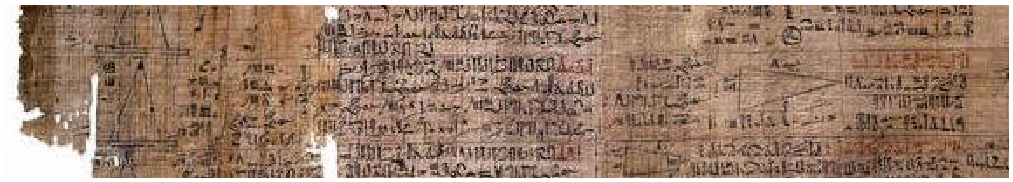

For a reference on Egyptian and Babylonian mathematics, here is a chapter by Victor Katz. (Linked from course website under Resources if you can't click it here.)

(A1) Use Egyptian algorithms to do the following arithmetic problems: 72×14, $126/14$, $100/12$, $5/8$.

Decompose the ratios $126 : 14$, $100 : 12$, and $5 : 8$ by Greek anthyphairesis. (Recall that *anthyphairesis* is a picture-based method of decomposing rectangles using squares. I'm just asking you to draw the pictures.)

Write continued fraction expressions for $126/14$, $100/12$, and $5/8$.

(A2) Because of the importance of doubling, and of unit fractions, Egyptian arithmetic often required expressions for twice a unit fraction, or $2/n$.

Write $2/103$ as the sum of two unequal unit fractions. Show all work.

Show that whenever n is a multiple of three, the vulgar fraction $2/n$ can be written as a sum of two unequal unit fractions, one of which is $1/2n$.

(B1) The "greedy algorithm" is the method that tries to reach a goal by taking the biggest available step at every stage. Prove that the greedy algorithm for Egyptian fractions always works! Here is a breakdown:

- Suppose that $1/n$ is the largest unit fraction less than or equal to a/b. Explain why
$$\frac{1}{n} \leq \frac{a}{b} < \frac{1}{n-1}.$$
- Find the difference $\frac{a}{b} - \frac{1}{n}$, and show that the numerator of the new fraction is strictly less than a.
- From this, explain how you can give an upper bound on the number of steps required for the Greedy Algorithm to terminate.

(B2) Using a calculator, show that the first few steps of the anthyphairesis pattern for the ratio $\sqrt{2} : 1$ match with $[1, 2, 2, 2, \ldots]$. Now figure out two whole numbers $a : b$ such that their anthyphairesis pattern would be *exactly* $[1, 2, 2, 2]$. What is the ratio a/b?