Masters of Theory

Cambridge and the Rise of Mathematical Physics

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A Mathematical World on Paper

The Material Culture and Practice-Ladenness of Mixed Mathematics

We are perhaps apt to think that an examination conducted by written papers is so natural that the custom is of long continuance.

W. W. ROUSE BALL, 1889[1]

3.1. The "Familiar Apparatus" of Pen, Ink and Paper

When James Clerk Maxwell delivered his inaugural lecture as the first Professor of Experimental Physics in Cambridge in 1871, he contrasted the new instruments and machines that would soon occupy the proposed physical laboratory with what he referred to as the "familiar apparatus of pen, ink, and paper" (Maxwell 1890, 2:214). Having been employed to deliver courses on experimental physics, a new departure for the university, Maxwell sought to reassure his audience that public provision for experimentation was now a necessity both for teaching and for research. Henceforth, students preparing for the Mathematical Tripos would be required not merely to master higher mathematics and physical theory but to exercise their "senses of observation" and their "hands in manipulation," activities that smacked too much of the artisan and the workshop to endear them to many of his new Cambridge colleagues. Pressing home his defense of the soon-to-be-built Cavendish Laboratory, Maxwell insisted that students and teachers of experimental physics would "require more room than that afforded by a seat at a desk, and a wider area than that of the black board." Maxwell's analogy between the material apparatus proper to mathematical training and that

1. Rouse Ball 1889, 193.

required to teach experimentation was ingenious. Few would deny, once they were reminded of the fact, that pens, paper, desks, and blackboards were everyday tools of the mathematician's trade; it followed that if the university wished to continue to "diffuse a sound knowledge of Physics" (214) it would have to accept the physical laboratory despite its connotations of industry and manufacture.[2]

Maxwell's rare allusion to the material culture of mathematical training provides an appropriate point of departure for this chapter. Historians of science have recently begun to explore the remarkable changes in scientific practice wrought by the so-called laboratory revolution of the late Victorian era, yet very little scholarly attention has so far been paid to the changes in *theoretical* practice wrought by the introduction of written examinations in mathematics a century earlier.[3] This may sound a somewhat surprising statement given the several scholarly studies that exist on the origins of written examination in Cambridge, but these studies are mainly concerned with the wider sociopolitical significance of competitive assessment rather than with the effects of its style and material medium on the content and practice of mixed mathematics. Teaching and assessing mathematical disciplines on paper now seems such a self-evidently correct practice that it is difficult to imagine that it was once otherwise. Even by Maxwell's day paper-based learning in mathematics was so commonplace that he could make a point simply by acknowledging the fact; and, as Rouse Ball remarked in the epigraph above, these pedagogical practices appeared so "natural" to mid-Victorian mathematicians that they were assumed, when considered at all, to have been in use for centuries.

At the beginning of the nineteenth century, however, the whole apparatus of paper-based learning and examining was not only new in Cambridge but seemed as peculiar to undergraduates as would galvanometers and Wheatstone's Bridge to Maxwell's first students at the Cavendish. And like the laboratory revolution, the introduction of written examinations dramatically altered the skills and competencies prized in undergraduate study. As we saw in chapter 1, mathematics had always been part of the education offered by the ancient universities but had generally been learned only to an elementary level and in connection with the practical use and construction of mathematical instruments. By the early Victorian period the written

2. On resistance to a physical laboratory in Cambridge, see Warwick 1994.

3. For an exception to the lack of work on the material apparatus of mathematical practice, see Brock and Price 1980. On the laboratory revolution, see Gooday 1990.

examination and its attendant pedagogical apparatus had rendered the study of mathematics a purely paper-based activity, and, as we saw in chapter 2, were producing young graduates whose technical mastery of mathematical physics far outshone the capabilities of earlier generations of students.

In the first half of this chapter I explore the impact and development of paper-based learning and examination in Cambridge from the perspective of some of the students first exposed to such practices. Their testimony is especially valuable as it reveals both the profound shift in student sensibilities that accompanied the introduction of written examinations as well as the change in the content of undergraduate studies prompted by the new culture of working on paper. Our understanding of the nature of college and public teaching circa 1800 is surprisingly thin, and the sources from which the history can be written are sparse. We must assume that university professors and college lecturers taught mainly by emulating their own teachers and by introducing whatever innovations seemed appropriate. What can be learned regarding the actual content and conduct of mathematical lectures and examinations has to be reconstructed from occasional remarks in the memoirs, correspondence, and diaries of students and teachers, and from occasional descriptions of important Senate House ceremonies published by the university.

My analysis of these sources will focus on two specific and closely related aspects of the shift from oral to written examination during the early decades of the nineteenth century. The first concerns the very different personal and intellectual qualities tested by oral and written examinations respectively and the ways in which student sensibilities were gradually reshaped to enable them to deal effectively with the latter form of assessment. The second concerns the introduction of what Maxwell referred to as the "apparatus of pen, ink, and paper" to mathematical education. As I have already noted, these now seem the obvious tools for teaching a mathematics-based discipline, but it was only with the introduction of written assessment that paper-based techniques of calculation began routinely to be employed in undergraduate study. Prior to this, mathematical education, like other branches of study institutionalized in the medieval university, was based mainly on private reading, oral debate, and catechetical lectures. The introduction of paper-based mathematical study gradually displaced these long-established pedagogical traditions, instituting instead tough regimes of competitive technical training. An important aspect of these regimes was the requirement that students be able both to *write out* their mathematical knowledge from memory and to solve difficult mathematical problems on paper. This aspect of the shift from oral to written study represented much more than a mere change in the medium by which mathematics was learned. Paper-based problem solving had hitherto been a technique employed by those engaged in original mathematical investigations, and many of the problems in analytical geometry and dynamics that gradually became standard student exercises were ones that had originally challenged the greatest mathematicians of the eighteenth century. The methods of mathematical reasoning proper to the solution of such problems could not easily be mastered by oral debate, but required long and assiduous practice on paper. The introduction of paper-based learning and examination therefore marked a major and profound period of cultural transformation in the history of the exact sciences, during which the working practices used by the founders of modern mathematical physics became central to undergraduate training in the universities.

In the second half of the chapter I explore the changing style and content of Tripos-examination questions within the new regime of private teaching and paper-based study. The first questions to which written answers were required were relatively simple exercises taken more or less directly from standard treatises. But, during the early decades of the nineteenth century, and especially during the analytical revolution, these questions gradually became both more technically demanding and more original. By the 1830s, the examiners were routinely drawing upon their own research to construct the most difficult problems, while students were increasingly assessed by their ability to solve such novel problems under severe constraints of time. These developments played a major role in altering the relationship between the books from which students learned and the examination questions by which they were assessed. Where examination questions had originated as examples from mathematical and natural philosophical treatises, a new generation of what might properly be called mathematical "textbooks" incorporated large numbers of these questions as exercises for the reader. This convergence between training and assessment reinforced the ideals of competitive written examination within an emergent textbook tradition and provided a virtually endless supply of exemplars for those who aspired to master mathematics and mathematical physics in the Cambridge style. These developments were to some extent an instantiation of pedagogical changes taking place at several European institutions during this period, and I conclude by considering the extent to which new forms of undergraduate training altered not only the academy but the wider study of mathematical physics.

3.2. Oral and Written Cultures of Examination

We saw in chapter 2 that the introduction of written examinations played a major role in changing the nature of undergraduate studies in Cambridge in the late eighteenth century. These examinations were instrumental in the rise of private tutors, provided what came to be seen as an impartial, efficient, and objective means of assessing student ability, and gradually altered the entire pedagogical culture of the university. But, despite their importance, we actually know surprisingly little about precisely when and why they were introduced. The first definite evidence that written answers to questions were being required in the Senate House examination dates from the early 1770s, but it is quite possible, even likely, that such answers had by this time been required for a decade or more.[4] The *reasons* for the introduction of written examinations are similarly elusive. It is generally agreed that the Senate House examination was conducted orally until at least the middle decades of the eighteenth century, but thereafter the extant evidence reveals only that written answers to questions gradually became an increasingly important mechanism for assessing the relative performances of the most able candidates.

One factor which almost certainly played a major role in this process was the high status enjoyed by Newton's works in Cambridge.[5] As I noted in chapter 2, it was originally in the form of natural theology that Newtonian science entered undergraduate studies, but from the middle decades of the century more technical aspects of his mathematics and mechanics began to be emphasized.[6] Part of the explanation for this important shift probably lies in the appearance in the 1740s and 1750s of a large number of introductory treatises on the fluxional calculus.[7] Although it was Newton who had originally invented and developed the fluxional calculus, he neither published an introductory text on the subject nor showed how it could be systematically applied to physical topics in natural philosophy. In the first half of the eighteenth century, Newton's mechanics and theory of gravitation was therefore taught in Cambridge in the geometrical form in which it was presented in the *Principia,* while optics, astronomy, pneumatics, and hydrostatics were

4. Rouse Ball 1889, 190. The first definite evidence comes from John Jebb's (2W 1757) account (reproduced by Rouse Ball) of the Senate House examination of 1772.

5. On the role of Newton's works in eighteenth and early nineteenth-century Cambridge, see Gascoigne 1988 and 1989, and Yeo 1988.

6. On the difficulties experienced even by the best Cambridge mathematicians in understanding mathematical aspects of Newton's works, see Stewart 1992, 101–8.

7. These books are discussed in Guicciardini 1989, chapter 4 and Appendix A.

generally taught as qualitative and experimental subjects.[8] The new books which appeared in the mid-eighteenth century offered a systematic introduction to the fundamental operations of the fluxional calculus and showed how it could be applied to the solution of a wide range of mathematical and physical problems.[9] These books appealed widely to the many so-called philomaths who frequented the coffee houses and mathematical societies which had sprung up in several English towns, and who regularly sent solutions to the tricky mathematical problems posed in such periodicals as the *Ladies' Diary.*[10] The strongly problem-orientated presentation in the treatises may well have been intended specifically to appeal to this audience, but this orientation also made it much easier for university students to master the fluxional calculus and its applications, helped to define a new field of mixed-mathematical studies, and provided Cambridge examiners with a ready supply of standard problems.[11] And unlike problems in natural theology, natural philosophy, and even elementary Euclidian geometry and Newtonian mechanics, the solutions to complicated problems involving the calculus were far more readily worked out on paper than stated or debated orally.

The earliest known examples of Senate House problems to which written answers were required date from the mid 1780s, but we can gain some insight into the nature and content of written examinations in Cambridge in the late 1760s from the first papers set for the Smith's Prize examination.[12] First held in 1769, this competition took place two weeks after the Senate House examination and was contested by a small handful of the top wranglers. It was intended, as I noted in chapter 2, specifically to test the candidates' knowledge of natural philosophy and mathematics.[13] From the first year the examination was held, moreover, the candidates were required to submit at least some of their answers in written form.[14] Examples of what appear to be

8. Guicciardini 1989, 65.

9. For a summary of the kinds of problems solved in these books, see the table of contents of William Emerson's *The Doctrine of Fluxions* (1743, 2d ed. 1757), reproduced in Guicciardini 1989, 143–46.

10. On the role of Newton's works in coffee-house culture, see Stewart 1992, 143–51.

11. Guiccardini 1989, 22–23, 64.

12. The problem papers for 1785 and 1786 are in the Challis papers (RGO-CUL) and are reproduced in Rouse Ball 1889, 195–96.

13. The written form of the Smith's Prize examination was based upon the Senate House examination and the fellowship examinations at Trinity College (discussed in section 3.3).

14. Commenting on the character of the examination as conducted by one of the first examiners, Edward Waring (SW 1757), an anonymous obituarist remarked that the students were occupied "in answering, *viva voce,* or writing down answers to the professor's questions." This com-

problem papers (with answers) set for the years 1769–72 provide a useful indication of the breadth and level of mathematical knowledge of the most able questionists (fig. 3.1).[15] Assuming that the professors who set the questions had an accurate sense of what the top three or four wranglers might reasonably be expected to tackle, these students must have had a very sound knowledge of arithmetic, geometry, algebra, pneumatics, optics, acoustics, astronomy, and Newton's *Principia*, as well as an advanced knowledge of the operations of the fluxional calculus and its application to mathematical and physical problems.[16] The Smith's Prize papers also indicate—as do most of the earliest-known Senate House question papers—that it was problems involving the fluxional calculus that were the most technically demanding and that most necessitated the use of paper and pen. It should be kept in mind that candidates for the Smith's Prize examinations were the very best of each year and would probably have gained their advanced mathematical knowledge through private study guided by one or more college fellows. The vast majority of students would have had a much narrower and more elementary grasp of mixed mathematics.[17]

The *content* of the questions to which written answers were first required was by no means the only factor driving their introduction; other, more

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mentator also judged that perhaps no other institution in Europe "affords an instance of so severe a process" (Anon. 1800, 48).

15. RGO 4/273 (RGO-CUL). The first page of this small collection of sewn sheets contains a list of candidates for the Smith's Prize for the years 1769–75 and 1777 (the candidates for 1777 being wrongly listed as those for 1776). The following pages contain questions (with answers) marked for the years 1769–72. It seems extremely likely that these questions were used in the Smith's Prize examination for those years (though they are not explicitly marked as such). The collection is kept in the Neville Maskelyne papers, but the handwriting strongly suggests that they were written by Edward Waring (whose papers are also in the collection), who was one of the examiners for these years.

16. The answers sketched in fig. 3.1 apply to the following questions : (13) To find the cube roots of 8. (14) What are the Geometrical, Arithmetical and Harmonical means between a and b? (15) To form an equation whose roots are 1, 2, 3, 4. (16) What surd multiplied by $\sqrt{2} + \sqrt{3}$ will make the product rational? (17) To find the length of a parabolic curve. (18) To determine the modular ratio. (19) Suppose the point A moves in the right line AT, and draws the body B by the string AB or TP of a given length, what curve will B describe? (20) To find the nature of the harmonical curve which a musical string forms itself into. (21) What was the origin of Fluxions, how does it differ from Archimedes's method of Exhaustions, Cavalieri's method of indivisibles, Dr. Wallis's Arithmetic of Infinites; and what advantages has it over them? (22) What force is necessary to make a Body describe a Conic Section? (23) The velocity, center of force and direction being given, to find the Trajectory described.

17. The questions just referred to are certainly more difficult than those that appear more than a decade later on the earliest known Senate House papers (note 12). Questions 19 and 23 (fig. 3.1), for example, require a higher knowledge of fluxional calculus and Newton's *Principia* respectively than do similar Senate House questions from the mid 1780s.

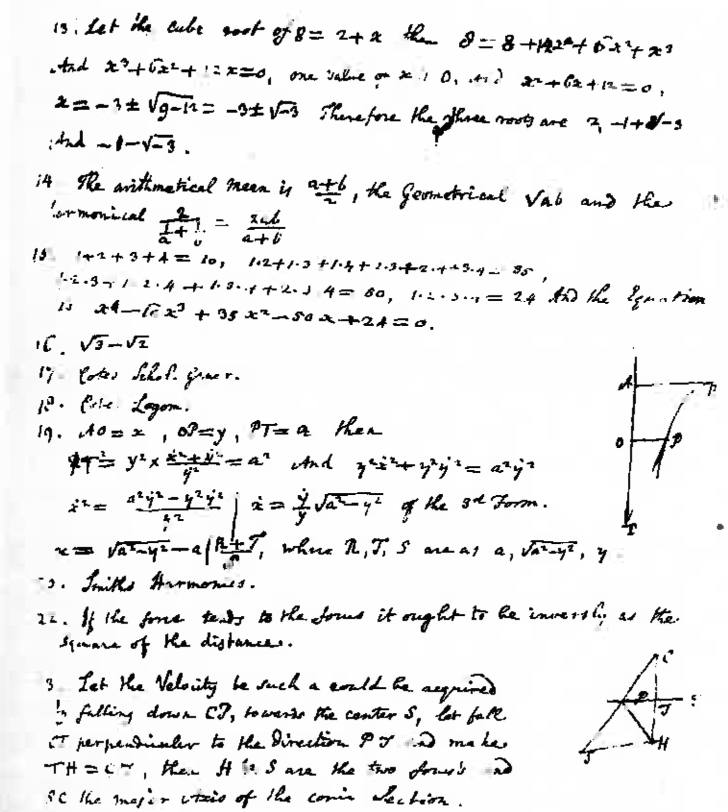
13. Let the cube root of 8 = 2 + x then 8 = 8 + 12x + 6x² + x³
And x³ + 6x² + 12x = 0, one value of x is 0, and x² + 6x + 12 = 0,
x = −3 ± √(9−12) = −3 ± √−3 Therefore the three roots are 2, −1 + √−3
and −1 − √−3.
14 The arithmetical mean is (a+b)/2, the Geometrical √ab and the
Harmonical 2/(1/a + 1/b) = 2ab/(a+b)
15 1 + 2 + 3 + 4 = 10, 1.2 + 1.3 + 1.4 + 2.3 + 2.4 + 3.4 = 35,
1.2.3 + 1.2.4 + 1.3.4 + 2.3.4 = 50, 1.2.3.4 = 24 And the Equation
is x⁴ − 10x³ + 35x² − 50x + 24 = 0.
16. √3 − √2
17. Cotes Schol. Gener.
18. Cotes Logom.
19. AO = x, OP = y, PT = a then
PT² = y² × (ẋ² + ẏ²)/ẏ² = a² And y²ẋ² + y²ẏ² = a²ẏ²
ẋ² = (a²ẏ² − y²ẏ²)/y² | ẋ = (ẏ/y)√(a² − y²) of the 3rd Form.
x = √(a² − y²) − a | (R + T)/S, where R, T, S are as a, √(a² − y²), y
20. Smiths Harmonics.
22. If the force tends to the focus it ought to be inversly as the
square of the distances.
23. Let the Velocity be such as could be acquired
by falling down CP, towards the center S, let fall
CT perpendicular to the Direction PT and make
TH = CT, then H & S are the two focus's and
SC the major axis of the conic Section.

