\omega t - kx)] + B\ exp([i(\omega t + kx)],\ -h < x < 0 \tag{6}$$

where the $(\omega t - kx)$ term indicates the upgoing wave and the $(\omega t + kx)$ term denotes the downgoing wave. The most general solution for displacement in the lower layer has the form:

$$u(z,t) = U_0\ exp[i(\omega t - \bar{k}x)] + D\ exp([i(\omega t + \bar{k}x)],\ x < -h \tag{7}$$

Where U_0 is the incident wave coming from below and D is the downgoing wave into the substrate. The boundary conditions for this problem are that displacements (Eq. 8) and stresses (Eq. 9) must be equal across the interface between the basement and overburden layer and that stresses must be 0 at the free surface (Eq. 10):

$$u(-h^+, t) = u(-h^-, t) \tag{8}$$

$$\sigma(-h^+, t) = \sigma(-h^-, t) \tag{9}$$

$$\sigma(0, t) = 0 \tag{10}$$

We start with the boundary condition that at the free surface, stresses are 0, or:

$$G\frac{\partial u}{\partial x}|_{x=0} = 0 \tag{11}$$

so

$$-kiAe^{i\omega t} + kiBe^{i\omega t} = 0 \tag{12}$$

$$A = B \tag{13}$$

Now we solve using the boundary conditions displacements must be equal across the interface, hereafter (1) and stresses must be equal across the interface, hereafter (2). Using B.C 1 and plugging in the equality in Eq 13, we know that

$$U_0\ e^{i(\omega t+\bar{k}h)} + D\ e^{(i(\omega t-\bar{k}h)} = A\ e^{i(\omega t+kh)} + A\ e^{(i(\omega t-kh)} \tag{14}$$

This reduces to

$$U_0\ e^{i\bar{k}h} + D\ e^{-i\bar{k}h} = A(\ e^{ikh}\ e^{-ikx}) \tag{15}$$

Now using B.C 2, we know

$$U_0 e^{i\bar{k}h} - De^{-i\bar{k}h} = \frac{Gk}{\bar{G}\bar{k}} A(e^{ikh} - e^{-ikh}) \tag{16}$$

Now adding equations 15 and 16, the "D" drops out and we get:

$$U_0 e^{i\bar{k}h} = \frac{Gk}{\bar{G}\bar{k}} A[(e^{ikh} - e^{-ikh}) + (e^{ikh} + e^{-ikh})] \tag{17}$$

Expanding the terms within the brackets after "A" using equation 4 we get:

$$A = B = \frac{U_0 e^{i\bar{k}h}}{cos(kh) + \frac{Gk}{\bar{G}\bar{k}} sin(kh)} \tag{18}$$

Now plugging U_0 into equation 6, and solving at the free surface x=0, we get:

$$u(0,t) = 2U_0 e^{i\omega t} \frac{cos(\bar{k}h) + isin(\bar{k}h)}{cos(kh) + i\frac{Gk}{\bar{G}\bar{k}} sin(kh)} \tag{19}$$

Solving this equation for the amplitude, we find

$$\frac{|u(0,t)|}{|u_0|} = \frac{2}{\sqrt{cos^2(kh) + (\frac{Gk}{\bar{G}\bar{k}})^2 sin^2(kh)}} \tag{20}$$

Now using eqs 1 and 2 and the relationship $f = \frac{\omega}{2\pi}$, we rearrange equation 20 to be in terms of v_s and frequency. We will use equation 21 to look at the soil response in Mexico City without damping.

$$\frac{|u(0,t)|}{|u_0|} = \frac{2}{\sqrt{cos^2(\frac{2\pi fh}{v_s}) + (\frac{v_s\rho}{\bar{v}_s\bar{\rho}})^2 sin^2(\frac{2\pi fh}{v_s})}} \tag{21}$$

Adding damping

We have solved for an undamped soil layer on an elastic basement, now we solve for the case of a damped soil layer over a damped basement. We start by rearranging equation 19 using eqs 1 and 2:

$$u(0,t) = 2U_0 e^{i\omega t} \frac{cos(\frac{2\pi fh}{\bar{v}_s}) + isin\frac{2\pi fh}{\bar{v}_s})}{cos(\frac{2\pi fh}{v_s}) + i\frac{v_s\rho}{\bar{v}_s\bar{\rho}} sin(\frac{2\pi fh}{v_s})} \tag{22}$$

