

PROBLEM SET 2

History of Math, Duchin

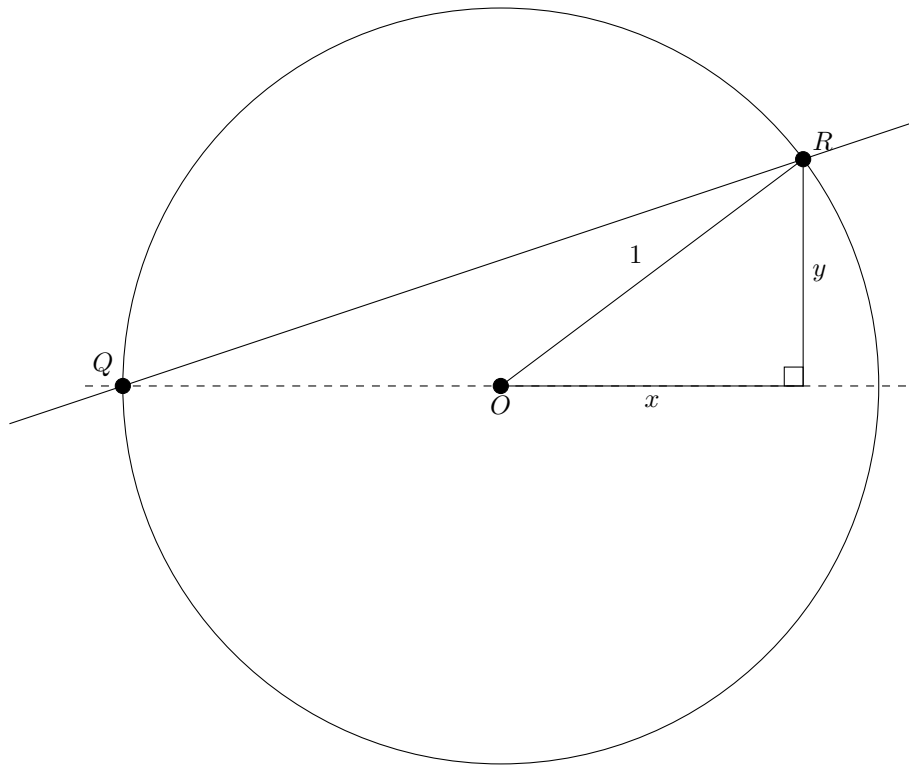
(B2) Show that if (a, b, c) is any Pythagorean triple, then

$$\frac{a}{c} = \frac{p^2 - q^2}{p^2 + q^2}, \quad \frac{b}{c} = \frac{2pq}{p^2 + q^2},$$

for some integers p, q . (Try this first. I'll add a hint later.)

Explain how this can be used to derive the Neugebauer-Aaboe interpretation of Plimpton 322 that Robson describes on p107-108.

Hint: I'm going to suggest a setup for showing that EVERY Pythagorean triple (a, b, c) has a p and q that work in the formulas above. It's just one approach! There are a ton of ways to do this problem.



If you scale an (a, b, c) right triangle down until its hypotenuse is 1, what are x and y ?

Based on its position in the picture, what are the coordinates of the point Q ?

If the line \overline{QR} has slope m , what is its equation?

Using the fact that R is on the circle and that line, solve for x and y in terms of m .

A hard step: Explain that m is rational if and only if R has rational coordinates.

Now let m range over all rationals and see what you learn about x and y .

If you use this—or any other source—that's totally welcome, but be sure to cite the source.