FIGURE 3.1. Outline solutions to Smith's Prize problems, probably by Edward Waring (SW 1757). The relevant questions are reproduced in footnote 16. The precise purpose of these solutions is unclear, but there were probably used either as an aide mémoire while marking examination scripts or as model answers by other examiners or Waring's own students. These are possibly the earliest extant examples of mathematical problems for Cambridge students to which written answers were required. RGO 4/273 (RGO-CUL) (By permission of the Syndics of Cambridge University Library.)

practical, considerations were also in play. During the latter eighteenth century, increasing student numbers together with greater emphasis on finely grading and ranking the candidates gradually made oral examination an excessively time consuming business and one that was vulnerable to accusations of partiality by the examiners (who might also be college or private tutors). The practice of reading out problems to which written answers were required eased the load of the examiners and facilitated a more impartial system of evaluation. Instead of engaging each student in lengthy discussion, an examiner could dictate a question to a large group of students and leave them to write out the answers as quickly as they could. Another important step in the same direction was taken in the late 1780s with the introduction of sheets of written problems which were handed out to the most able candidates. These sheets were taken by the student to a convenient window-seat in the Senate House, where he wrote out as many solutions as he could in the time allotted. After 1791 these problems sheets were printed and, in 1792, specific numbers of marks began to be allocated to specific questions.[18]

The gradual shift from oral to written examination described above should not be understood simply as a change in the *method* by which a student's knowledge of certain subjects was assessed. The shift also represented a major change both in *what* was assessed and in the skills necessary to succeed in the examination. Consider first the major characteristics of the oral disputation. This was a public event in which a knowledge of Latin, rhetorical style, confidence in front of one's peers and seniors, mental agility, a good memory, and the ability to recover errors and turn the tables on a clever opponent were all necessary to success. The course of the examination was regulated throughout by its formal structure and by the flow of debate between opponent, respondent, and moderator. The respondent began by reading an essay of his own composition on an agreed topic and then defended its main propositions against three opponents in turn according to a prescribed schedule (fig. 3.2).[19] The public and oral nature of the event meant that the processes of examination and adjudication were coextensive. The moderator formed an opinion of the participants' abilities as the debate unfolded and the nature and fairness of his adjudication were witnessed by everyone present. The examination was also open-ended. The moderator could prolong the debate until he was satisfied that he had correctly assessed the abilities of opponents and respondents, and would generally quiz the respon-

18. Gascoigne 1984, 552.

19. For a account of a disputation in the latter eighteenth century, see Rouse Ball 1889, 166–69.

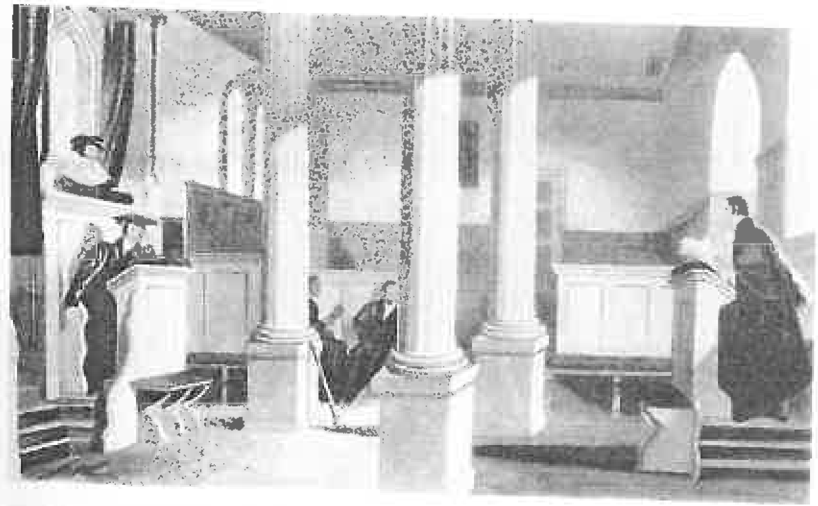

FIGURE 3.2 A rare depiction of a disputation as seen by the audience. The student being examined (the "respondent") is shown (right) reading out his essay. On the left are the first "opponent" (below), who will shortly oppose the propositions advanced by the respondent, and the moderator (above). The second and third opponents are shown sitting in the background, one listening to the respondent's essay, the other apparently reading his notes. Huber, *English Universities*, 1843. (By permission of the Syndics of Cambridge University Library.)

dent himself. Finally, once the disputation was completed, the only record of the examination was the recollections of those involved.

The economy of a written examination was quite different in several important respects. First, the oratorical skills mentioned above became irrelevant, as the examination focused solely on the reproduction of technical knowledge on paper and the ability to marshal that knowledge in the solution of problems. Second, the process of examination was separated from that of adjudication, a change that destroyed the spontaneity of the public Act. Without the formal procedure of a disputation and the rhythm of debate between opponent and respondent, each candidate was left to work at his own pace with little sense of how well or how badly he was doing. And, once the written examination was completed, the student had no opportunity either to wrangle over the correctness of an answer or to recover errors. The advent of written examinations also brought substantial change to the role of the moderators. Instead of overseeing and participating in a public debate, they became responsible for setting questions, marking scripts, and policing the disciplined silence of the examination room (fig. 3.3). This brings us to a third important difference between oral and written assessment. As I noted above, setting identical questions to a group of students made it much

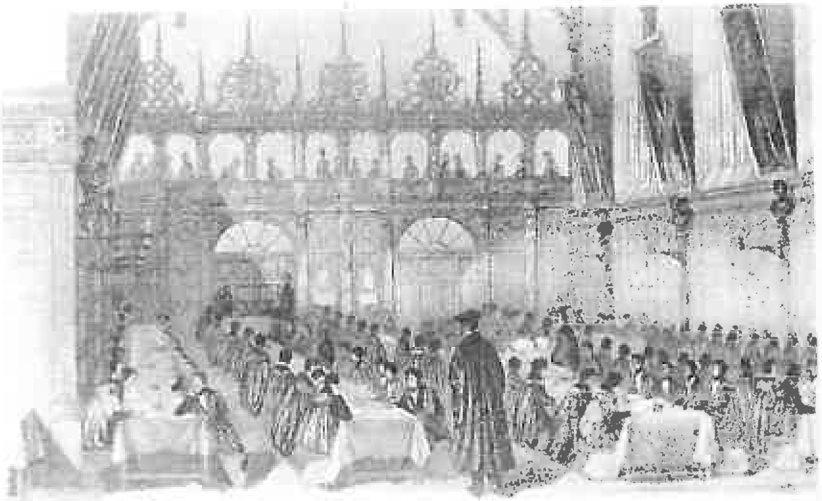

FIGURE 3.3. A written examination in progress in the Great Hall of Trinity College, circa 1840. This rare, possibly unique image, strikes a powerful comparison with figure 3.2. Oral wrangling has given way to paper-based problem solving while the moderator's role has altered from adjudicating a public disputation to preserving the disciplined silence of the examination room. The status of the shadowy figures watching the examination from the gallery is unclear, but they may represent a transitional remnant of the audience at a public disputation. Huber, *English Universities*, 1843. (By permission of the Syndics of Cambridge University Library).

easier directly to compare and rank the students, especially if each question was marked according to an agreed scheme. Furthermore, the written examination scripts provided a permanent record of each student's performance which could be scrutinized by more than one examiner and reexamined if disagreements emerged. These differences between written and oral assessment altered the skills and competencies required of undergraduates and completely transformed their experience of the examination process. In the balance of this section I explore that changing experience over the first half of the nineteenth century.

It is a measure of the relative novelty yet rapidly increasing importance of written examinations circa 1800 that this was the moment when the university published the earliest firsthand accounts of the manner in which they were conducted. In 1802, the annual Cambridge calendar, which described the constitution and customs of the university, provided a detailed commentary on the progress of an examination in the Senate House. Unlike earlier accounts of this examination, known from student correspondence and private journals, this one focused not on the *oral* questioning of candidates but on the peculiarities of providing written answers to questions. Following

some brief remarks on the ceremony that announced the start of the examination, the anonymous commentator discussed the seating of the various classes of students (as determined by the disputations) at appointed tables, emphasizing that upon each table "pens, ink and paper are provided in abundance." The commentary then offered the following account of the method by which questions were posed and answered:

> The young men hear the proposition or question delivered by the examiners; they instantly apply themselves; demonstrate, prove, work out and write down, fairly and legibly (otherwise their labour is of little avail) the answers required. All is silence; nothing heard save the voice of the examiners; or the gentle request of some one, who may wish a repetition of the enunciation. (Rouse Ball 1889, 198)

This commentary, like those on other university customs, was partly intended to inform younger undergraduates of the conduct proper to university ceremonies. The new written form of the Senate House examination must have posed special problems in this respect, as most colleges did little to prepare their students for such examinations, and, unlike oral disputations, undergraduates could not witness the procedure for themselves.[20]

I have already noted that one peculiarity of written examinations is that they lacked the prescribed order and interactive rhythm of the oral disputation. The unusually breathless narrative of the above account was probably intended to convey this point to undergraduates and to impress upon them the importance of working as quickly as possible without the prompting of an examiner. The separation of examination and assessment also raised the possibility that a student might not fully appreciate that the act of writing the examination script did not of itself constitute the completion of the examination. Thus our commentator reminded undergraduates that "although a person may compose his papers amidst hurry and embarrassment" he must remember that his answers "are all inspected by the united abilities of six examiners with coolness, impartiality and circumspection" (Rouse Ball 1889, 199). One final point of difference, hinted at by reference to the "embarrassment" a candidate might experience, was the great breadth and detail of technical knowledge expected of candidates providing written answers. Although in an oral disputation the respondent could not anticipate precisely what objections his opponents would raise, he could choose and

20. On the introduction of written examinations in Cambridge colleges, see Gascoigne 1984, 556–57.

prepare the propositions he wished to defend. In written examinations by contrast, a student could be questioned on any topic considered appropriate by the examiners and had to tackle problems based on those topics. In the above commentary the student was warned accordingly that in a written examination "no one can anticipate a question, for in the course of five minutes he may be dragged from Euclid to Newton, from the humble arithmetic of Bonnycastle to the abstruse analysis of Waring" (Rouse Ball 1889, 199). Even the most able students had therefore to prepare themselves for the fact that they might be partially or entirely unable to tackle some of the questions posed.

During the first three decades of the nineteenth century, undergraduates were gradually familiarized with the techniques of written assessment by the introduction of regular college examinations. The first examinations of this kind had been introduced in the latter eighteenth century by St. John's and Trinity in an effort to stem the rising prestige of the Senate House examination and preserve the independence of college teaching.[21] The power of the examiners to choose the topics examined in the Senate House meant that any college wishing its students to do well was obliged to teach those subjects that appeared regularly on the Tripos papers. Efforts to resist this centralization of power seem eventually to have backfired, as the longer-term effect of college examinations was simply to drive the level of technical performance in the Senate House ever higher. By 1831 all colleges had introduced regular undergraduate examinations of their own, and, as we shall see in chapter 4, these examinations appear to have been almost as competitive as the Senate House examination itself. College examinations established a student's reputation among his immediate peers and provided an important indication of how well he could be expected to do in the Senate House.

By the early 1840s, the use of written examinations had become so commonplace in Cambridge that it was only to visitors from overseas that the system seemed sufficiently peculiar to warrant special comment. One such American visitor was Charles Bristed, who, as we saw in chapter 2, studied at Trinity College in the early 1840s. Bristed identified the emphasis on written examinations as one of the most striking characteristics of a Cambridge education and suggested that the "pen-and-ink system of examination" had been adopted "almost entirely at Cambridge, in preference to *viva voce*, on the ground, among others, that it is fairer to timid and diffident men" (Bristed 1852, 1:97). This explanation of the demise of public disputations,

21. Gascoigne 1984, 556–57.

probably common by the early 1840s, is somewhat implausible since, as I have already noted, oral and written examinations were not simply alternative methods of testing the same abilities. Bristed's remark nevertheless highlights the extent to which the new examinations had altered student sensibilities by the early Victorian period. The ideal of a liberal education prevalent in Georgian Cambridge was, as we have seen, one in which students were supposed to learn to reason properly through the study of mathematics and to acquire appropriate moral and spiritual values through the subservient emulation of their tutors. This kind of education, which had long been seen as an appropriate preparation for public life, valued the qualities of good character, civility, and gentility above those of introspection, assertiveness, and technical expertise.[22] The oral disputation formed an integral part of this system of education as it tested a student's knowledge through a public display of civil and gentlemanly debate. Timidity and diffidence were indeed qualities that handicapped a student in this kind of examination, though not because they prevented him from answering technical problems in mathematics, but because they were seen as undesirable qualities in a future bishop, judge, or statesman. During the early decades of the nineteenth century, the increasing reliance on written examinations as a means of ranking undergraduate performance began to undermine the ideal of a liberal education from within. As we saw in chapter 2, ambitious students aiming for high placing in the order of merit abandoned college and professorial lectures during the 1820s and 1830s, turning instead to the leading mathematical coaches. By the early Victorian era, as Bristed's comments make clear, the now defunct oral disputations had come to be regarded as *unfair* examinations because they required students to display qualities beyond the purely technical.

It is nevertheless important to notice that although the written examination was not a *public* ritual, it was by no means an emotionally neutral one. Candidates for written examinations might have been spared the trials of stage fright and of making intellectual fools of themselves in front of their peers, but they were subject to a whole new set of pressures and anxieties.[23] These subtle changes in student sensibilities were made very visible by those who sought to preserve the broader ideals of a liberal education. Most vocal among these was William Whewell, who pointed explicitly to competitive examinations as the source of what he saw as some new and highly undesirable traits in undergraduates. According to Whewell, these examinations

22. Rothblatt 1982, 5.
23. On the anxieties generated by oral examination, see Rothblatt 1982, 7.

aroused in students a desire for "distinction" and "conquest," and evoked a "play of hopes and fears, sympathy and novelty"; all sensibilities which in Whewell's opinion encouraged them to become *active* instead of properly *passive* participants in the educational process. As he revealingly stated in 1837, the competitive written examination made any candidate who was tipped to do well "one of the principal actors in the piece, not a subordinate character, as he is in the lecture room" (1837, 59). Whewell saw the oral disputation as the form of examination that best preserved the social order of the lecture room; it was a public event during which the candidate addressed an audience and remained visibly under the authority of the moderator. In a written examination each candidate worked alone and introspectively to the sole end of out-pointing his competitors. As we shall see in chapter 4, to succeed in these circumstances a student had not only to work for several years at the very limit of his intellectual ability but to deal with the emotional pressures induced by the build up to a few days of gruelling examination upon which the success or failure of his entire undergraduate career depended.

Bristed's undergraduate experience confirms that sitting a written examination could be just as much an emotional trial as could participating in a public disputation. His initial discussion of the relative pedagogical merits of written and oral assessment suggests that he supposed the former to be a more natural and less stressful form of testing mathematical knowledge. But once he had actually experienced a written examination for himself, he acquired a strange virtual nostalgia for the oral disputation. I have just noted the frantic preparations made by ambitious students prior to the examination, but Bristed claimed that even entering and working in the examination hall could be stressful and disorientating experiences. He recalled that the early impression of Trinity Great Hall imbibed by most freshmen was that of a friendly dining room in which a common collegiate spirit was developed through convivial meal-time conversation. That impression was quickly dispelled when he entered the hall for his first college examination: "The tables were decked with green baize instead of white linen, and the goodly joints of beef and mutton and dishes of smoking potatoes were replaced by a profusion of stationery" (1852, 1:97). According to Bristed, the atmosphere in the hall became yet more disconcerting once the examination got under way. The enforced silence was broken only by the ominous footfall of the examiners "solemnly pacing up and down all the time," while the peculiar sound of "the scratching of some hundred pens all about you [made] one fearfully nervous" (fig. 3.3). Bristed also lamented both the lack of personal interaction between student and examiner characteristic of written examinations, and the permanency and precision of the written script. He observed that any "little slips" made *viva voce* might be recovered, allowed for, or even pass unnoticed, whereas "everything that you put down" in a written examination would be "criticised deliberately and in cold blood. Awful idea!" (1852, 1:98).