Now we plug in equation 3:

$$u(0,t) = 2U_0 e^{i\omega t} \frac{cos(\frac{2\pi fh}{\bar{v}_s(1+i\bar{\zeta})}) + isin\frac{2\pi fh}{\bar{v}_s(1+i\bar{\zeta})})}{cos(\frac{2\pi fh}{v_s(1+i\zeta)}) + i\frac{v_s(1+i\zeta)\rho}{\bar{v}_s(1+i\bar{\zeta})\bar{\rho}} sin(\frac{2\pi fh}{v_s(1+i\zeta)})} \tag{23}$$

Solving this equation for the amplitude, we find:

$$\frac{|u(0,t)|}{|u_0|} = \frac{2}{|cos(\frac{2\pi fh}{v_s(1+i\zeta)}) + i\frac{v_s(1+i\zeta)\rho}{\bar{v}_s(1+i\zeta)\bar{\rho}} sin(\frac{2\pi fh}{v_s(1+i\zeta)})|} \tag{24}$$

According to Kramer (1996), equation 24 cannot "be expressed in a very compact form when soil damping exists" (page 265), so we evaluate equation 24 using Matlab.

Evaluating undamped and damped solutions using Mexico City soil parameters

We have derived 2 equations, eqs 21 and 24, which model uniform, undamped soil on rigid rock and uniform damped soil on elastic rock respectively. We will now evaluate each transfer function using the Mexico City soil parameters in figure 3 and a depth h = 70 meters. Starting with the undamped case (Fig 3), we see that resonance peaks are at

$$f_s = \frac{v_s}{4h} \tag{25}$$

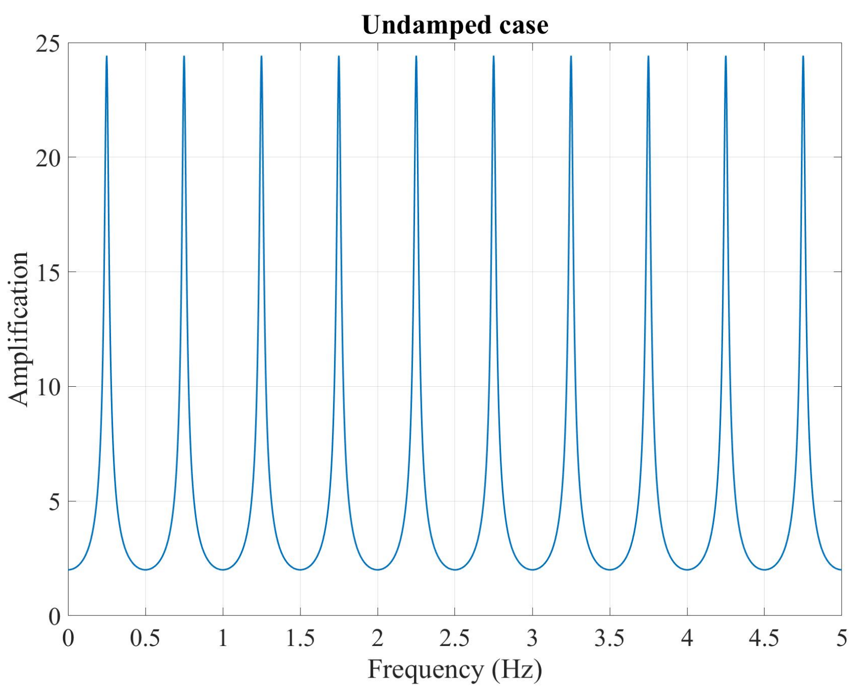

Figure 4: Undamped case transfer function with the Mexico soil parameters

For the damped case, the amplifications at all frequencies are significantly lower than in the damped case and the higher mode amplifications decrease as the modes get higher (Fig. 5).

