# Are District Compactness Scores an Indicator of Gerrymandering?

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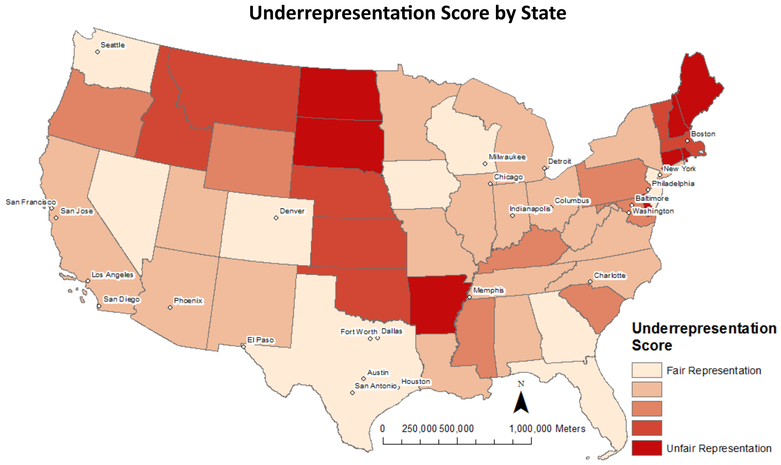
## Abstract

After each Census, States are required to redraw their legislative districts for the House of Representatives. In many instances, incumbent parties are accused of suppressing minority voting demographics by drawing districts in a way which underrepresents them through a process known as gerrymandering. Presumably, gerrymandering would result in irregularly-shaped, non-compact districts. The extent to which minorities are underrepresented can be expressed by an "underrepresentation score," a difference between the number of representatives a state actually has for each party and the number it would theoretically have if constituents were represented fairly, normalized by population. Here, this score for each state is compared to two measures of compactness, the Polsby-Popper score of perimeter complexity, and the Roeck score of dispersion. Results show that, although the Polsby-Popper and Roeck scores are highly correlated, both are a very poor indicator of a state's underrepresentation score. Furthermore, the investigation shows that compactness scores are not significantly affected by a change in projection. Low compactness should not be used as the sole indicator of underrepresentation of minority constituents. There are simply too many other factors at play which may be responsible for some groups' underrepresentation, only one of which is intentional gerrymandering.

## Calculating Underrepresentation

**Underrepresentation Score by State**




Using the numbers of votes cast for Democratic and Republican candidates, a votes sum is calculated for one of the two parties in each respective state. By then dividing the number of votes for the Democratic candidate by the total number of votes cast, a proportion is calculated which is a reasonable approximation of the proportion of individuals that politically align with the Democratic Party, and the rest are assumed to align with the Republican Party. This proportion is then multiplied by the number of House representatives that are apportioned to each state in the 113$^{th}$ congress; the product is the number of Democratic representatives each state would send to the House of Representatives if Democratic constituents were to be fairly represented. Based on the difference between the number of Democratic representatives that a state should send to the House for fair Representation and the number the state *actually* sent to the house is then calculated. This number is then divided by the total number of representatives which normalizes the score for each state. The absolute value of this number is the calculated and shows there constituents of any political alignment are underrepresented and is referred to as the **underrepresentation score**.

## Calculating District Compactness

One measure of irregularity surrounding districts is known as compactness. There are a number of ways to measure compactness, but two of the most widely used are a district's perimeter complexity and its dispersion around its center. To measure the perimeter complexity, the **Polsby-Popper score** can be used and to measure the dispersion of a district, the **Reock score** can be used. The Reock compactness score is the ratio of a district's area to that of its minimum circumscribing circle. The Polsby-Popper compactness score is the ratio of a district area to the area of a circle whose circumference equals the district perimeter.