The comments by Whewell, Bristed, and others cited above provide some insight into the new cultural values associated with the written examination in early Victorian Cambridge. The qualities of civility, gentility, and public presence valued in the older ideal of a liberal education had been superseded by those of competitiveness, individual coaching, solitary study, and the reproduction of technical knowledge on paper. The pressures and anxieties experienced by the students during the actual examination had also altered substantially. A public disputation lasted at most a couple of hours and during this time the respondent was required to display a range of abilities in defending a prepared topic.[24] By the early 1840s, the written Senate House examination lasted six days (papers being sat morning and afternoon) and focused narrowly on the writing-out from memory of mathematical proofs, theorems, and definitions, and the solution of progressively difficult mathematical problems.[25] Moreover, the written examination was considered just as much a test of moral qualities as had been the oral disputation. The protracted intellectual and emotional stress induced by consecutive days of examination proved an unendurable trial to some students, and, as we shall see in chapter 4, the ability to remain calm and to reason quickly and accurately under these conditions gradually became a new hallmark of good character.

3.3. A Mathematical World on Paper

Another extremely important change in undergraduate mathematical studies prompted by the introduction of written examinations was the shift it engendered from a culture of reading and oral discourse to one of writing on paper. The use of paper and pen as a medium of mathematical learning is now so commonplace that, like written examinations themselves, it is accepted as the natural and obvious way of training students in the technical disciplines. At the beginning of the nineteenth century, however, when this practice was still relatively new, it made a sharp contrast both with earlier techniques of learning in the university and with contemporary ones in el-

24. Rouse Ball 1889, 182.
25. Glaisher 1886–87, 15.

ementary schools. We have seen that, until the mid-eighteenth century, undergraduates continued to be assessed by oral Acts, and that in preparation for these Acts they learned from a range of sources: by private reading guided by a tutor, by attending college and, occasionally, professorial lectures, by attending formal disputations and debates, and by discussions with their tutors and peers.[26] In connection with these activities students also copied out, annotated, and summarized important texts, but, in a pedagogical culture of oral teaching and examination, they were required neither to *write out* what they had committed to memory nor to solve technically difficult problems that could only be tackled with the aid of paper and pen. The kinds of natural-philosophical proposition that students were required to defend from the *Principia* reflected the oral mode of examination. Rather than reciting long mathematical proofs and demonstrations or solving problems, students were required to attack or to defend such qualitative propositions as "whether the cause of gravity may be explained by mechanical principles" or "whether Newton's three laws of nature were true."[27]

During the second half of the eighteenth century, following the introduction of the Senate House examination, the emphasis in undergraduate studies began to shift away from qualitative natural-philosophical and natural-theological aspects of Newton's work towards more technical aspects of mixed mathematics. It was in this context, as we have seen, that students taking the Senate House examination were first required to provide *written* answers to orally dictated questions. This was not in fact the very first use of written examinations in the university, as Richard Bentley (3W 1680) had required written answers to questions from candidates for fellowships at Trinity College since 1702. In this case each candidate was given a different set of questions to tackle, but it is nevertheless likely that Bentley's innovation provided the model for the subsequent introduction of written examinations in the Senate House.[28] Once such a system had been introduced, moreover, it was a straightforward matter to make the questions more numerous and more difficult year by year as the competition intensified and the students' level of technical competence rose. By the first decade of the nineteenth century, when the written papers had become the most important factor in determining a student's place in the order of merit, the most difficult questions had become too technical and too complex to be solved orally.[29] From this point on, learning and examining mathematics were primarily paper-based activities, and the formal disputations and oral questioning in the Senate House gradually declined in importance until they were discontinued completely in the late 1830s.[30]

26. For a discussion of undergraduate pedagogical practice in sixteenth-century Cambridge, see Sherman 1995.

27. Gascoigne 1989, 177.

28. Rouse Ball 1889, 193. This was probably the first occasion in Europe on which written answers to mathematical questions were required. It is likely that the questions were relatively elementary since, as I noted in chapter 1, Bentley himself had difficulty understanding technical aspects of the *Principia*; see Guicciardini 1999, 125.

We know very little about the methods by which school children and undergraduates were instructed in mathematics in the early decades of the nineteenth century, but there is no doubt that those arriving to study in Cambridge at this time were surprised by the emphasis upon working on paper.[31] When J. M. F. Wright arrived at Trinity College in 1815, he was admonished by the tutor who examined him when he confessed his inability to "write down" his mathematical knowledge on paper. The tutor then warned him sternly that at Cambridge:

> all things—prizes, scholarships, and fellowships, are bestowed, not on the greatest readers, but on those who, without any assistance, can produce most knowledge on paper. You must hence-forth throw aside your slate . . . and take to scribbling upon paper. You must "*write out*" all you read, and read and write some six or eight hours a day. (Wright 1827, 1:96)

This experience was echoed by Wright's contemporary Solomon Atkinson (SW 1821), who, as we saw in the previous chapter, arrived at Queens College in 1818. This very ambitious and largely self-taught son of a Cumbrian farm laborer hoped that he would be able to impress the president of the college, Isaac Milner (SW 1774), sufficiently to gain entrance as a sizar.[32] Atkinson had mastered a good deal of arithmetic and geometry before arriving at the university and, although unable to recall every proposition in Euclid by number, greatly pleased Milner by being able to scrawl the proof of the "proposition about the square of the hypothenuse" [sic] on a "slip of paper" (Atkinson 1825, 496). Although well informed about undergraduate studies at Cam-

29. Compare, for example, the problem papers for 1785 and 1802 reproduced in Rouse Ball 1889, 195, 208–9.

30. Ibid., 183.

31. For a discussion of elementary mathematical education in the nineteenth century, see Howson 1982, chap. 6.

32. A "sizar" was student who paid only a fraction of the usual college fees in return for acting as a college servant. See Searby 1997, 70–71.

bridge, Atkinson was still astonished by the emphasis on writing-out mathematical knowledge, and he reflected that most freshman would be "shocked" by the "University practice of scribbling everything on paper." To an impoverished student from a laboring community, this unrestrained use of paper to acquire skill with a quill pen together with speed and accuracy in mathematical technique seemed wanton. Paper was an expensive, hand-made commodity in the early nineteenth century, and one which peaked in price around 1810.[33] Charting the development of what he termed "the progress of a poor man's extravagance," Atkinson noted how what seemed at first the very expensive process of mastering mathematics in the Cambridge fashion soon became commonplace. In the case of textbooks, for example, he noted that at first a poor student was unwilling to spend what seemed to him the extravagant sum of five shillings on a "Treatise on Conics or Algebraic Equations" whereas, by the end of the second year, he would not "scruple to bestow 10 or 15 guineas" in the form of a college prize on a book of little or no direct value to his studies. Atkinson also discussed the student's changing perception of the value of paper, noting that the freshman felt "more reluctance at wasting an inch square of white paper" than he would subsequently feel "in throwing half a dozen sheets into the fire" (1825, 507).

The Cambridge emphasis on mastering mathematics on paper is highlighted from a different perspective by one of Wright and Atkinson's most brilliant younger peers, George Airy. Although Airy displayed some ability for mathematics early on in his school career, he was initially identified as a potential scholar by his excellent memory for Greek and Latin poetry. It was only after it had been decided that he would be sent to Cambridge that, as we saw in chapter 2, Airy was taught mathematics privately by a recent graduate of the university, Mr. Rogers. One of the important things that Airy learned from Rogers was that he had not only to grasp the gist of required theorems and techniques, but had also to be able to "write out any one of the propositions which [he] had read in the most exact form." It was this ability to reproduce his mathematical knowledge on paper that Airy remembered as standing him in particularly good stead when he arrived at the university (Airy 1896, 19–20).

The wide gap between the pedagogical practice anticipated by most freshmen and that actually encountered in Cambridge at this time is further

33. Dykes Spicer 1907, 86–87. An average quality paper cost around one shilling and sixpence a pound in 1810, while a farm day-laborer earned at best around ten shillings a week. The cost of a couple of pounds of paper a week would therefore have seemed considerable to a boy like Atkinson.

underlined by Wright's account of his first mathematical lecture at Trinity College. I mentioned above that a great deal of mathematical teaching from the late sixteenth century focused on the use of practical mathematical instruments. It was widely considered that a basic knowledge of the techniques of accounting, surveying, navigation, military fortification, gunnery, and so on was important in the education of a young gentleman.[34] This tradition appears to have persisted beyond Cambridge in the nineteenth century to the extent that most of the roughly one hundred students present at Wright's first lecture supposed that they would be required accurately to construct geometrical figures. Most had accordingly come equipped with a case of mathematical instruments for the purpose. But the lecturer, Mr. Brown, immediately informed the class that these instruments were superfluous as "the theory we are about to expound" required nothing but freehand sketching and writing.[35] Brown demonstrated to the class, from his rostrum, how the only figure that needed sometimes to be drawn with tolerable accuracy, the circle, could be produced by using one hand as a pair of compasses while rotating the paper with the other. Wright's account of his first Trinity lecture also reveals another important difference between elementary methods of teaching mathematics and that emergent in Cambridge. In addition to the case of mathematical instruments, most students had brought with them—as had Wright to his first interview at Trinity—a "slate" on which to draw and erase geometrical figures and calculations. However, in the Trinity lecture room each student was provided with an ample supple of "pens, ink and foolscap [paper]."[36] Trinity students were required from the very beginning of their undergraduate studies to master the art of writing rapidly and accurately with an ink quill on paper. Brown accordingly announced that "those gentlemen . . . who have slates before them, will be pleased to bring them no more, paper being the only thing scribbled upon, in order to prepare you for the use of it in the Examinations."[37]

The arrival of paper and pens in the Trinity College lecture room was, as Brown's comment makes clear, a concession to the rapidly increasing importance of written examinations in Cambridge, but it should not be taken as an indication that the teaching methods employed by college lecturers in

34. Turner 1973; Feingold 1997, 372–79.

35. Wright 1827, 1:118.

36. Ibid., 116. The use of slates in classrooms remained commonplace well into the mid-Victorian period. See Howson 1982, 121.

37. Wright 1827, 1:118. For additional comments on taking notes in the Trinity lecture room, see Bristed 1852, 1:23.

general had altered a great deal from the previous century. According to Wright (1827, 1:120–50), even at Trinity the primary function of a college lecturer was still to inform students what they should *read* and to test their recall and understanding of that reading by catechetical inquisition. Although by the 1820s the formal disputations played only a minor role in deciding a student's place in the order of merit, they nevertheless remained an important hurdle to be cleared, and college lecturers continued to prepare their classes accordingly.[38] In order to find out whether students had learned the definitions, proofs, and theorems they were required to know, the lecturer would go round the class asking them in turn to enunciate propositions and even to solve simple problems orally.[39] The paper and pens provided in the lecture room were not therefore central to the teaching process, but enabled students to take notes as they saw fit during the oral exchanges between the lecturer and individual members of the class.[40] A rare firsthand account by Charles Prichard of mathematical lectures at St. John's College reveals that paper and pens had not entered that college's lecture rooms even in the late 1820s. Prichard confirms that lectures remained "*viva voce*" and "strictly catechetical" in his time, the students sitting "round the walls of the room" with nothing in their hands "excepting an unannotated copy of some classical author." The only visual aid employed by the mathematical lecturer was "a cardboard, on which diagrams were drawn relating to the mathematical subject before us" (Prichard 1897, 36). This cardboard was handed from student to student as the lecturer went round the class and seems to have functioned as a kind of primitive blackboard. Instead of going through a geometrical demonstration in purely oral form, the student could refer to a diagram that was visible to the teacher and to the other students.

These oral exchanges between individual students and the lecturer could be of considerable help to other members of the class, especially when it came to gaining a firm grounding in more elementary mathematics. As Prichard recalled, the value of each of the "logomachies between the tutor and the undergraduate" depended mainly on the ability of the student being quizzed. If he were dull the interaction was at best amusing but if he were bright and well informed it was "frequently very instructive to those who came seriously to

38. Rouse Ball 1889, 183. For an account of a tough disputation in 1826, see DeMorgan 1872, 305.

39. Wright (1827, 1:147) notes that students practiced the enunciation of such demonstrations aloud in their rooms.

40. Wright only implies that this was the use of paper and pens in the Trinity lecture room, but it is confirmed by Airy (1896, 23–25) and Bristed (1852, 1:23).

learn" (1897, 36).[41] But in the main, college lectures were run at the pace of the least able members of the class, and the most ambitious students soon resorted to private tutors. Atkinson was very disappointed at the elementary level of mathematical lectures at Queens College, claiming that apart from informing reading men of the "proper subjects of study" they were "so much time wasted."[42] Trinity College appears to have been something of an exception in encouraging students to take notes on paper in the latter 1810s, and it is likely that this practice would have given Trinity men a significant advantage over their peers from other colleges when it came to the Senate House examination.[43]

The rapid rise of the private tutor made paper-based learning in mathematics commonplace from the 1830s, and explicit references to the practice become correspondingly sparse thereafter. Occasional remarks by students in private correspondence nevertheless suggest that the imperative to learn with pen and paper still surprised some freshmen well into the Victorian era. Thus in a letter to his father in October 1840, even the well-heeled Francis Galton remarked: "I waste paper fearfully, i.e. scribble over both sides of it innumerable x, y's and funny looking triangles." Galton added that his "bedder" told him it was always easier to keep the room of a "reading gentleman" because she was never short of paper with which to light the fire (Pearson 1914, 1:143). The young William Thomson (2W 1845), who went on to become one of Cambridge's most original mathematical physicists, was similarly struck by the material apparatus of Cambridge mathematics. A few weeks after he began studying with the outstanding private tutor, William Hopkins, he recorded in a letter to his father that he had taken to doing all his mathematical work with a quill pen because they were used in examinations and one "must get into the habit of being able to write with them" (Thompson 1910, 1:31). This habit of learning mathematics through writing out and problem solving on paper was also one which shaped the style of successful wranglers when they undertook original investigations. In his second undergraduate term at Cambridge, George Airy commenced what he called the "most valuable custom" of always having "upon [his] table a quire of large-

41. Prichard considered this form of catechetical instruction preferable to the professorial style of simply reading lectures.

42. Atkinson 1825, 501. It is very likely that there was a wide disparity in the quality and level of lectures delivered at different colleges.

43. Trinity prided itself on the quality of its college lectures, and there is some evidence that Trinity students were marginally less reliant on private tutors than were students from other colleges at the beginning of the Victorian period. See Searby 1997, 131.

sized scribbling paper sewn together: and upon this paper everything was entered: translations out of Latin and into Greek, mathematical problems, memoranda of every kind (Airy 1896, 25). Airy also used his scribbling paper to "put [his] lecture notes in order" and to write out summaries of the more difficult mathematical topics with which he was expected to be familiar. On one occasion, having wrestled in vain with a textbook explanation of the phenomenon of "Precession," he "made out an explanation for [himself] by the motion round three axes." This manuscript, with "some corrections and additions," was subsequently printed as one of Airy's famous *Mathematical Tracts* (1826). William Thomson was famous for always carrying a "copy-book" which he would produce at "Railway Stations and other conveniently quiet places" in order to work on his current mathematical problems. He had begun keeping a research notebook as an undergraduate, and, as we shall see in chapter 4, would sometimes set to work on occasions that seemed highly inappropriate to his companions.[44]

Some very distinguished Cambridge mathematicians of the mid-Victorian period also preserved a copy-book style of teaching long after blackboards had become commonplace in college lecture and private teaching rooms. J. J. Thomson (2W 1880) recalled that when the Sadeirian Professor of Pure Mathematics, Arthur Cayley (SW 1842) gave lectures in the late 1870s he "sat at the end of a long narrow table and wrote with a quill pen on large sheets of foolscap paper" (Thomson 1936, 47). This, as Thomson remarked, "made note-taking somewhat difficult," but he insisted that the lectures were nevertheless "most valuable" in teaching students "not to be afraid of a crowd of symbols." Having launched into a difficult problem using the first method that came to mind, Cayley would frequently develop "analytical expressions which seemed hopelessly complicated and uncouth." Confident of his own manipulative powers, Cayley "went steadily on" and, to the satisfaction of the students, "in a few lines had changed the shapeless mass of symbols into beautifully symmetrical expressions, and the problem was solved" (1936, 47).[45] Thomson also recalled that although the Lowndean Professor of Astronomy and Geometry, John Couch Adams (SW 1843), delivered his lectures orally, they were "read from beautifully written manuscripts which he brought into the lecture room in calico bags made by his wife." Adams's lectures in fact combined both teaching and research, as the

44. Smith and Wise 1989, 180; Thompson 1910, 2:616. Maxwell too "always" carried a "notebook" in which he worked out "solutions of problems" (Campbell and Garnett 1882, 368).

