> Howbeit, for eaſie alteratiō of *equations*. I will pro-
> pounde a fewe exāples, bicauſe the extraction of their
> rootes, maie the more aptly bee wroughte. And to a-
> uoide the tediouſe repetition of theſe woordes : is e-
> qualle to : I will ſette as I doe often in woorke vſe, a
> paire of paralleles, or Gemowe lines of one lengthe,
> thus: =====, bicauſe noe.2. thynges, can be moare
> equalle. And now marke theſe nombers.

(A1) al-Khwārizmī writes: *I have divided ten into two parts; I have multiplied the one by ten and the other by itself, and the products were the same.* The first part of the sentence means that he is dealing with two numbers that add up to ten. Solve this problem, which indicates that al-Khwārizmī was comfortable dealing with irrational numbers.

Modern techniques would produce two solutions to this problem, but al-Khwārizmī would only have recognized one. Which one and why?

(A2) In Greek mathematics, some scholars (like Ptolemy, working in the +2nd century in Alexandria) studied trigonometric functions, particularly in connection with problems in astronomy. They did not use the modern sine and cosine, though, but rather mainly a function called the *chord*, later adopted in India and written about in the *Aryabhatiya.*

The chord of an angle α, denoted $\mathrm{crd}(\alpha)$, is the distance between two points on a circle of radius R if the angle between them at the center of the circle is α.

Let R be fixed. Work out $\mathrm{crd}(\alpha)$ in terms of α, R and the modern sine function. Work out $R\sin(\alpha)$ in terms of $\mathrm{crd}(\alpha)$.

(A3) Give a geometrical proof that $a^2 - b^2 = (a-b)(a+b)$.

(B1) Using the notation $e^{i\theta} = \cos\theta + i\sin\theta$ and the usual algebra of complex numbers and exponentiation, derive the double-angle formulas and triple-angle formulas.