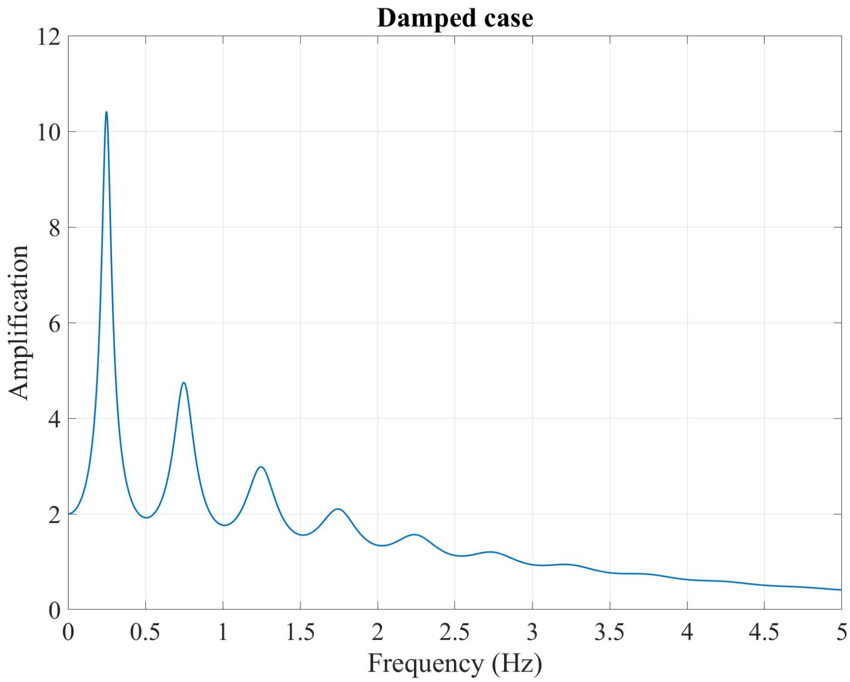

Figure 5: Damped case transfer function with the Mexico soil parameters

Comparing model to data

Following the 1985 Michoacán earthquake, the government of Mexico City installed the RACM network (http://www.cires.org.mx/) in partnership with the National Autonomous University of Mexico (UNAM) to provide ground motion data for basic research to assess and mitigate vulnerability within the Mexico City basin. We selected the site - reference pair CE32 - TP13 to calculate the simple spectral ratio between the two (Fig. 6). The simple spectral ratio is the empirical measurement of the analytical transfer functions derived above.