45. Cayley's teaching method was probably taken from William Hopkins, who had coached him to the senior wranglership in 1842.

manuscript from which he read contained his "own unpublished researches on Lunar Theory" (1936, 47–48).

By the 1880s, the link between paper-based mathematical training and subsequent research practice was so well established that most commentators were concerned not by the novelty, but by the quality, of such training. When George Darwin (2W 1868) delivered his inaugural lecture as Plumian Professor of Astronomy and Experimental Philosophy in Cambridge in 1883, he drew the attention of his audience to the importance of teaching students to solve mathematical problems neatly and systematically on paper. Drawing upon his experience as a Tripos examiner, he expressed the opinion that too little attention was paid both to "style" and to the "form on which the successful, or at least the easy, marshalling of a complex analytical development depends." This was, he argued, a serious oversight, as some otherwise outstanding Cambridge mathematicians "seem never to have recovered from the ill effects of their early training even when they devote the rest of their life to original work" (Darwin 1907–16, 5:5). In order to illustrate his point, Darwin offered a comparison between, on the one hand, the exemplary copy-book work of William Thomson and John Couch Adams, the latter's manuscript being "a model of neatness in mathematical writing," and, on the other, James Clerk Maxwell (2W 1854), who "worked in parts on the backs of envelopes and loose sheets of paper crumpled up in his pocket." Darwin deplored this slovenly practice which, he suggested, was responsible for the fact that the first edition of Maxwell's well-known *Treatise on Electricity and Magnetism* "was crowded with errata, which have now been weeded out one by one." In Darwin's opinion, Maxwell had only been saved from more serious error "by his almost miraculous physical insight, and by a knowledge of the time when work must be done neatly" (1907–16, 5:6).

George Darwin was by no means the first person to comment on Maxwell's shortcomings as an analyst, but it is significant that he attributed them to poor training.[46] The reputations of many successful Cambridge mathematical physicists of the Victorian period were based on their finely honed skills of mathematical manipulation and problem solving, skills developed

46. One of Maxwell's examiners at Edinburgh University, J. D. Forbes, complained of Maxwell's "exceeding uncouthness" as a mathematician. Forbes confided to William Whewell that the "Drill" of Cambridge was "the only chance of taming [Maxwell]," but added that Maxwell was "most tenacious of physical reasonings of a mathematical class, & perceives them far more clearly than he can express." Maxwell's private tutor at Cambridge, William Hopkins, later remarked in a similar vein that although it appeared "impossible for Maxwell to think incorrectly on physical subjects," in his analysis he was "far more deficient." Forbes to Whewell, 2 May 1852, Add. Mss A.204/103 (WHP-TCC); and Glazebrook 1896, 32.

through years of progressive training under the tutelage of a handful of brilliant mathematical coaches. This was especially true of men like Couch Adams and Darwin, whose outstanding work in planetary motion and geodesy (discussed further below) relied heavily upon their ability to solve extremely difficult and analytically complicated mathematical-physical problems. The culture of reading and oral disputation prevalent in undergraduate studies a century earlier was neither intended nor suited to prepare graduates for this kind of original investigation, but, following the transformation of that culture wrought by the switch to working on paper, and with the rise of a research ethos during the last third of the nineteenth century, this became virtually its only purpose. In the eighteenth century a graduate who wished to make an original contribution to mixed mathematics would have had either to train himself to manipulate symbols reliably or to work under the guidance of a more senior member of the university. By the time of Darwin's inaugural address, it was taken for granted that undergraduate training should develop the paper-based skills of mathematical manipulation necessary for research, and that the career even of a mathematician of Maxwell's undoubted brilliance could be blighted if that training was inadequate.

3.4. Bookwork, Problems, and Mathematical Textbooks

Having explored the change in material culture and student sensibilities that accompanied the gradual switch from oral to written examinations, I turn now to the structure and content of the examination questions themselves and to their relationship with the wider pedagogical apparatus of undergraduate mathematical teaching. Just as the introduction of paper-based examination in mathematics was a gradual and historically specific process, so too the form of questions to which written answers were required was peculiar to Cambridge and changed over time. The first printed question-sheets, which, as we have seen, appeared in the 1780s, contained only short and relatively simple mathematical questions, generally only one or two lines in length. The likely purpose of these questions was, as I noted above, to provide a convenient, efficient, and uncontroversial method of comparing the abilities of the most able students, but the introduction of these questions marked the beginning of two very important changes in the way mixed-mathematical studies were approached in Cambridge.

Before outlining these changes, however, we should note that, almost from the time of their introduction, questions requiring written answers

were divided into two kinds.[47] The first kind, which from the early nineteenth century would be known as "bookwork" questions, required students to write out standard definitions, laws, proofs, and theorems from memory.[48] These questions were answered by all the students and, until the late 1820s, were dictated orally by the examiners.[49] The second, and more difficult kind of question, was the "problem." These questions were also dictated in the 1770s and 1780s but, from the mid 1790s, were given to the more able students in the form of printed sheets. It was these problem sheets that provided the most severe test of a candidate's technical ability and, from around 1800, effectively decided his place in the order of merit.[50] The precise origin of these two types of question is unclear, but it seems extremely likely that they reflected the style of presentation of such classical mathematical texts as Euclid's *Elements* and, especially, Newton's *Principia*. Newton began the *Principia* by stating the definitions and laws which would apply throughout the text, and then developed the detailed geometrical-mechanical content of his argument as an extended series of propositions. Each proposition consisted either of a theorem concerning the motion of bodies, or of a specific problem that was solved by the application of the foregoing theorems. Given the *Principia*'s extremely high status in eighteenth-century Cambridge, it seems reasonable to conclude that, at least in mechanics, the bookwork questions reflected the canonical status of Newton's propositional theorems while the problem questions reflected the ingenious application of the theorems as exemplified in the propositional problems.

Returning now to the changes wrought by the introduction of written examination papers, the first concerns the status of the various physical-mathematical definitions, laws, and theorems taught to undergraduates. We have seen that when students were examined by oral disputation on, say, Newton's *Principia*, they were generally required to *defend* qualitative propositions against the carefully contrived objections of several opponents.[51]

47. Little is known of the form and content of questions posed and answered orally in the mid eighteenth century, but the range of topics covered was roughly that of early written questions. See Rouse Ball 1889, 190–92.

48. It is unclear when the term "bookwork" was introduced, but it was definitely in use by the latter 1810s. See Wright 1827, 2:62.

49. Rouse Ball 1889, 199, 212. The new regulations of 1827 were the first to require that all examination papers be printed and given to the students.

50. Ibid., 194–96. The earliest examples of the problems set orally to candidates in the Senate House examination date from the mid 1780s.

51. Ibid., 182. Although the opponents were the undergraduate peers of the respondent, they were supplied with difficult objections by their college tutors.

These verbal encounters implied that there *were* seemingly plausible objections to Newton's celestial mechanics and required the student to locate the fallacies in such objections from a Newtonian perspective.[52] In providing *written* answers to questions, by contrast, students were required either to reproduce as bookwork the laws and propositional theorems found in such books as the *Principia*, or to *assume* their truth in tackling questions on the problem papers.[53] The move from oral disputation to written examination was therefore accompanied by a far more dogmatic approach to the physical foundations of mixed mathematics.

The second way in which written examinations altered the kind of mathematical knowledge prized in Cambridge concerns the technical application of physical principles and mathematical methods. Although many problems requiring written answers were at first sufficiently short and simple to have been solved orally—in this sense being a straightforward substitute for the more time-consuming oral examination—they gradually developed to become a test of quite different skills. In the mid 1780s, for example, a typical question might require the student to find the "fluxion" or "fluent" of a simple algebraic expression in one variable, or to calculate the velocity with which a body must be thrown to "make it become a secondary planet to the earth."[54] The steps in the solution of these simple questions could have been rehearsed orally by a well-prepared student, but, once the practice of requiring written answers had been established, it was a straightforward matter to increase the quantity and complexity of such questions to a level that made the solutions virtually impossible to obtain without the aid of paper and pen. By the turn of the century, typical questions could require the student to tackle one of the following: to find the fluxions or fluents of three or more complicated expressions; to solve one or more cubic equations; to extract several cube roots; or to sum several complicated arithmetic and/or geometrical series. All of these exercises would have been impossible for the vast majority of students to tackle orally.[55] The rise of written examinations,

52. Ibid, 180–81. The *Principia* makes an interesting contrast with Euclid's *Elements* in this respect since, as Rouse Ball points out, students were seldom allowed to defend propositions from the latter on the grounds that they were too difficult to argue against.

53. Ibid., 174–78, 195. Compare the specimen disputation of 1784 and the written problem papers for 1785, both reproduced by Rouse Ball. The 1785 paper reveals that half the problems set drew either explicitly or implicitly upon Newton's mechanics and fluxional calculus.

54. Ibid., 195. Questions 3 and 8 from the paper of 1785. Finding the "fluxion" or "fluent" of an expression were equivalent respectively to differentiating or integrating the expression.

55. Ibid. See paper for 1802 (208–9). By 1802, the Senate House problem papers set to the most able students were at least as difficult, perhaps more so, as the Smith's Prize questions circa 1770

especially problem papers, during the last two decades of the eighteenth century therefore brought about a very significant change in the skills possessed by successful graduates. Instead of preparing to defend general principles verbally against standard objections, to recite short proofs and theorems from memory, and to solve simple problems orally, they were required to reproduce lengthy proofs on paper, and to learn how to apply them to technically complicated examples. It was this important shift from oral to paper-based examination that opened the way for a subsequent dramatic rise in the level of mixed-mathematical knowledge and problem solving skill demanded of the more able undergraduates.

Despite the gradual increase in the number and complexity of problems set from the 1770s to the early 1800s, their form remained stable and their content relatively elementary. As Rouse Ball (2W 1874) noted in the late 1880s, many of the "so-called problems" set at the beginning of the nineteenth century counted as little more than bookwork when compared with the problems set in his day (1889, 209). Why, then, did the form of problem questions remain relatively stable until roughly the end of the second decade of the nineteenth century but thereafter begin to become much more demanding? The simple answer to this question is that it was the analytical revolution circa 1820 that marked the beginning of a quarter of a century of rapid inflation in the conceptual and technical difficulty of Tripos problems, but the onset of this inflation needs to be understood in the context of a more subtle change that had been under way since the turn of the century. This change concerns the relationship between Tripos problems and the textbooks commonly used in undergraduate teaching. We saw in chapter 2 that until the end of the second decade of the nineteenth century, the teaching of undergraduates in all but the first few terms of residence remained substantially in the hands of college lecturers. Since the majority of these teachers had little interest in making original contributions to mixed-mathematical studies, they were heavily reliant, especially in teaching their more able pupils, on textbook expositions of standard topics and the accompanying worked examples.

The term *textbook* needs to be treated with considerable caution in this context, as most of the books from which advanced mixed-mathematical topics were studied from the 1770s until at least the 1830s were somewhat different from the textbooks familiar to the modern reader. Rather like the classical mathematical texts expounded by college lecturers—Euclid's *Elements* and Newton's *Principia*, for example—they made few concessions to pedagogy and were generally aimed as much at the experienced mathematician as at the student.[56] Even in the mid 1830s, as Augustus DeMorgan (4W 1827) pointed out (1835b, 336), there was no book (or small collection of books) that covered anything like a syllabus of undergraduate mathematical studies at Cambridge, so that any student who tried to study without the guidance of a tutor would end up with a very idiosyncratic view of mathematics. The word "textbook" never in fact appeared in the title of teaching texts in this period and was used mainly (and then only occasionally) to refer informally to whichever mathematical treatises happened to contain sections that approximated closely to courses typically given by college lecturers.[57] These books sometimes reproduced worked examples of some of the principles and propositions with which they dealt, but seldom gave systematic accounts of the range of problems that could be solved using specific techniques and certainly gave no problems for the students themselves to tackle as exercises.[58] The only exception to all but the last of these points were the treatises on the principles and applications of fluxions, but since books on other branches of mixed mathematics could not assume that the reader was familiar with the fluxional calculus these latter books generally presented their subject matter in a fairly nonmathematical form.[59] Moreover, even problems that required the use of the fluxional calculus tended to be straightforward reproductions of the worked examples given in the treatises.[60] The relative simplicity and uniformity of the problems set on the annual Tripos papers around 1800 can therefore be understood as a direct reflection of the kind of examples discussed in the mathematical treatises most commonly recommended by college lecturers.[61] These examples formed the basis of college teaching, informed students of the type of question likely to be encountered in examinations, and could be reproduced more or less from memory.

56. The expository material in the earlier sections of many "treatises" appears to have been intended more to lead the reader to the original contributions of the author than as a general introduction to the topic in question.

57. DeMorgan 1832, 277. Books such as Evans 1834 were "textbooks" in the traditional sense that they were based on manuscripts used by undergraduates and "printed with the view of saving to the student the time and trouble, which it has hither to been necessary to bestow in copying them" (Preface).

58. For an example of a book widely used in undergraduate studies, see Wood 1790–99.

59. These books were only partial exceptions, however, as they too tended to lead up to the original contributions or expositions of the author. See Guicciardini 1989, 125.

60. Ibid., 125–26. Late eighteenth-century textbooks on fluxional calculus intended specifically for Cambridge undergraduates were much more elementary than those published mid-century.

61. Rouse Ball 1889, 209; Guicciardini 1989, 125. Rouse Ball attributed the elementary level of questions circa 1800 to the "inferior and incomplete" nature of the current textbooks

During the first two decades of the nineteenth century a number of factors conspired slowly to alter the relationship between Tripos problems and textbook examples. We saw in chapter 2 that the rules restricting the use of private teaching were relaxed in 1807 and 1815, and that an extra day of mathematical examination was added to the Tripos in 1808. These developments, together with rising student numbers and increasing competition for high placing in the order of merit, meant that in order to tax the abilities of the most able candidates the examiners began to devise more novel variations on the standard problems. Over the first two decades of the nineteenth century, the Tripos problems gradually became a little more difficult, or, at least, more diverse, than the standard examples given in the textbooks.[62] This process altered the role of the examiners and accelerated the changes discussed above. The ability to set novel but neatly soluble problems now became an important part of the examiner's art, while the need to solve such problems quickly and accurately drove students to spend long hours working through past examination papers and to seek the help of private tutors to guide their efforts. Furthermore, it was the accumulating archive of locally manufactured problems drawn from the annual Senate House examination, rather than the standard examples in treatises, that gradually became exemplary of the breadth and level of performance expected of would-be wranglers.

A public manifestation of this very important shift was the commencement by the university of regular publications of complete collections of Tripos problems. The prime purpose of the first of these, which contained the problem-papers for the years 1801 to 1810 inclusively, appears to have been to counter criticism of the quality of undergraduate studies in the university.[63] Those contributing to the "din of obloquy" against Cambridge were invited to try solving the problem papers for themselves; but the collection was also intended to assist undergraduates in "acquiring a facility of investigation" that would eventually enable them to "command success" in the Senate House.[64] A second volume of past-papers, covering problems from the previous twenty years, appeared in 1820, the latter papers containing the

62. Compare the difficulty of the problems set in the Triposes of 1802 and 1819 reproduced in Rouse Ball 1889, 200, and Wright 1827, 2:63, respectively.

63. Anon. 1810; Williams 1991, 119.

64. Anon. 1810, xi. Wright considered himself extremely lucky as an undergraduate to have been lent a collection of past college examination papers by a professor. These papers were "not purchasable" and enabled him to get "direct by the nearest route, to academical distinction" (Wright 1827, 1:152).

first problems set by examiners such as Peacock and Gwatkin to promote the incipient analytical revolution.[65] The gradual establishment of the Tripos problem as exemplary of the kind of question that a would-be high wrangler had to be able to solve also altered the kind of mixed-mathematical knowledge prized in Cambridge. The growing archive of questions both defined an effective syllabus of mathematical studies and provided exercises through which students could hone their problem-solving skills.[66] Unlike textbook examples, Tripos problems were not worked exercises, but required the students to work out for themselves how to apply the physical principles and mathematical methods they had learned. This change naturally encouraged the more ambitious students to seek the assistance of private tutors. The best of these tutors had prepared manuscripts that not only explained the highly compacted proofs and problem solutions given by Newton in Book 1 of the *Principia,* but showed how these and other examination problems could be tackled using the techniques of the fluxional calculus. None of these manuscripts has survived, but it is a measure of their utility that when several young wranglers published technical exegeses of relevant sections of the *Principia* in the 1820s and 1830s, they found these manuscripts the most useful resource for explaining Newton's work.[67] The availability of these pedagogically useful manuscripts and books increased the technical ability of the undergraduates, a development which, in turn, encouraged the examiners to contrive ever more ingenious and difficult variations on the propositional problems given by Newton. By the 1830s, as we shall shortly see, it was the ability to solve highly original problems and to recapitulate the production of new mathematical knowledge (in the form of problems) that had become the most severe test of the top students. Before discussing this development we must examine the impact of the analytical revolution on the paper world of Cambridge mathematical studies.