The equation for the Polsby-Popper score is:

$$\frac{\text{Area of the district}}{\text{Area of circle whose circumference is district's perimeter}}$$

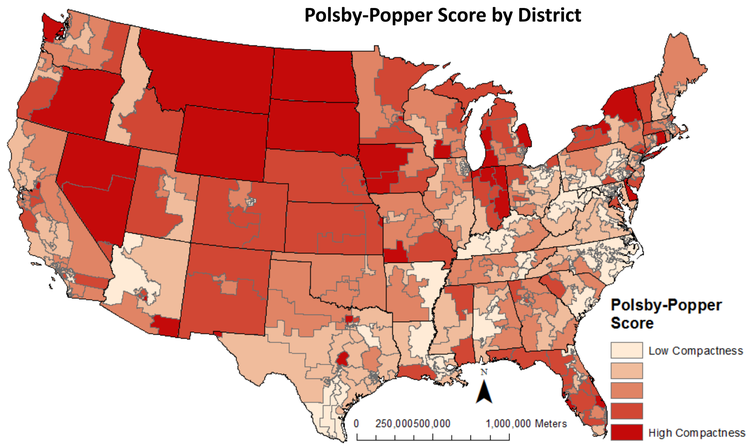
The equation for the Roeck score is:

$$\frac{\text{District Area}}{\text{Minimum Bounding Circle Area}}$$

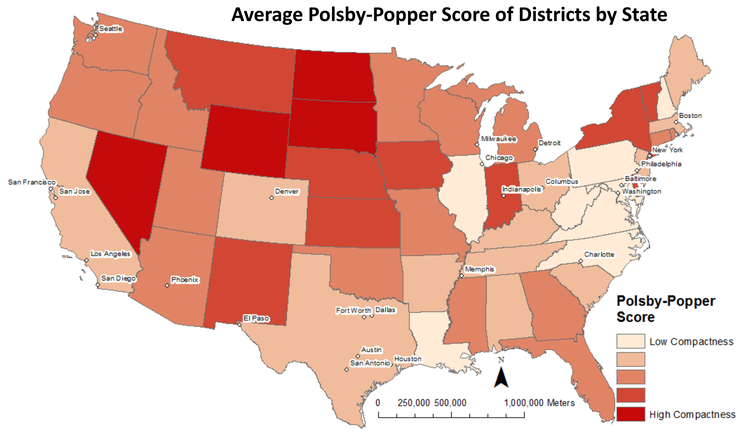
## Compactness by District and by State

A Polsby-Popper and Roeck compactness score was calculated for each of the 435 US House of Representative Districts. Below is a graphical representation of the Polsby-Popper score for each state. Low compactness scores are thought be a indicator of potentially gerrymandered states.