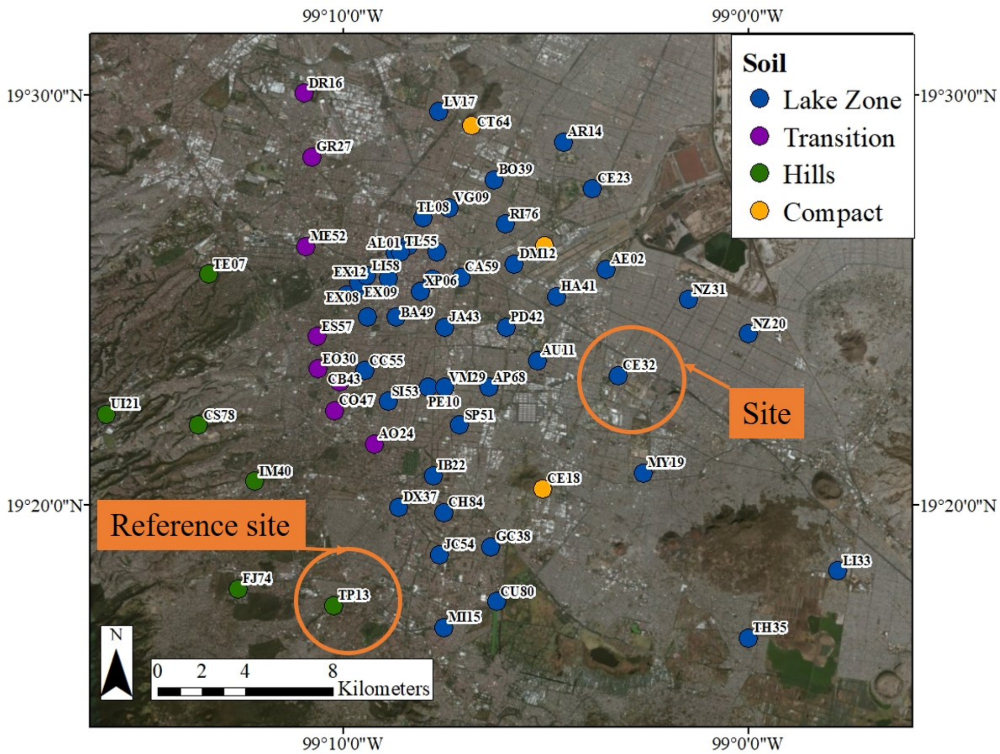

Figure 6: Damped case transfer function with the Mexico soil parameters

We used 37 events that were recorded at both CE32 and TP13 since 1985 and computed the simple spectral ratio for each of those events. We've provided an example of the waveform from the 2017 Puebla event (Fig 7) We then average all 37 simple spectral ratios together and plotted the resulting empirical transfer function with the theoretical damped layer case (Fig 8).

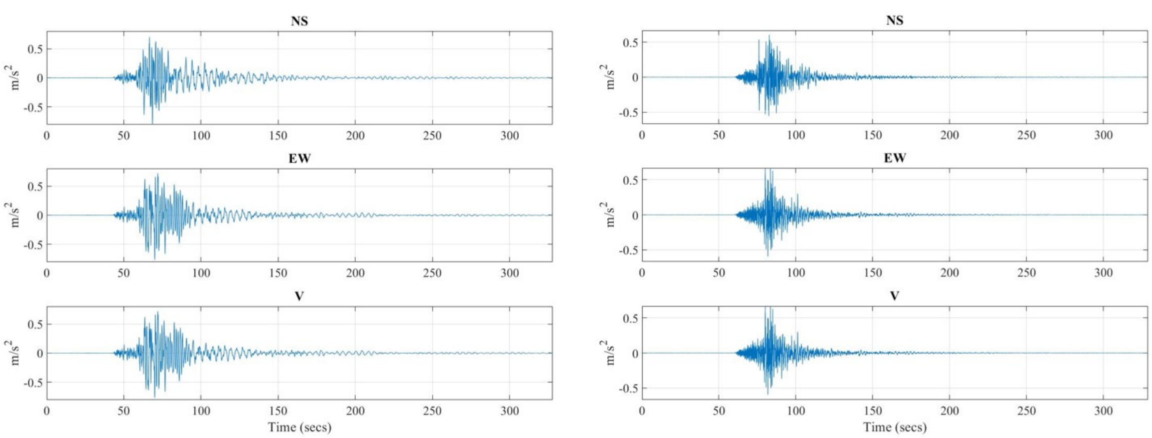

Figure 7: Waveforms for the 2017 Puebla event in Mexico City. The left figure is the NS, EW and V records at CE32, the soft site. The right figure is the NS, EW and V records at TP13, the hard reference site. Note the lower frequency on the soil site compared to the reference site.