3.5. Problems, Textbooks, and the Analytical Revolution

The problem-solving tradition of the early nineteenth century presented both potential obstacles and major opportunities to supporters of the analytical revolution. In addition to providing new textbooks and getting ques-

65. Anon. 1820. See, for example, 355–56 for questions on differential equations and D'Alembert's principle.

66. On examination questions as the best guide to the "staple of mathematics at Cambridge," see DeMorgan 1835b, 336.

67. See the prefatory comments in Carr 1821, Wright 1828, and Evans 1834.

tions involving analysis onto the Tripos papers, these men realized that it was expedient to present the new mathematics in a manner that could easily be incorporated within this tradition. It is noteworthy, for example, that the first book intended to introduce analytical methods to Cambridge, Robert Woodhouse's *Principles of Analytical Calculation* of 1803, was aimed at accomplished mathematicians rather than undergraduates and contained no examples of how these methods were applied to practical problems. These shortcomings would have severely limited the book's value to students and made it hard for tutors and examiners to formulate difficult but neatly soluble problems in analytical form.[68] When Babbage, Herschel, and Peacock translated Lacroix's *Elementary Differential Calculus* in 1816, they addressed this issue directly by assuring the reader that a "collection of examples and results" would shortly follow "indicating such steps in the processes as cannot be expected to be discovered by an ordinary student" (Peacock 1820, iv). In order to appreciate the full significance of this remark it is important to keep in mind that the inclusion of Continental mathematical methods in the Tripos papers of the late 1810s represented rather more than a mere switch of notation in the calculus. The examination papers set by Peacock and Gwatkin, for example, assumed that the students were familiar with a range of new analytical techniques including D'Alembert's principle and the theory of differential equations.[69] The reformers appreciated that numerous special techniques were required to apply these analytical methods to the solution of difficult problems and that such techniques were only learned through practice with graded examples. As Peacock astutely remarked in the preface to his book of worked examples in differential and integral calculus in 1820, many families of then standard problems had originally taxed the abilities of the greatest mathematicians of the eighteenth century and appeared straightforwardly soluble to contemporary readers only because the solutions had "been borrowed with little scruple or acknowledgement by succeeding authors" (1820, iv).[70]

The problem-solving approach to the new mathematics was further promulgated through the latter 1820s by the publication of William Whewell's *Treatise on Dynamics* in 1823. Whewell's book is especially relevant to our present concerns because in addition to providing a concise guide to dynamics in analytical form, it also discussed the introduction of the new

68. Subsequent analytical works by Woodhouse that did include examples were used extensively as a source of examination questions. See Becher 1980b, 393–94.

69. Wright 1827, 2:63–92.

70. Peacock had Leonard Euler especially in mind with this comment.

mathematics from an expressly pedagogical perspective.[71] Although Whewell is frequently, and correctly, depicted as a supporter of the analytical revolution in the 1820s, it is important to remember that, as we saw in the previous chapter, he envisioned a rather different role for analysis in Cambridge to that proposed by more radical reformers such as Babbage and Peacock. The latter, at least initially, saw analysis as a pure-mathematical discipline in which analytical techniques were studied in isolation from their physical applications.[72] Whewell's vision was more conservative in two respects: first, he introduced analytical methods as useful tools in the formulation and study of Newtonian dynamics; and, second, he presented this mixed-mathematical approach to analysis in the form of progressive propositions each followed by concrete examples. As far as Whewell was concerned, the abstract formulations of analytical mechanics due to analysts such as Lagrange had made dynamics a form of pure analysis from which such physically intuitive notions as force had all but disappeared. These formulations were not well suited to the mixed and problem-solving style of Cambridge mathematics and, in any case, were virtually impenetrable to most undergraduates and, no doubt, to many tutors. Rather than exploring the generality of such formulations, descending only occasionally to specific physical examples, Whewell proposed to start with physically intuitive propositions and then gradually to build via examples and problems to more generalized equations. The advantage of this approach, he argued in his *Treatise* (1823, vi), was that it made an analytical account of the "Newtonian System" accessible at some level to undergraduates of all abilities while providing an appropriately problem-based foundation for those brilliant students who would eventually go on to master the more abstract formulations.

One of the most striking aspects of Whewell's *Treatise,* especially when compared with other contemporary books intended to introduce Continental mathematical methods to Cambridge, is the several ways in which it was written explicitly for pedagogical purposes. First, the book was aimed specifically at undergraduates. That this was unusual at the time is highlighted by the fact that Whewell felt it necessary to acknowledge that his aim was to expound such books as Laplace's *Mécanique céleste* at "the level of the ordinary readers" and to warn any "mathematician" who consulted the book that he should expect to find little "except what he will consider as elementary."

71. Whewell had already published a treatise on analytical mechanics (1819), but this book was more elementary and less sophisticated from a pedagogical perspective.

72. Becher 1995. Peacock's *Examples* (1820) focused on problems in analytical geometry rather than physics.

Second, Whewell had taken some trouble to divide the material presented into progressive propositions which could easily be followed by students. By "breaking up the reasoning into distinct and short propositions," he assured the reader, the subject had been "rendered more easily accessible" (1823, v–vi, ix–x). Not only was it easier to follow a book that developed a topic proposition by proposition, but it was also clear to a student precisely at what point he failed to follow the argument. Third, Whewell thought it was the duty of the textbook writer not merely to reproduce the presentation of a Continental author or to offer an idiosyncratic compendium of analytical techniques and formulae, but to act as a reliable, informed, and selective guide to the most interesting and important results obtained by *all* authors to date. According to Whewell, students currently wasted a great deal of time "searching and selecting through a variety of books," engaging throughout in "unprofitable and unsystematic reading." The *Treatise on Dynamics,* he hoped, would "make the extent of our [mathematical] *encyclopedia* less inconvenient" (1823, iii). Fourth, Whewell made sure that each of the propositions he discussed was "elucidated by a considerable collection of mechanical problems, selected from the works of the best mathematicians, and arranged with their solutions under the different divisions of the science" (vi). His purpose was both to display the kinds of problem that could be tackled using analytical mechanics and to teach students the many "artifices which have been employed" (ix) in solving such problems. This emphasis on the use of analysis to solve difficult problems was calculated both to facilitate the inclusion of analytical methods within the Cambridge mixed-mathematical tradition and to make it easy for examiners to set, and for students to tackle, questions requiring analytical methods on the Senate House examination papers. Lastly, and following on from the previous point, Whewell thought it appropriate to use the problem-solving approach to analytical mechanics as a vehicle for teaching "the application and utility of some of those particular cases and branches of analysis, which might otherwise be considered as merely subjects of mathematical curiosity" (x).[73] In Whewell's book, analytical techniques would be taught not because they were fundamental to the study of pure analysis, but because they were useful mathematical methods for solving specific physical problems.

Whewell's presentation of analytical dynamics can be understood both as a political statement about the proper role of mathematics in undergrad-

73. For example, problems in which force varied "inversely with the distance" required students to "obtain the *definite integral* of exp $(-x^2)$," while the "motion of a complex pendulum" required a knowledge of "*simultaneous integration* of *n* differential equations."

uate studies and as an expedient for integrating analysis into the Cambridge mixed-mathematical tradition as smoothly as possible. It should also be read as an insightful commentary on the contemporary difficulty of teaching relatively advanced mathematics to large groups of mixed-ability students. Whewell was perhaps the first Cambridge author to write a book at this level specifically for undergraduates and to devote such attention to pedagogical issues. An ideal textbook presentation, according to Whewell, developed the student's mathematical knowledge systematically and progressively, presented relevant results obtained by a wide range of authors within the traditional divisions of mixed mathematics, illustrated every division with numerous worked examples, and introduced advanced analytical methods as and when they were required to solve specific problems. The various readings of Whewell's book referred to above were thus not only consistent, but mutually supportive. Whewell realized that a carefully designed textbook could be a powerful resource in his campaign to incorporate the power of analysis within the Cambridge mixed-mathematical and problem-solving traditions. For our present purposes the most important aspect of Whewell's book is that offered able undergraduates a concise account of the key elements of analytical dynamics, and made it easier for them to tackle difficult dynamical problems on the examination papers. Ambitious students no longer needed to struggle unguided through the original works of Continental authors, wrestling simultaneously with a wide range of disparate physical principles and analytical techniques. Drawing upon the "synthetical" style of such classical mathematical texts as Euclid's *Elements*, Whewell presented a carefully chosen selection of the works of such diverse authors as D'Alembert, Euler, Lagrange, Laplace, Maclaurin, Newton, Poisson, and Simpson, as a single, consistent discipline, developed in easy-to-follow propositions (1823, viii). As books of this kind recast the traditional branches of mixed mathematics in analytical form over the next two decades, examiners could exploit the analytical power of the calculus to devise a virtually endless supply of increasingly ingenious and technically demanding problems.[74]

Despite the evident utility of many of the new textbooks, not all were as accessible as Whewell's *Dynamics* and none (Whewell's included) contained worked examples of actual Tripos problems.[75] Both of these factors almost

74. For examples of other important texts introducing physical subjects in analytical form
Hamilton 1826, Airy 1826, Hymers 1830, and W. H. Miller 1831.

75. On the idiosyncratic style of many textbook writers and the resulting confusion on the part of the reader concerning the details and relative importance of various mathematical subjects, see DeMorgan 1835b, 336.

certainly contributed to the complete takeover of undergraduate teaching by private tutors during the latter 1820s and 1830s. There was an unwritten rule in the university that examination questions should not be set on new topics until they had been covered in a treatise suitable for use by Cambridge students.[76] In practice, however, many of the books that technically fulfilled this condition continued to be written, as DeMorgan had pointed out, in a style more suited to the needs of experienced mathematicians than to those of undergraduates.[77] It became a major responsibility of the private tutor to master and to teach the new topics covered in recent textbooks and to show how the techniques discussed were used in the solution of ever more intricate problems.

One contemporary indication both of the rising status of Tripos problems in the 1820s and of the role of private tutors in teaching students how to solve them was the publication in 1825 by J. M. F. Wright of the first collection of *solutions* to Cambridge mathematical problems.[78] College lecturers, teaching large, mixed-ability classes, had neither the time nor the inclination to drill the brightest students in the art of problem solving, and Wright, who, as we saw in chapter 2, made it his business to make the aids of the private tutor accessible to all students, believed that a volume of solutions to the published Tripos papers would enable poor but keen students to master problem solving on their own. As he advised in a second volume of *Hints and Answers* to questions set in college examinations, published in 1831, on those occasions when the student's "own patient efforts [had] entirely failed," a reference to his solutions would be "equally advantageous with the help of a tutor" (1831, iii). Like a private tutor, moreover, Wright's volumes not only provided model solutions but referred, where appropriate, to the sections of various textbooks in which the physical principles and mathematical methods in question were most clearly discussed. A bright student who found all or part of a solution difficult to follow was thus directed immediately to the best source of further instruction. Wright did not publish any more volumes of problem solutions after 1831, nor were his innovative publications initially emulated by members of the university.[79] This was probably because, by the 1830s, those ambitious students who could have

76. Rouse Ball 1889, 128.

77. Airy's *Mathematical Tracts* (1826), for example, did not discuss the solution of specific problems except in the case of the calculus of variations. See Whewell's comments on Airy's *Tracts* in Becher 1980a, 26, and DeMorgan's (1835a) review of Peacock's *Treatise on Algebra*.

78. Wright 1825. These solutions were intended to complement the volume of examination papers published by the university.

79. Wright 1831 was the last of his volumes of problem solutions.

made most use of such volumes were already working with private tutors. The content of the most difficult Tripos problems was in any case changing so rapidly at this time due to the influx of new analytical topics that past papers from all but the last couple of years would have been of little use to the most able students.[80]

The analytical revolution of the 1820s and 1830s witnessed a dramatic increase both in the level of mathematical knowledge required of the most able students and in the difficulty of the problems they were expected to solve. Where the final paper for the Tripos of 1819 contained twenty-four questions, some of which required no more than the calculation of logarithms, a description of how to measure refractive index, or an account of the principle and construction of an achromatic telescope, the final paper of 1845 contained only eight questions, none of which could be tackled without an advanced knowledge of one of either algebra, mechanics, integral equations, the wave theory of light, potential theory, the calculus of variations, differential geometry, or the mathematical theory of thermo-optics.[81] This enormous increase in technical competence was generated by the new paper-based pedagogical economy which had developed in Cambridge over the first third of the nineteenth century. In the vanguard of this training revolution were private tutors such as Hopkins and Hymers, but we should also note the important role played by new textbooks and the increasingly positive input being made by examiners. The more elementary textbooks by Lacroix, Whewell, and others enabled students quickly to master a wide range of basic analytical techniques that were generally applicable to most branches of mathematical physics. These books provided a progressive survey of useful mathematical methods taken from numerous far less accessible memoirs and treatises, and were designed to prepare students for higher studies in general, rather than the specific contributions of the author.

Remarking on the importance of the many excellent elementary mathematical textbooks employed in France at this time, DeMorgan observed that such books had "caused the road to be smoothed and levelled till the examiners are able to ask and obtain such a degree of mathematical acquirements from candidates of sixteen years of age, as would have made a prodigy a century ago" (1835b, 333).[82] This comment neatly captures the constructive

80. DeMorgan 1835b, 336; Becher 1980a, 19–23.

81. The Tripos examination papers for 1817 and 1845 are reproduced in Wright 1827, 2:87–92, and Bristed 1852, 2:426–28, respectively.

82. DeMorgan highlighted the pedagogical shortcomings of mathematical treatises by remarking that it "needs many treatises to make a good book" (1835b, 333).

power of a well-written textbook to organize, summarize, and communicate a previously disparate collection of mathematical methods and applications. Its analogical reference to road transport also highlights the pedagogical similarity between the teaching methods employed by good private tutors and the attributes of a good textbook. Both accelerated the learning process by selecting and explaining the most generally applicable techniques due to many authors, teaching in a well-ordered and progressive style (only assuming a knowledge of what has actually been covered), and giving numerous examples to build confidence and technical expertise. The Tripos examiners also made an increasingly positive contribution to this process in the 1810s and 1820s. Whereas the questions set in the latter eighteenth century were closely based on examples in well-known mathematical and natural-philosophical treatises, those set in the early decades of the nineteenth century offered ever more novel and ingenious variations on standard exercises. The publication of collections of these questions enabled students to hone their problem solving skills to a very high degree, especially when set and marked by private tutors as illustrative of techniques discussed in the new textbooks.

3.6. Cambridge Mathematics and "Tacit" Knowledge

Before pursuing the development of Tripos questions into the early Victorian period, it will be useful briefly to consider some of the ways in which, even by the early 1830s, the mathematical skills and competencies of Cambridge-trained undergraduates differed substantially from those possessed by students trained at another site. In order to make this comparison I draw upon an essay written by DeMorgan in 1835, in which, although ostensibly reviewing a recently published treatise on algebra, he wrote a wide-ranging commentary on the Cambridge system of mathematical training in general and the relationship between Cambridge textbooks and Tripos examination problems in particular. DeMorgan was one of the few people ideally placed to notice and to reflect upon the peculiarities of the Cambridge system. As an undergraduate at Trinity College in the mid 1820s he had first-hand experience of Cambridge pedagogy, while his subsequent career teaching mathematics at the recently established University College London had made him acutely aware of the difficulties of using Cambridge textbooks to teach non-Cambridge students. What is particularly striking about DeMorgan's remarks is the subtle relationship he detected between the Cambridge system of examination and the structure and content of the textbooks written by Cambridge authors.

DeMorgan began his essay with the interesting observation that books written *by* Cambridge authors *for* Cambridge undergraduates were virtually incomprehensible to students at other institutions (1835a, 299).[83] He attributed this strange phenomenon to two principal features of the Cambridge system of written examinations. The first was the setting of numerous *bookwork* questions. Conscious that they might be asked to write out any proof, theorem, or principle with which they were supposed to be familiar, Cambridge undergraduates committed all such material to memory. As a result, the authors of Cambridge textbooks troubled neither to recapitulate material covered in more elementary books nor to reproduce steps in a derivation or example with which a Cambridge undergraduate should be familiar. Non-Cambridge readers, with a more tenuous grasp of elementary mathematics, found the arguments impossible to follow and had no means of knowing even what additional material they should refer to in order to made the arguments comprehensible.