**Polsby-Popper Score by District**




**Average Polsby-Popper Score of Districts by State**




## Compactness Examples

A compact district: Indiana's 1st

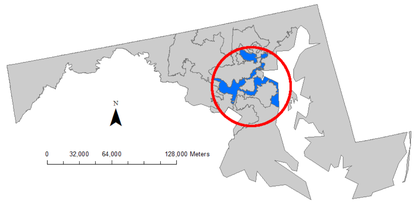
Polsby-Popper Score: 0.578895

Roeck Score: 0.470389


A disperse District: Maryland's 3rd

Polsby-Popper Score: 0.03291

Roeck Score: 0.218429




## Results

| State | Under-rep. Score | Compactness | |
|---|---|---|---|
| | | Polsby-Popper | Roeck |
| AL | 0.1713 | 0.193 | 0.380 |
| AK | 0.3068 | 0.064 | 0.145 |
| AZ | 0.1141 | 0.299 | 0.457 |
| AR | 0.4221 | 0.199 | 0.392 |
| CA | 0.1660 | 0.237 | 0.382 |
| CO | 0.0472 | 0.245 | 0.390 |
| CT | 0.4190 | 0.273 | 0.441 |
| DE | 0.4196 | 0.456 | 0.304 |
| FL | 0.0112 | 0.300 | 0.395 |
| GA | 0.0238 | 0.260 | 0.450 |
| HI | 0.3632 | 0.227 | 0.166 |
| ID | 0.3639 | 0.255 | 0.394 |
| IL | 0.1449 | 0.166 | 0.317 |
| IN | 0.1828 | 0.431 | 0.470 |
| IA | 0.0546 | 0.389 | 0.421 |
| KS | 0.3424 | 0.408 | 0.413 |
| KY | 0.2082 | 0.191 | 0.350 |
| LA | 0.1488 | 0.144 | 0.352 |
| ME | 0.4398 | 0.232 | 0.428 |
| MD | 0.2542 | 0.113 | 0.271 |
| MA | 0.3770 | 0.225 | 0.395 |
| MI | 0.1013 | 0.299 | 0.381 |
| MN | 0.1171 | 0.331 | 0.412 |
| MS | 0.2029 | 0.264 | 0.409 |
| MO | 0.1409 | 0.270 | 0.438 |
| MT | 0.3591 | 0.480 | 0.468 |
| NE | 0.2954 | 0.384 | 0.461 |
| NV | 0.0293 | 0.523 | 0.470 |
| NH | 0.5345 | 0.163 | 0.266 |
| NJ | 0.0073 | 0.198 | 0.389 |
| NM | 0.1506 | 0.350 | 0.429 |
| NY | 0.1797 | 0.349 | 0.421 |
| NC | 0.1477 | 0.120 | 0.299 |
| ND | 0.5492 | 0.512 | 0.533 |
| OH | 0.1897 | 0.189 | 0.348 |
| OK | 0.2993 | 0.251 | 0.355 |
| OR | 0.2725 | 0.312 | 0.426 |
| PA | 0.2416 | 0.171 | 0.327 |
| RI | 0.4060 | 0.284 | 0.287 |
| SC | 0.2733 | 0.209 | 0.385 |
| SD | 0.4882 | 0.558 | 0.494 |
| TN | 0.1396 | 0.203 | 0.389 |
| TX | 0.0117 | 0.197 | 0.361 |
| UT | 0.1082 | 0.277 | 0.356 |
| VT | 0.3316 | 0.366 | 0.373 |
| VA | 0.1617 | 0.162 | 0.274 |
| WA | 0.0668 | 0.278 | 0.379 |
| WV | 0.1126 | 0.138 | 0.303 |
| WI | 0.0710 | 0.293 | 0.451 |
| WY | 0.2580 | 0.773 | 0.618 |

## Conclusions

The results of this investigation show that the two measures of compactness, the Polsby-Popper score and the Roeck score are highly correlated. These two scores were calculated for each district and the correlation coefficient between them was 1.0000 ($p < 0.0001$). While these scores may seem somewhat different, they are both similar in that what they are measuring the difference in attributes between a given shape and those of a similarly-sized, regular shape. In other words, they both measure the amount of space near a polygon that would be part of that polygon if it were not irregular. The Polsby-Popper and Roeck scores measure this differently, one by its perimeter and one by its area, but since a shape's area and perimeter are covariant, there is not much to be gained from using one over the other in these types of analyses.

Both of these scores are a very poor indicator of a state's underrepresentation score. There is no apparent relationship between a state's underrepresentation score measurement of representation fairness and its gerrymandering as measured by compactness of that state's districts. The conventional wisdom is that gerrymandering is a method by which minority votes are suppressed. While this may be true, this analysis shows that neither the Polsby-Popper score of perimeter complexity nor the Roeck score of dispersion is an accurate method of quantifying gerrymandering. Non-significant R-squared values show that both scores explain no more than 3% of underrepresentation.

Additionally, there is no indication from this study that either party is more responsible for inducing underrepresentation on their opposing demographics, although further investigation into compactness scores stratified by Democratic states vs. Republican states was not performed and may yield significant results. Furthermore, the investigation shows that compactness scores are not significantly affected by a change in projection. While scores do change slightly, they do not change significantly enough to affect results. Since low compactness is a poor indicator of underrepresentation, there should be further work in establishing a method of accurately quantifying the effects of gerrymandering on underrepresentation. This, however will be difficult because there are many other factors that contribute to underrepresentation of minority constituents.

In the literature, there are other ways to measure irregularity of districts which were not tested here, those include the Schwartzberg measure, the convex hull measure, and bizarreness measures. In the future, it may be worth investigation whether these can individually, or in some combination predict underrepresentation or gerrymandering.