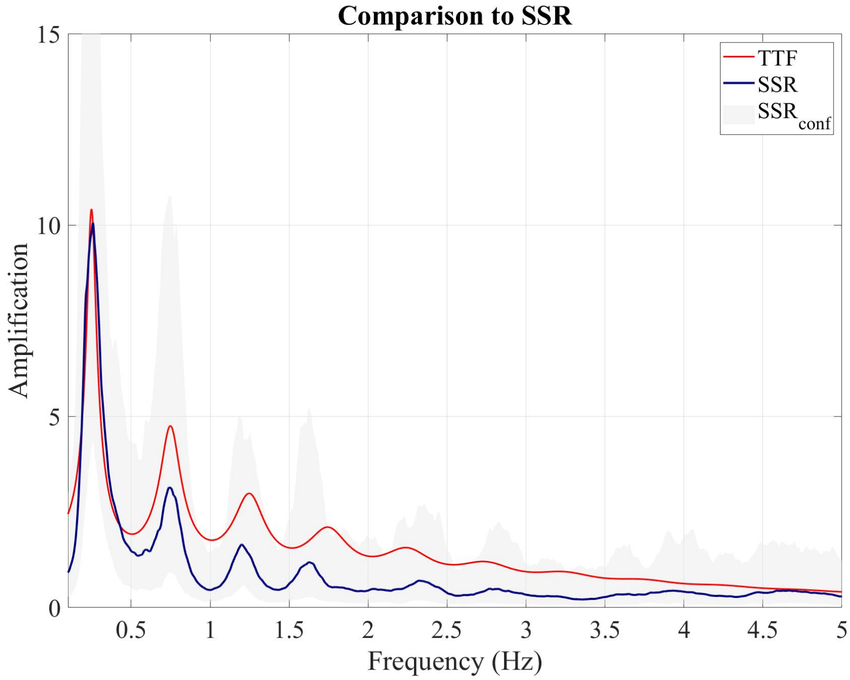

Figure 8: Comparison of the simple spectral ratio to the analytical solution of the damped layer over elastic halfspace case. The grey fill is the 95 % confidence interval of the mean found from averaging the 37 spectral ratios together.

Conclusions

The damped soil layer over an elastic halfspace provides a good model for vertically propagating shear waves in Mexico City. The correlation between ETF and TTF from 0 to 5 hz is 0.6762, above the threshold of 0.6 proposed in Thompson et al. (2012) for a site that is well modeled by the SH1D transfer function. This is remarkable considering the simplicity of the model. Presumably, as we add more soil layers to the model, the correlation between it and the ETF would improve even more and get well up above that 0.6 threshold. The model predicts the fundamental mode well but overpredicts the higher modes. This could be due to incorrect input parameters, either an incorrect impedance contrast or damping. This could be refined by fitting the analytical solution to the data using a minimization technique and varying the impedance contrast, damping or both.

References

Kramer, S.L. (1996). Geotechnical Earthquake Engineering, Prentice Hall, Upper Saddle River, N.J.

Mayoral, J.M., Castañon, E., Alcantara, L., Tepalcapa, S. (2016). Seismic Response Characterization of High Plasticity Clays. Soil Dynamics and Earthquake Engineering,84 (2016) 174-189.

Romo, M.P., Jaime, A., Reséndiz, D (1988). The Mexico City Earthquake of September 19, 1985- General Soil Conditions and Clay properties in the Valley of Mexico. Earthquake Spectra, Vol 4, No. 4, 731-752.

Stephenson, B. Lomnitz, C. (2005). Shear-wave velocity profile at the Texcoco strong-motion array site, Valley of Mexico. Geofisca Internacional 44(1): 3-10.

Thompson EM, Baise LG, Tanaka K, Kayen RE. (2012) A taxonomy of site response complexity. Soil Dynamics and Earthquake Engineering. 41(2012): 32-43.

Code listing

Undamped case)

```
%% Solution for undamped case, equation 19, figure 4
% Marshall Pontrelli
% 5/8/2020
close all
clear all

%%
h = 70; % meters
vs = 70; % shear wave velocity of the overburden
r = 1.5; % density of overburden
vs_bar = 475; % shear wave velocity of basement
r_bar = 2.7; % density of basement
imp = (vs*r)/(vs_bar*r_bar); % impedance contrast

freq_vec = linspace(0, 5, 100000);

for i = 1:length(freq_vec)
    f = freq_vec(i);
    amp(i) = 2/ sqrt((cos(2*pi*f*h/vs))^2 + imp^2*(sin(2*pi*f*h/vs))^2);
end
undamp = figure;
plot(freq_vec,amp, 'LineWidth',1.5)
xlabel('Frequency (Hz)')
ylabel('Amplification')
title('Undamped case')
set(gca, 'FontName', 'Times New Roman', 'FontSize', 24)
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
grid on
box on

saveas(undamp,'Fig4.jpg')
```