The second characteristic of Cambridge examinations that made Cambridge textbooks opaque to non-Cambridge readers was their emphasis on problem solving. DeMorgan noted that Cambridge students were "supposed to acquire facility of application" such that they could solve "problems which are given by the examiners, and which are *not to be found in the books*." This remark underlines two very important aspects of Cambridge pedagogy to which I have already referred: first, that by the early 1830s it was the problems on examination papers, rather than exercises in textbooks, that defined the standard to which ambitious students aspired; and, second, that solving such problems was a highly skilled activity only acquired through extended practice. Given that a Cambridge undergraduate was supposed to begin cultivating these skills as soon as possible, DeMorgan concluded, it was "even desirable" for him "that the elementary works should not lead him all the way, but should indicate points between which he may be expected to travel for himself" (1835a, 299). Another difference between Cambridge and non-Cambridge students, therefore, was that the former were not only expected to find their way through the merest sketch of an example, but were taught to regard such exercises as useful preparation for tackling difficult problems in examinations. Conversely, regular practice at

83. The article was published anonymously but DeMorgan 1882, 402, reveals him as the author.

solving past examination problems prepared Cambridge students to deal with sketchy examples in textbooks. DeMorgan did not point out, perhaps because he was unaware of the fact, that it was the job of the private tutor to provide the intermediate steps when necessary and that these tutors could also supply model answers to past examination questions if one of their pupils was stuck.[84] What he did note with some insight was that the many steps in Cambridge books left as exercises to the reader fostered in Cambridge students a singular attitude towards mathematical problems. When a Cambridge student tackled a textbook exercise or past examination problem he "[did] not permit himself to believe in impossible difficulties; but consider[ed], or should consider, that he *must* conquer whatever any one who knows the public examinations thinks it expedient to write" (1835a, 299–300).

These fascinating and unique comments on Cambridge mathematical training take us to the heart of much of what can be labeled "tacit" in Cambridge pedagogy by the early 1830s. First, the requirement that ambitious students be able to *write out* on demand virtually all the formal proofs, theorems, and definitions they were supposed to know gave them an encyclopedic mathematical knowledge unmatched by students at other British institutions. Second, the emphasis on problem solving required the students to develop a rare facility for applying physical principles and mathematical methods to the branches of physical science studied in Cambridge. But, since the many examination questions through which this facility was developed were not reproduced in the textbooks, they were unknown to non-Cambridge students. It is also worth noting in this context that the inability of non-Cambridge students to solve Tripos problems did not derive simply from their lack of access to examples. As we shall see in chapter 5, even when Tripos questions became widely available in popular textbooks, they remained insoluble to non-Cambridge readers because the techniques by which they were tackled were known only to Cambridge tutors.[85] Lastly, and closely connected with the previous point, Cambridge students approached any problem set by a Cambridge examiner in the certain belief that they *could* and *must* solve it using the mixed-mathematical methods in which

84. DeMorgan might have been unaware of rapid expansion in private tutoring that had taken place since his undergraduate days in the mid 1820s.

85. Tripos problems were, strictly speaking, available in published form from 1810, as were Wright's solutions for the years 1801–20. However, these volumes seem to have been little known or used beyond Cambridge.

they had been trained. Students accordingly devoted a great deal of time and effort to trying to solve difficult problems, and readily attributed any failure to do so to their own insufficiencies.

These attributes of Cambridge mathematical education are appropriately labeled "tacit," not because they were inescapably invisible or intangible, but because they had come to be taken for granted within the university, and made the exercises and problems by which undergraduates were trained and tested extremely difficult for students at University College to understand. Furthermore, I have made these attributes visible by pointing to the pedagogical incommensurability that existed between these two geographically separated places of mathematical training. It was DeMorgan's comparative perspective that enabled him to identify important and site-specific characteristics of the Cambridge system, characteristics of which Cambridge tutors were probably not themselves consciously aware (this being the reason that these attributes are appropriately labeled "tacit"). Comparisons of this kind are extremely useful because they highlight the crucially important connection between local regimes of training and the specific kinds of mathematical skill and competence that such regimes produce. If students at University College had been able, without the aid of the pedagogical resources specific to Cambridge, to solve difficult Tripos problems with the same speed and ease as their Cambridge peers, then it would be virtually impossible to establish a relationship between training and technical competence. But, as DeMorgan's remarks make clear, the style and content of Cambridge textbooks embodied the pedagogical ideals and practices of the university to such an extent that they were of little use at other sites.

Even more interesting, however, is DeMorgan's conclusion that the purpose of the emphasis on problem solving in Cambridge was to foster mathematical *originality* in the best students. There was, he remarked, "certainly no place where original effort [was] so much the character of education" as at Cambridge, no undergraduate achieving marked success "until he became capable of original mental exercise" (1835a, 300). The hallmark of this originality was that a student could rapidly solve a large number of difficult problems, each of which was not exactly like (but not completely unlike) any problem he had previously encountered. An obvious implication of this aspect of Cambridge pedagogy is that tacit skills acquired in undergraduate training played a major role in shaping the kinds of original investigation in which graduates of the Mathematical Tripos subsequently engaged. Discussion of this important issue is taken up in chapter 5, in which I also explore the conditions under which the locally specific skills generated by a Cambridge training could be propagated to new and distant sites. In the balance of this chapter I return to the development of Tripos problems in the mid-Victorian period, paying special attention to the forging of a new relationship between Tripos problems, textbooks, and the research undertaken by Tripos examiners.

3.7. Research, Problems, and Senate House Solutions

An inextensible heavy chain
Lies on a smooth horizontal plane
An impulsive force is applied at A,
Required the initial motion of K.

JAMES CLERK MAXWELL, FEBRUARY 1854[86]

We have seen that during the first two decades of the nineteenth century, the growing archive of Tripos questions became both definitive of the range of topics undergraduates were required to master and constitutive of the practical exercises through which technical proficiency and problem-solving skills were acquired. Up to and including the early 1820s, these problems remained relatively elementary, generally being little more than variations on the standard examples given in the commonly used mathematical treatises. But, during the late 1820s and 1830s, the upward spiral of technical proficiency, driven by a combination of private teaching, intense competition, and the regular introduction of new and advanced analytical methods, gradually altered the job of the examiners. In order to stretch the abilities of the most able and well-prepared candidates, some examiners began to draw upon their own research as a source of advanced Tripos questions. Remarkable though this development may sound, it was in many respects a predictable outcome of the new pedagogical economy discussed above. We have seen that it was widely believed in Cambridge that the best way of teaching mathematics, including the new analytical methods, was through practical examples and problems, and, by the mid 1830s, some of the first generation of young college fellows to have been taught higher analysis this way were beginning both to undertake their own research and to be appointed as Tripos examiners.[87] It is not surprising therefore that these men sometimes

86. These opening lines of a "A Problem in Dynamics," penned by Maxwell shortly after completing the Tripos of 1854, enunciate a problem that is solved in rhyme in subsequent verses. The poem offers a rare and amusing insight into the way Maxwell tackled such problems. The poem is reproduced in Campbell and Garnett 1882, 625–28.

87. In the mid 1830s, the examiners were on average only six years from graduation.

approached research as a problem-solving exercise and could easily turn such exercises into Tripos problems.

The best-known case of a new theorem being announced in this way is that due to George Stokes, who set the derivation of what would become known as "Stokes's theorem" as a question in the Smith's Prize examination of 1854.[88] This example is sometimes seen as an amusing oddity but it is actually quite typical of the way Cambridge examiners tested the mettle of the most able students throughout the Victorian era. An early example of a Cambridge mathematician famous for his "unrivalled skill in the construction and solution of problems," especially those requiring the "application of complicated analysis," is Thomas Gaskin (2W 1831).[89] A private tutor and textbook writer through the 1830s and 1840s, Gaskin possessed such "extraordinary power" in constructing problems that he acted six times as a Tripos examiner.[90] As his obituarist pointed out, Gaskin not only based his questions on his own research, but made it "his custom to put any new theorem that he discovered in the form of a problem, rather than in that of a paper in a mathematical journal."[91] A Cambridge mathematician of the next generation who routinely announced new results in the form of Tripos problems was Joseph Wolstenholme (3W 1850). Despite the fact that he published a number of original mathematical papers in standard journals, Wolstenholme's fame as a mathematician rested chiefly upon the "wonderful series of mathematical problems" he composed as an examiner, many of which announced "important results, which in other places or at other times would not infrequently have been embodied in original papers."[92] Wolstenholme acted seven times as a Tripos examiner from the 1850s to the 1870s, and was so proficient at the "manufacture of problems" that he was able to pass on questions superfluous to his own needs to less able colleagues.[93]

As these remarks suggest, the most advanced Tripos problems were understood, at least in Cambridge, to embody new results every bit as important

88. Cross 1985, 144. Arthur Cayley (SW 1842) regularly published solutions to the (sometimes original) questions he set for the Smith's Prize. See *Quarterly Journal of Pure and Applied Mathematics* 8 (1867): 7–10; *Oxford, Cambridge and Dublin Messenger of Mathematics* 4 (1868): 201–26; 5 (1871): 40–64, 182–203; and *Messenger of Mathematics* 1 (1872): 37–47, 71–77, 89–95; 3 (1874) 165–83; 4 (1875): 6–8; 6 (1877): 175–82.

89. Routh 1889, ii.

90. Gaskin was a Tripos examiner in 1835, 1839, 1840, 1842, 1848, 1851.

91. Routh 1889, iii.

92. A. R. Forsyth (SW 1881) quoted in the *DNB* entry on Joseph Wolstenholme.

93. Wolstenholme 1878, v. Wolstenholme acted as a Tripos examiner in 1854, 1856, 1862, 1863, 1869, 1870, and 1874.

as those appearing in the standard research journals. Thus Edward Routh insisted that some of the most original problems set in mechanics were frequently "so good as to rank among the theorems of science rather than as among the examples," while Isaac Todhunter, defending the examination as a unique test of intellectual originality, claimed that the Tripos papers "abound in new results which are quite commensurate in importance and interest with the theorems previously established and studied."[94] By the late Victorian era, William Shaw (16W 1876) was prepared to go as far as to claim that, in the case of mathematical physics, the "original contributions to the subject were not papers in the Royal Society Proceedings or the mathematical journals but the questions set in the problem papers of the Tripos."[95] Shaw's claim is somewhat exaggerated, as many Cambridge mathematicians did publish important work in the standard mathematical journals, but there is one sense in which his comment might be taken literally. The announcement of a new result in the form of an examination question was an invitation to those who read the paper to try to prove the result (along the lines sketched by the examiner) for themselves. This was true not just for the students sitting the examination but for students from subsequent years and for senior members of the university who wished to master such problems either for teaching purposes or as a matter of interest or pride. One great advantage of this method of publication, therefore, was that it virtually obliged both the rising generation of wranglers as well as many established Cambridge mathematicians to explore the technical details of the examiners' original investigations.

Such explorations could also result in original questions becoming the inspiration for a research publication. If an examiner did not publish a solution to an interesting problem, solutions were sometimes proposed in such publications as the *Cambridge Mathematical Journal (CMJ)*. Established in 1837 by the founding editor, Duncan Gregory (5W 1837), and strongly supported in its early volumes by Archibald Smith (SW 1836) and Leslie Ellis (SW 1840), the *CMJ* regularly contained short articles proposing solutions to questions from past Tripos papers.[96] An interesting example of this kind is discussed by Routh (1898a, 80), who revealed that a special technique (the method of "infinitesimal impulses"), devised and taught by William

94. Routh 1891, v; Todhunter 1873, 7. Routh's textbooks reveal numerous new results first announced in the form of Tripos problems. See, for example, Routh 1892, 80, 141, and 1898a, 265, 361.

95. Shaw, "Twice Twenty and Four," Box 3, WNS-CUL.

96. On the establishment and early development of the *CMJ*, see Smith and Wise 1989, chaps. 5 and 6.

Hopkins for solving certain difficult questions in dynamics, had been made public by one of Hopkins's pupils in the form of a Tripos problem in 1853. Following the examination, three recently graduated wranglers, Arthur Cayley (SW 1842), P. G. Tait (SW 1852), and William Steele (2W 1852), each published an elegant solution to the problem. Routh himself on at least two occasions took Tripos problems, possibly of his own manufacture, as the starting point for research papers.[97] In these cases it appears that a line of investigation begun with an examination problem in mind turned out to lead to more general results or techniques that warranted publication in a journal.

On some occasions, examination questions could aid or inspire research on much more wide-ranging topics. William Thomson found inspiration of this kind while actually sitting the Tripos of 1845. In the late summer of 1844, just prior to the beginning of his final term's coaching, Thomson encountered a serious difficulty in attempting to establish the equilibrium conditions on which a mathematical theory of the distribution of electrostatic charge on a conductor could be built. In the Senate House, Thomson tackled a problem set by Harvey Goodwin (2W 1840) which "suggested some consideration about the equilibrium of particles acted on by forces varying inversely as the square of the distance." Developing these considerations in the days immediately following the examination, Thomson was able to establish the equilibrium condition he sought.[98] A question set in 1868 by Maxwell further illustrates the role of the Tripos papers as a medium for the local circulation of new results. Maxwell's problem concerned the dynamical interaction between waves traveling through the ether and the atoms of a material medium. One of the expressions to be derived by the candidates predicted that, under certain conditions, an "irregularity of refraction" would occur, a prediction that Maxwell retrospectively claimed as an explanation of the phenomenon of the anomalous dispersion of light.[99] In 1899, when Lord Rayleigh (SW 1865) edited a paper he had written in 1872 on the reflection and refraction of light for inclusion in his collected works, he stated that Maxwell's question "may have been in [his] mind when the text of this paper was written."[100] Rayleigh's vague and belated acknowledgement of his possible debt to Maxwell's problem suggests that he, like many other Cambridge mathematical physicists of the period, found it extremely difficult to keep track of

97. Marner 1994, 19–20.

98. For a technical discussion of these events, see Smith and Wise 1989, 213–16.

99. Harman 1990–95b, 11–12.

100. Rayleigh 1899–1920, 1:156. Maxwell drew attention to his question when refereeing a paper by Rayleigh in 1873.

the many results and techniques picked up from examination questions that subsequently found their way into his research. As we shall see in chapter 6, one of the problems Rayleigh himself set for the Tripos of 1876 began J. H. Poynting (3W 1876) on a line of investigation that eventually led to some very important new results in electromagnetic theory.[101]

Yet another important route by which recent research could become the basis of a Tripos problem was that an examiner could borrow a new technique or result from another Cambridge mathematician. An excellent example of this kind from the late 1850s, and one that beautifully illustrates just how intimate the relationship between undergraduate teaching, new mathematical results, and Tripos examining could become, was brought to light in the autobiographical reminiscences of James Wilson (SW 1859). A few days before the commencement of the Tripos examination of 1859, Wilson took an evening stroll to a Cambridge bookseller to read the papers. When he arrived, Wilson noticed the latest edition of the *Quarterly Journal of Pure and Applied Mathematics (Quarterly Journal)* lying on the counter. Flicking through the pages, he came across an article by the previous year's senior wrangler, George Slesser (SW 1858), on "the solution of certain problems in Rigid Dynamics by a new artifice, referring the data to movable axes." Wilson mastered the method of moving axes out of curiosity, but thought no more about it until a few days later when he was sitting an examination paper. He then had the "singular good fortune" to come upon an extremely difficult problem in rigid dynamics that was very similar to an example given in Slesser's paper and which could easily be solved using Slesser's method. Wilson applied the method to the problem and quickly "got the result stated in the paper."[102]

At first sight this seems, as it did to Wilson, a remarkable piece of happenstance, but a little research quickly reveals the predictable course of events by which Slesser's work ended up as a Tripos problem. Slesser was the first student coached to the senior wranglership by Edward Routh (SW 1854), and, after graduation, Slesser began research on the theory of moving axes, a subject he had been taught by Routh. We know that the method had originally been shown to Slesser by Routh (whose career is discussed in chapter 5), as the latter seems to have been annoyed that one of his pupils had developed and published a technique apparently shown to him confidentially

101. Rayleigh considered his Tripos problems so original that he published them; Rayleigh 1899–1920, 1:280–86.

102. Wilson and Wilson 1932, 46; Slesser 1858. George Slesser became Professor of Mathematics at Queen's College, Belfast, in 1860 but died in 1862 aged just 28.

as a useful device for solving a certain class of Tripos problems. Routh had himself published in the *Quarterly Journal* on the subject of moving axes and when he explained the technique in a textbook some years later he noted, rather pointedly, that the general equations had first been published by Slesser to whom "two special cases . . . had been previously shown by the author, together with their application to the motion of spheres."[103] Slesser, it seems, had abused his privileged access to Routh's special techniques—perhaps because he had not discussed his intended research with his old coach—but the episode nevertheless highlights the way coaches could, albeit sometimes unwittingly, shape the early research interests of their pupils. The path from Slesser's training to his first publication was thus extremely smooth, and the further step by which his research found its way onto an examination paper was equally direct. One of the editors of the *Quarterly Journal*, Norman Ferrers (SW 1851), was also an examiner for the Tripos of 1859. Having received and accepted Slesser's paper for publication, Ferrers turned its central technique and one of its special cases into a difficult Tripos problem.[104]

I shall return in chapter 5 to the role of examination questions as models of original mathematical research, but for the moment it is sufficient to note that it was the increasing diversity, novelty, and difficulty of the problems set through the 1830s that prompted the first widespread publication of solutions to Tripos problems. We have seen that Wright's attempts to use problem solutions for pedagogical purposes in 1825 did not catch on during the latter 1820s and 1830s, but, by the early 1840s, two new factors greatly increased the demand for the publication of solutions in some form. First, by this time all but the most elementary examination problems had ceased to be obvious variations on the examples routinely reproduced in textbooks. Any undergraduate hoping to obtain a respectable honors degree had to be able to tackle a wide range of problems case by case, a skill that could only be learned through practice with numerous examples. Those students who were fortunate enough to study with one of the leading private tutors were almost certainly supplied with answers and model solutions to selected Tripos problems, but even these tutors could not guarantee that the solutions they provided followed the mathematical principles or precise line of argument envisaged by the examiners. There was thus a growing market in Cambridge through the latter 1830s for books containing answers and model solutions to past Tripos problems, especially if the solutions were supplied by the examiners themselves. The second difficulty posed by the absence of any formal mechanism for the publication of Tripos solutions concerned the examiners' recently acquired habit of introducing new analytical methods, and even completely novel mathematical results, in the form of Tripos problems. This practice made the more advanced Tripos papers comparable in some respects to research articles or sections of a mathematical treatise. As Edward Routh subsequently pointed out, the more difficult Tripos problems set in mechanics each year provided an excellent guide to "recent directions in dynamical thought" in the university (1898a, vi). Yet apart from occasional articles in research journals, those who studied the Tripos papers prior to the mid 1840s had little or no indication of the techniques of proof or solution envisaged by the examiners.

103. Routh 1892, 4, 162. Routh observed of Slesser's original contribution that he "uses moving axes, and his analysis is almost exactly the same as that which the author independently adopted."

104. Compare Slesser 1858 and *Cambridge Examination Papers*, Mathematical Tripos, Friday (morning) 21 January 1859, question 5. Ferrers was a Tripos examiner eleven times from 1855 to 1878.

The first steps toward the resolution of these difficulties were taken in the latter 1830s, when new textbooks began to contain increasingly large numbers of worked examples on each topic. This was clearly an attempt to help students master the many applications of the mathematical methods discussed, but it is important to note that even these books did not reproduce actual Tripos problems, and rarely provided additional, unsolved problems (with answers) for students to tackle for themselves.[105] It was only in the early 1840s that the enormous archive of past examination questions began to be exploited to provide numerous exercises for aspiring wranglers. The first example of this practice occurred in 1841 when John Coombe (4W 1840) published a complete collection of solutions to the Tripos problems of 1840 and 1841. This collection, written with the assistance of the examiners, was explicitly intended to provide exercises for high-flying students during their final term of Tripos preparation. With this aim in mind, Coombe offered solutions that illustrated "general principles applicable to a class of problems" so that students were prepared to tackle new variations on the problems they had mastered. Coombe also noted that he had deliberately provided solutions to problems from the most recent Tripos examinations because they offered the best guide to "the present character of the Senate House Examination" (Coombe 1841, iii–iv).

During the next decade the use of past Tripos problems for pedagogical purposes rapidly became commonplace. When William Walton (8W 1836)

105. Circa 1840 textbooks sometimes left the proof of a few propositions or results as exercises for the reader. See, for example, Gregory 1841.

published a new collection of student exercises in mechanics in 1842, it was considered a special promotional feature of his book that many of the problems had been "selected from the Cambridge Senate-House papers."[106] Two years later, in 1844, Matthew O'Brian (3W 1838) published one of the first Cambridge mathematical treatises (in this case on coordinate geometry) to contain "various Problems and Examples, many of which are taken from the Senate-House Problems . . . added at the end of each chapter" (O'Brian 1844, iv). That same year, 1844, O'Brian was a moderator in the Mathematical Tripos, and he and his co-moderator, Leslie Ellis (SW 1840), established another new tradition in the university by publishing a complete set of solutions to the problems they had set (Ellis and O'Brian 1844). In the introduction to their volume, O'Brian and Ellis drew attention to the fact that although the chief value of a mathematical problem was to illustrate a general mathematical principle or analytical process, the solutions to Tripos problems that had occasionally been published hitherto "did not indicate the point of view contemplated by the proposer." It was in order to provide a set of exemplars for future reference that the moderators had on this occasion been "induced to publish a collection of their own solutions" (1844, Preface).

Seven years later a complete collection of Tripos-problem solutions for the years 1848 through 1851 was published by Norman Ferrers (SW 1851) and J. Stuart Jackson (5W 1851) (Ferrers and Jackson 1851). It is unclear whether the examiners from these years played a significant role in the preparation of the volume, but this project has to be understood in the context of the reforms of 1848. As we saw in chapter 2, this was the year in which the examination was extended from six to eight days and divided into two parts. It was also the year in which the Board of Mathematical Studies was established, its first job being to prescribe a stable syllabus for the Mathematical Tripos. The volume by Ferrers and Jackson provided a definitive guide both to the kinds of problems examiners were henceforth likely to set and to the most appropriate forms of solution to such problems. As the authors of the solutions to the Senate House problems of 1860, Edward Routh and Henry Watson (2W 1850), noted, it was very important that problem solutions were provided by an authoritative source, preferably the "framer," so the that the student learned "the manner in which he is expected to proceed in the Senate House."[107]

106. Walton 1842, Preface. Walton's book also contained a small number of problems (with answers) for students to solve for themselves.

107. Routh and Watson 1860, Preface. Routh and Watson were the moderators for the examination of 1860.

Further evidence of the demand for definitive solutions to Tripos questions in the wake of the reforms of 1848 is provided by the publication of a book of solutions to "Senate-House 'Riders'" for the years 1848 to 1851 by Francis Jameson (6W 1850). In view of the often uncomprehending way in which less able students reproduced from memory the proofs and theorems required in bookwork questions, the Board of Mathematical Studies recommended in 1849 that every such question be accompanied in future by a "rider" which required the student to apply the bookwork to solve a short problem.[108] The rider carried around half the marks of the question and was probably intended also to reduce the effect of cheating among the less able candidates.[109] These questions were known from around 1850 as "bookwork-and-rider" questions, the problems that constituted the riders frequently being as ingenious and as difficult as many of the questions set on the easier problem papers. As Jameson sternly advised his undergraduate readers, no student should consider a proposition or piece of bookwork thoroughly mastered until he had "diligently practised examples connected with it, so as to be able, when called upon in an Examination, to apply it readily to any required purpose" (1851, viii).

It was, as we have seen, a mark of originality in Cambridge students that they could apply their mathematical knowledge to the rapid solution of novel examination problems. Indeed, by the early Victorian period it was this ability that had become the definitive product of the new paper-based pedagogy peculiar to the university. The closest we can now come to glimpsing in real time the skills of speed and application that an able student had to display is to study exemplary problem-solutions actually written by candidates in the Senate House. My comments on Poynting's answer to Rayleigh's question of 1976 (discussed in chapter 1) showed the sort of information that can be gleaned from examination scripts; here I analyze one more example in order to look more closely at the technical skills demanded by typical Tripos problems. The following solution by William Garnett (5W 1873) was written in answer to a difficult bookwork-and-rider question set by Maxwell for the Tripos of 1873.[110] Based on rigid dynamics (a topic at the heart of mixed

108. Jameson 1851, v.

109. It was fairly common until at least the mid 1850s for some of the least-able candidates to steal paper from the Senate House a few days before the examinations and to write out some propositions they thought likely to appear. These were then smuggled into the Senate House and "shown up" at the end of the examination. On this and other common methods of cheating, see Harrison 1994, 75–77.

110. This example is also chosen as it might equally have been set as an easy problem. Both Maxwell and Lord Rayleigh used old examination scripts as rough paper when drafting their sci-

mathematical studies), Maxwell's question concerned a wheel on an axle suspended by three strings, one wound round the wheel and two wound the opposite way round the axle on each side of the wheel (fig. 3.4).[111] The student was required to determine the relationship between the physical dimensions of wheel and axle such that when the string around the wheel was drawn up or let down, the tension in the other two strings remained the same. This part of the question, whose detailed form was unlikely to have been anticipated even by the best coach, tested the student's ability (i) to visualize the arrangement described, (ii) to analyze the kinematics and dynamics of the arrangement, (iii) to decide what kinematic and dynamical relationships satisfied the special conditions given, and (iv) to show that the conditions led to the required expression without making an error in algebraic manipulation.[112]

Garnett's attempt at this question is typical of those submitted by the top six wranglers. Maxwell allotted a total of 51/66 marks for the bookwork and rider respectively. Garnett scored full marks for the bookwork, crossing out only a couple of minor errors in two pages of neat, textbook-like prose. His attempt at the rider was slightly less successful. He made a three-dimensional sketch of the arrangement described by Maxwell but did not take sufficient care in defining the length of the axle. It is not clear in Garnett's sketch whether the length "a" of the axle includes the part running through the wheel (as Maxwell's result requires) or whether it is a measure only of the parts protruding on either side of the wheel (as Garnett's calculation of the total mass "M" assumes). Garnett correctly identified the forces keeping the arrangement in equilibrium, the accelerations generated by raising the string round the wheel, and the dynamical conditions that should lead to the required result. In setting up the equations from which the required expression could be derived, however, he made a second, more serious error (penultimate line on fig. 3.4) by associating the additional tension (τ) produced by pulling on the string around the wheel with the acceleration (f) of the string itself (rather than the acceleration of the center of

entific papers and books. Numerous answers to Tripos problems are to be found on the backs of their manuscripts.

111. *Cambridge University Examination Papers*, Mathematical Tripos, Thursday (morning) 2 January 1873, question v.

112. Maxwell's rhymed solution to the chain problem (see epigraph to this section) follows a similar sequence of operations. Having sketched the problem and labeled the key variables, he commences his analysis with the lines: "In working the problem the first thing of course is / To equate the impressed and effectual forces." He then writes down the dynamical and kinematic relationships, solves the resulting equations, and works out several special cases (Cambell and Garnett 1882, 625–28).

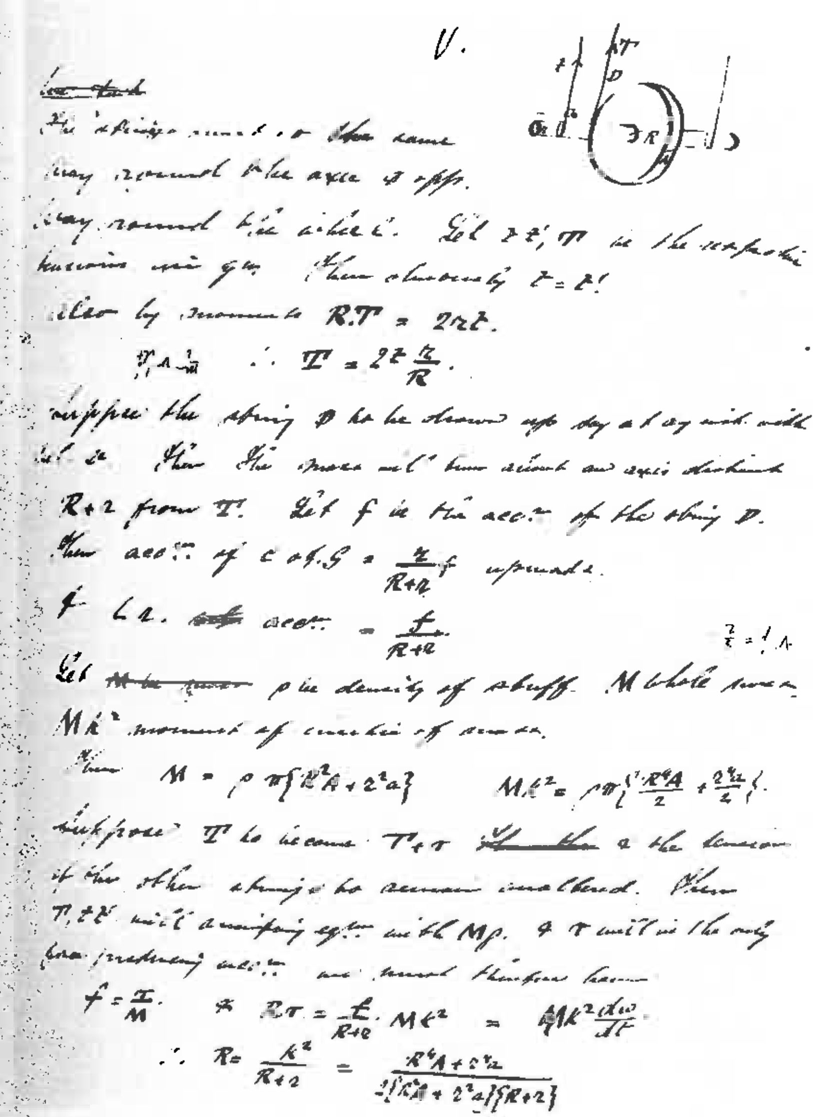

FIGURE 3.4. A mathematical world on paper. The first page of Garnett's attempt to solve Maxwell's problem. As with Poynting's attempt at Rayleigh's problem (fig. 1.3) Garnett's solution begins with a sketch of the mechanical arrangement and the labeling of the important variables. Garnett's work also displays his corrections and errors as he struggles in real time to obtain the required result. The two equations jotted upside down are rough calculations made by Maxwell at a later date. Box 5 Add 7655/vk/8, (JCM-CUL). (By permission of the Syndics of Cambridge University Library.)

mass of wheel and axle together).[113] This error, together with the incorrect definition of the length of the axle, meant that Garnett could not obtain the required expression. When this became clear after a few lines of algebra he noted that he had probably made "an error in estimating the angular acceleration of the mass" (which he had not) before moving on the next question. Maxwell deducted ten marks overall for Garnett's errors, leaving him with a total of 51/56 for bookwork and rider respectively.[114]

The questions set on the most difficult problem papers were far more complicated and mathematically demanding than the example just discussed, but scripts like Garnett's nevertheless provide a rare glimpse of a top wrangler deploying the skills that the new pedagogical apparatus was designed to produce.[115] The fact that Garnett attempted all nine questions on the paper and was unwilling to waste time trying to locate his errors suggests that he spent little more than ten minutes on the rider. This would have left him very little time to ponder or experiment with solutions; having launched upon a line of analysis, he was bound to see it through and then to move straight on to the next question. This example highlights that fact that the kind of originality required of mathematics undergraduates was largely defined by the form of the problems they were expected to solve. They were not required to invent and deploy new physical principles or mathematical methods, nor even to analyze novel or unfamiliar physical phenomena. They were required, rather, to show that they could understand the enunciation of a well-formulated problem, analyze the physical system described using the principles and techniques they had been taught, and use that analysis to generate specific mathematical expressions and relationships. It was, as we have already seen, this aspect of undergraduate training that enabled some recent graduates to publish solutions to the more interesting Tripos problems. Indeed, the ability to solve problems rapidly and elegantly was so well developed in some students that they could even outperform their examiners while working in the Senate House. The standard of excellence by which examination questions were marked was set by the proofs given in the best textbooks and the solutions to problems provided by the examiners. It sometimes happened, however, that an outstanding student offered one or more solutions that were judged better than those provided by the examiners. In these cases the student could score more than full marks for the question (and even for the whole paper) and have his original answers published.[116] If the examiners produced a set of solutions to their Senate House problems, they would generally use those student answers they considered superior to their own.[117]

113. Garnett has correctly calculated the acceleration [rf/R + r)] of the center of mass in the middle of the page.

114. vk/8/8 (xii), JCM-CUL. The bulk of the marks were probably deducted for the error in dynamics as the definition of the axle is somewhat ambiguous.

115. The top two wranglers scored full marks for the question, but below the seventh wrangler the marks tail off dramatically. Only seven of the 41 students who attempted the question scored more than half the marks, no one below the fourteenth wrangler scoring any marks at all for the rider (vk/8/8 [xii], JCM-CUL).