Damped case)

```
%% Solution for damped case, equation 24, figure 5
% Marshall Pontrelli
% 5/8/2020
close all
clear all

%%
h = 70; % meters
vs = 70; % shear wave velocity of the overburden
r = 1.5; % density of overburden
z = 0.07;% damping of overburden
vs_bar = 475; % shear wave velocity of basement
r_bar = 2.7; % density of basement
z_bar = 0.05; % damping of basement
```

```
imp = (r*vs*(1+1i*z))/(r_bar*vs_bar*(1+1i*z_bar)); % impedance contrast

freq_vec = linspace(0, 5, 100000);

for j = 1:length(freq_vec)
    f = freq_vec(j);
    amp(j) = 2/(abs(cos(2*pi*f*h/(vs*(1+1i*z))) + imp*1i*sin(2*pi*f*h/(vs*(1+1i*z)))));
end

damp = figure;
plot(freq_vec,amp, 'LineWidth',1.5)
xlabel('Frequency (Hz)')
ylabel('Amplification')
title('Damped case')
set(gca, 'FontName', 'Times New Roman', 'FontSize', 24)
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
grid on
box on

saveas(damp,'Fig5.jpg')
```

Figure 8 and correlation)

```
%% Solution for damped case, equation 24, figure 5
% Marshall Pontrelli
% 5/8/2020
close all
clear all

%%
h = 70; % meters
vs = 70; % shear wave velocity of the overburden
r = 1.5; % density of overburden
z = 0.07;% damping of overburden
vs_bar = 475; % shear wave velocity of basement
r_bar = 2.7; % density of basement
z_bar = 0.05; % damping of basement

imp = (r*vs*(1+1i*z))/(r_bar*vs_bar*(1+1i*z_bar)); % impedance contrast

freq_vec = linspace(0, 49.990, 50000);

for j = 1:length(freq_vec)
    f = freq_vec(j);
    amp(j) = 2/(abs(cos(2*pi*f*h/(vs*(1+1i*z))) + imp*1i*sin(2*pi*f*h/(vs*(1+1i*z)))));
end

damp = figure;
xlabel('Frequency (Hz)')
ylabel('Amplification')
title('Comparison to SSR')
set(gca,'FontName', 'Times New Roman', 'FontSize', 24)
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
xlim([0.1 5])
ylim([0 15])
grid on
box on
hold on

%% load data for plot
statname = 'CE32';
```

```
% Load SSR
load(strcat('C:\Users\mpontr01\Box\Data\Ground motion\Mexico CIty\SSR_Shape_statistics\',...
    statname, '\',statname, '-TP13'))

ahatfSSR = data.complex.ahatf';
sigmaSSR = data.complex.sigma';
confinthighSSR = data.complex.confinthigh';
confintlowSSR  = data.complex.confintlow';

% load frequency vector
load('C:\Users\mpontr01\Box\Data\Ground motion\Mexico ...
    CIty\Processed_data2\AE02\AE0219900511234349')
fax_HzN = data.processing.filtereddata.freq_vec;

% Now some inputs
upbound = 10;
lowbound = 0.1;
[¬, lowbound] = min(abs(fax_HzN - lowbound));
[¬, upbound] = min(abs(fax_HzN - upbound));


% now plot confidence intervals
fr = fax_HzN(lowbound:length(fax_HzN))';
cohr = confinthighSSR(lowbound:length(fax_HzN))';
colr = confintlowSSR(lowbound:length(fax_HzN))';
x_plot =[fr, fliplr(fr)];
y_plot = [cohr, fliplr(colr)];

% now plot confidence interval
SSRconf = fill(x_plot, y_plot, 1,'facecolor', [0.9 0.9 0.9], 'edgecolor', 'none', ...
    'facealpha', 0.4);
hold on

% now plot TTF
TTF = plot(freq_vec,amp, 'LineWidth',1.5,'Color', 'r');
hold on

% Now plot SSR
SSR_plot = plot(fax_HzN,ahatfSSR, 'LineWidth', 2, 'Color', [0 0 0.5]);

% legend
legend([TTF,SSR_plot, SSRconf], 'TTF', 'SSR', 'SSR_{conf}', 'location', 'northeast')

%% Now correlate
amp2 = amp - mean(amp);
amp2 = amp2(1:5000);
ahatfSSR2 = ahatfSSR - mean(ahatfSSR);
ahatfSSR2 = ahatfSSR2(1:5000);
[c,lags] = xcorr(amp2, ahatfSSR2,'normalized');
[maxr, I] = max(c);
lagmax = lags(I)/5000;
r = c(2500);
datamat = [r,maxr,lagmax];

% and save
saveas(damp,'Fig8.jpg')
```