Collections of solutions to Senate House problems and riders were published at regular internals during the mid-Victorian period, generally appearing in the wake of alterations to the Tripos syllabus in order to show students and tutors how problems and difficult riders on new topics were likely to be framed and were properly solved.[118] From the mid 1840s, it also became increasingly commonplace for Cambridge textbook-writers to reproduce large numbers of past Tripos problems in illustration of virtually every branch of mixed mathematics. Thus Isaac Todhunter (SW 1848), one of the most influential Cambridge textbook writers of the mid-Victorian period, selected his examples "almost exclusively from the College and University Examination papers," sometimes reproducing more than seventy problems at the end of a chapter.[119] Some tutors and examiners produced books of selected Tripos and college examination problems (with or without solutions), collected under the various divisions of the Tripos syllabus, for students to work through either by themselves or under the guidance of a private tutor.[120] And, as we shall see in chapter 5, these textbooks and books of problems helped to carry the techniques and ideals of Cambridge mathematics into schools and institutions of higher education throughout Britain and her Empire. The use of the examination papers as vehicles for the announcement of new mathematical methods or results also accelerated the ongoing change in relative status between Tripos problems and textbook examples. Where examination questions had once been mere variations on the standard problems discussed in mathematical treatises, they now became definitive of many of the techniques and examples discussed in textbooks. Look-

116. If the student scored more than full marks, he was said to have "beaten the paper" (Wright 1827, 1:290). Bristed reports that Robert Ellis beat a paper in the Tripos of 1840 (Bristed 1852, 1:238–39).

117. See, for example, Walton and Mackenzie 1854, Preface; and Greenhill 1876, vii.

118. In addition to those mentioned above, volumes of compete solutions to problems and riders were also published by Walton and Mackenzie (1854), Campion and Walton (1857), Walton et al. (1864), Greenhill (1876), and Glaisher (1879).

119. Todhunter 1852, v. Todhunter acted as a Tripos examiner in 1865 and 1866.

120. For examples of books of college and Tripos examination problems, see respectively Morgan 1858 and Wolstenholme 1867.

ing back in the late 1880s on the development of mathematics in early and mid-Victorian Cambridge, Edward Routh pointed out that the problems set in the Senate House had "generally [been] absorbed into the ordinary textbooks, and become the standard examples by which successive generations of students acquire[d] their analytical skill" (1889, iii). Referring to the papers of just one prolific examiner, Thomas Gaskin, Routh noted that it now seemed remarkable "how many of [Gaskin's] problems [had] been taken, and may be recognized as old friends" (1889, iii).

The inclusion of illustrative exercises and problems at the end of chapters in textbooks of mathematical physics is now so commonplace as to seem unexceptional, but it is important to appreciate that this pedagogical device is of relatively recent origin and was introduced in a specific historical context. The books from which students learned higher mathematics in Cambridge made no use of these devices prior to the 1840s, and, when they were introduced, they were taken from an extant and local tradition of competitive written examination. This tradition already enshrined the belief that mathematics was best learned through problems and examples, and ranked student performance by the ability to solve such problems under severe constraints of time. The introduction of Tripos problems to mathematical treatises was an important step in the invention of the modern technical textbook and one which assumed and embodied the ideal of the written examination as the natural means of testing student ability. The immediate outcome of this innovation was to raise the standard of technical performance in Cambridge still higher by making a wide selection of worked exemplars and student exercises far more readily available. In the longer term, the appropriation of examination questions as textbook exercises and end-of-chapter problem sets played an important role in shaping undergraduate notions of the nature of mixed mathematics and, as we shall see in chapter 5, of how it was properly advanced through research.

3.8. The Material Culture and Practice-Ladenness of Theory

Commenting more than twenty years ago on the rapid mathematization of physical theory which took place in Europe from the mid-eighteenth century, historian Enrico Bellone noted that "the 'paper world' of theory experienced a very lively growth between 1750 and 1900."[121] Few would disagree

121. Bellone 1980, 5. Bellone emphasizes that mathematical theories of the physical world generated an increasingly autonomous intellectual space over the latter eighteenth and nineteenth centuries.

with this statement, but Bellone, like other historians of mathematical physics following his lead, uses the expression "paper world" to refer to an abstract intellectual space rather than to a process of practical reasoning involving the material apparatus of pen, ink, and paper. In this chapter I have explored the historical development of the material culture of mathematical work by highlighting the importance of paper-based calculation not merely as a record of what goes on inside a theoretician's head, but as a skilled activity that developed coextensively and inextricably with technical innovations in mixed mathematics. It is by charting the gradual institutionalization of these skills in undergraduate studies that what might be termed the "practice-ladenness" of mathematical theorizing can most effectively be brought to light.[122]

Historians of seventeenth-century university education have rightly pointed out that one reason the new physico-mathematical sciences of that period did not initially flourish at postgraduate level in Oxford and Cambridge is that these were primarily undergraduate institutions whose collegiate structure and liberal educational ideals were conducive neither to effective communication between professors and undergraduates nor to the specialized study of technical subjects.[123] The vast majority of undergraduates were hopelessly ill equipped—as were their tutors—to deal with Newton's celestial mechanics or the fluxional calculus and had little incentive to master the technicalities of this new knowledge. They were therefore extremely unlikely to try to contribute to the development of these subjects after graduation. What is clear from this chapter is that even if there had been a widespread desire to foster the new subjects in undergraduate studies, such a change could not have been accomplished simply by altering the syllabus. As long as the bulk of formal teaching was based on reading, tutorial discussion, catechetical lectures, and public disputations, training students to a level of technical competence from which they might follow a professor's lectures on anything but elementary mathematics remained virtually impossible.[124] Those few undergraduates who did reach a level from which they could embark upon original mathematical investigations shortly after graduation acquired their learning mainly through private study aided by those senior members of the univer-

122. For a similar approach to the practice ladenness of mathematical computation, see Warwick 1995b.

123. See, for example, Feingold 1997, 447–48.

124. For examples of the increasing long and complicated, paper-based analytical calculations being undertaken by Continental analysts in the mid-eighteenth century, see Greenberg 1995.

sity who recognized and encouraged their exceptional enthusiasm and ability. The propagation of the advanced mathematical sciences therefore remained capricious in the early modern universities in this period in the sense that it relied almost entirely on the personal interests, abilities, and reading of individual students as well as the goodwill and expertise of a handful of tutors.

What made the skills acquired by these students more visible and accessible in the latter eighteenth century was the change in pedagogical practice that accompanied the introduction of written assessment. As success in the Senate House examination came to depend upon the reproduction of mathematical knowledge on paper, the majority of students began to adopt learning techniques that had once been the preserve of a tiny minority of enthusiasts and academicians. As we saw in chapter 2, the rise of competitive examination prompted large numbers of students to turn to private tutors who were prepared, for a fee, to pass on their own technical expertise through personal instruction, varied explanations, progressive teaching, expository manuscripts, and graded exercises. The gradual institutionalization of once informal and rare processes of technical education also opened the way to further pedagogical developments that would enable paper-based mathematical training to transcend the boundaries of its eighteenth century origins. As we have seen, the relative speed of the analytical revolution in Cambridge from the late 1810s to the early 1830s was driven by the combined effects of competitive examination, private teaching, new textbooks, and innovative problem setting by examiners; the breadth and level of technical competence achieved by the top wranglers toward the end of this period would have been unimaginable just two decades earlier.

The outcome of this pedagogical revolution is best summarized as a change in the *scale* of mathematical training at one specific site. The doubling of the number of students achieving honors in the Mathematical Tripos during the analytical revolution was accompanied by a sharp increase in the breadth and level of mathematical knowledge that had to be mastered in just ten terms. The size, competence, and eminence of the mathematical community generated by this system also attracted students with mathematical ability from all over Britain, a development that further heightened both the atmosphere of competitive study and the standard of technical performance. By the latter 1820s, Cambridge was fast becoming not just the most important center of mathematical study in Britain, but an almost obligatory passage point for British students who aspired to make original contributions

to mixed mathematics.[125] When a new journal was started at the beginning of the 1860s to encourage original publications from young mathematicians at the three centers of mathematical learning in Britain—Oxford, Cambridge and Dublin—it turned out to be unsatisfactory because, despite the best efforts of the editors, more than 90 percent of the contributions came from Cambridge.[126] The journal was refounded in the early 1870s with several new Cambridge-based editors and thereafter became an important vehicle for the early publications of many young wranglers.[127]

The fact that the Cambridge training system was problem oriented and paper based also shaped the skills possessed by Cambridge graduates. Long hours spent solving novel examination problems—which, from the 1840s, were embodied in a new tradition of textbooks—gave them a degree of originality in the application of mixed mathematics that was not possessed by students from other institutions. The inclusion of original results and new mathematical techniques in the problems set by some examiners also enabled the most able undergraduates to become familiar with the topics and methods being employed by at least some, especially Cambridge-trained, research mathematicians.[128] Most important to our current concerns, however, is that the development of the new pedagogical economy from a play of written questions and answers led to paper-based calculation becoming the common currency of private teaching, private study, and competitive assessment. The importance of the material culture of mathematical work lies not so much in the simple materiality of pen, ink, and paper, as in the many activities that are bound together by their use. Mastering mixed mathematics was not just a matter of learning the appropriate notation and writing out proofs and theorems until they could be reproduced from memory—though these activities

125. The few outstanding analysts whose careers appear to confute this claim actually support it on closer inspection. The great Irish mathematicians William Rowan Hamilton and James MacCullagh were bringing French analytical mathematics to Dublin in the 1820s, and the fact that they did not establish a major school was partly due to the lack of a training system comparable to that in Cambridge. The early studies of the Nottingham miller and analyst, George Green, were almost certainly supported by two Cambridge-trained mathematicians, John Toplis and Edward Bromhead. See Cannell 1993.

126. *The Oxford, Cambridge and Dublin Messenger of Mathematics* 1 (1862): 1.

127. *Messenger of Mathematics* 1 (1872): Preface; A. R. Forsyth 1929–30, x.

128. The emphasis on "physico-mathematics" remained very strong in Britain in the mid-nineteenth century. The subjects recommended by the editors of the *Oxford, Cambridge and Dublin Messenger of Mathematics* as especially worthy of investigation were the moon's motion, the propagation of sound, the wave theory of light, and the application of quaternions to physical problems.

were important—but of learning through the very act of repeated and closely supervised rehearsal on paper to manipulate mathematical symbols according to the operations of, say, algebra or the calculus, to apply general laws and principles to numerous physical systems, and to visualize, sketch, and analyze a wide range of geometrical and physical problems.

It is in the relationship between this complex pedagogical economy and the specific range of skills, competencies, and attitudes that it produced that what I referred to above as the practice-ladenness of mathematical theorizing is most clearly seen. William Garnett's solution to Maxwell's wheel-and-axle problem (fig. 3.4) provides a real-time example of a small fraction of these abilities in action. Readers with a training in mathematical physics might find these abilities too elementary to be of any real significance, but it is precisely their taken-for-granted status that I wish to emphasize. Histories of mathematical physics are generally concerned with novel and innovative contributions to the discipline, but without the shared skills and competencies generated by years of intense training, it would not be possible to sustain a large community of expert practitioners within which highly technical, innovative work, could be generated, assessed, disseminated, and advanced.

The above remarks also raise a number of important issues concerning collaborative activity within and between research communities, especially with respect to some of Thomas Kuhn's influential remarks on the role of "paradigms" and "normal science" in the development of mathematical physics. One of the most important insights of Kuhn's philosophy of science, at least for this study, is the recognition that a physicist's knowledge cannot adequately be described through the formal statement of the laws and principles he takes for granted and the mathematical methods he employs. Kuhn argued that a physicist's knowledge is intimately linked to innumerable other skills and competencies acquired not through miraculous insight into the essential meaning of, say, Newton's laws of motion, but through years of practice at solving canonical exemplars in each of the areas where Newton's laws had been successfully applied (Kuhn 1970, 46–47, 188–89). This admirably practice-oriented approach to mastering science nevertheless sits somewhat uneasily with Kuhn's more idealistic notion of a ubiquitous paradigm that can guide the development of a technical discipline as a normal-scientific activity. Kuhn was fond of referring to the exercises found at the end of chapters in mathematical textbooks to support his claim that physics students learned by practice rather than by precept, but he also acknowledged that such books did not become "popular" until the early nineteenth century (1970, 10). Prior to this, he suggested, it was Newton's

Principia that acted as a canonical text for the development of celestial and terrestrial mechanics. Those who read the *Principia* were supposed to have acquired, from the laws, proofs, theorems, and solved problems in the book, the specific tools and techniques necessary for the normal-scientific development of "Newtonian mechanics" during the eighteenth century.

The development of celestial and terrestrial mechanics over the eighteenth century has long stood as the classic example of normal scientific activity, a process that allegedly witnessed the "mopping up" of problems implicit in Newton's *Principia*. Yet several historians of the mathematical sciences have insisted that this is a mistaken view which derives from a simplistic understanding of the work in question and an artificial separation of the histories of mathematics and mechanics. Men like Euler and D'Alembert were not just formalizing and extending Newton's work in a more appropriate mathematical language, but were actively and simultaneously creating and applying new mathematical methods and mechanical principles. It is a mark of the novelty of their work that they often failed to follow each other's technical arguments and sometimes disagreed strongly about the legitimacy of each other's problem solutions.[129] In short, the range of mathematical methods and mechanical principles they invented and employed is far too disparate and diverse to be meaningfully understood to have originated in the examples given in the *Principia*.[130] While we might therefore acknowledge that the *Principia* was a paradigmatic text to the extent that it offered, in Kuhn's words, "a criterion for choosing problems that . . . can be assumed to have solutions" (1970, 37), it did not provide the very technical apparatus through which those solutions were eventually accomplished. What is missing from Kuhn's account of normal science is a historical explanation of the way a community or subcommunity of practitioners comes to agree on what can be taken for granted as the foundational principles and techniques to be deployed in the solution of problems. Kuhn's only comments on this crucial issue were a few vague references to the importance of textbooks, but the content and use of such books cannot be properly analyzed in isolation from the pedagogical regimes in which they were used.

129. Trusdell 1984, 98–101; Guicciardini 1989, 141; Greenberg 1995, 621. Greenberg suggests that eighteenth-century mechanics has served as the "paradigm of a 'paradigm.'"

130. See Greenberg 1995, 620–24, and Garber 1999, 35–53. The different variational principles used by Continental mathematicians to tackle mechanical problems were quite alien to the Cambridge understanding of Newton's mechanics and could barely be expressed in the notation of the fluxional calculus. See Rouse Ball 1889, 97, and Guicciardini 1989, 89–91, 131, 141. Guicciardini discusses the difficulties experienced by Edward Waring (SW 1757) in understanding Euler's works.

What began to bring a measure of collective order to the disparate works of many Continental analysts at the end of the eighteenth century was the new regime of teaching and examination instituted at the Ecole Polytechnique in Paris.[131] It is not my purpose here to discuss pedagogy at this major center, but there are many important similarities between the training regimes instituted circa 1800 at the Ecole Polytechnique and Cambridge respectively that can usefully be emphasized. Although the former was organized according to military rather than monastic discipline and brought the professors into regular and effective contact with the students, it shared with Cambridge strong emphases on the teaching of small classes of students of roughly equal ability and on the thorough mastery of mathematical technique by supervised rehearsal on paper.[132] Likewise when Franz Neumann was struggling in the mid 1830s to establish the first physics seminar in Prussia, he felt that his efforts were being undermined by, among other things, the lack of a paper-based system of training and examining for physics students. Eventually Neumann himself developed a common system of graded exercises that introduced students to a hierarchy of essential mathematical skills and techniques, and (in the absence of problems in textbooks) began to construct his own problem sets through which his students could learn their craft.[133]

The common thread linking these otherwise very different centers of education in mathematical physics is a new concern with training students in the minutiae of analytical technique and its application to the solution of exemplary physical problems. Such skills were a prerequisite not just to higher studies in mathematics and mathematical physics but also, when held collectively, to the kind of collaborative activity that Kuhn identified as normal science. On the model presented here, then, normal-scientific activity is the product not of a ubiquitous paradigm originating in a canonical text, but of specific and localized pedagogical regimes. It was the emergence of these regimes in the early nineteenth century that generated communities within which the rapid growth of the paper world of mathematical-physical theory could take place. It is also implicit in this model that different training regimes will produce students with different technical skills, interests, and priorities, but this does not imply that their respective mathematical knowledge will be radically incommensurate. There was certainly a good deal of overlap in the range of competencies inculcated at each of the institutions mentioned above, due in large measure to the common ancestry of many of the mathematical methods and problem solutions taught. To what extent the production and interpretation of new knowledge in mixed mathematics in late nineteenth- and early twentieth-century Cambridge was specifically tied to the pedagogical regime by which wranglers were trained is a question that is explored in the second half of this study.

131. Grattan-Guinness 1990, 1: chap. 2; Hodgkin 1981.

132. For a detailed contemporary account of the teaching methods used at the Ecole Polytechnique, see Barnard 1862, 59–87. Barnard actually remarked upon the similarity between the Polytechnique and a large Cambridge college.

133. Olesko 1991, 111, 113–14. Neumann remarked that without such training the study of physics was reliant on the vagaries of "inclination and talent." The German academic, V. A. Huber, who compared undergraduate studies in Cambridge with those at German universities in the mid 1830s, noted that while "English examinations" in classics were "somewhat superior to our own" those "in the Mathematics [were] altogether beyond us." He attributed much of he superiority of Cambridge training to the use of "private study" (as opposed to formal lectures) and to the "method of paper examination" (Huber 1843, 2:358–63).