Throughput-Optimal Half-Duplex Cooperative Scheme with Partial Decode-Forward Relaying

Ahmad Abu Al Haija and Mai Vu,
Department of Electrical and Computer Engineering
McGill University
Montreal, QC H3A 2A7
Emails: ahmad.abualhaija@mail.mcgill.ca, mai.h.vu@mcgill.ca

***Abstract*—We study a cooperative communication system consisting of two users in half duplex mode communicating with one destination over additive white Gaussian noise (AWGN). Cooperation is performed between the two users by partial decode-forward relaying over 3 time slots with variable duration. During the first two slots, each user alternatively transmits and decodes while during the last time slot, both users cooperate to partially forward information to the destination. We establish the achievable rate region of this scheme. Then using the Lagrangian method, we analyze the optimal power allocations and the optimal time duration that maximize its throughput for the symmetric channel. Results show a significant improvement of the rates compared to the classical multiple access channel (MAC) when the inter-user channel quality is better than that between each user and the destination.**

I. Introduction

Cooperative communications have received substantial research during the last few years as cooperation provides an efficient way to increase throughput and reliability of communication systems. These characteristics allow cooperative communications to meet the high throughput requirements of image and video services offered by communication networks. Cooperative communication is possible in both wired and wireless networks.

One example of cooperative communication is multiple access with generalized feedback [1]. In [2], this scheme is applied to a cellular networks operating over fading channels which shows that the cooperation leads to higher achievable rate region and better outage probability. Another example is the relay channel introduced in [3]. The capacity region for the degraded relay channel is derived in [4] along with several schemes for the general case such as decode-forward and compress-forward. These schemes however assume full duplex operation. Only recently, attention has been paid to the more practice half duplex constraint. References [5], [6] studied performance of half duplex cooperative systems in terms of the outage and the error probability, respectively. Outer capacity bounds for half duplex cooperative relay and interference channels have also been analyzed in [7], [8], [9].

Furthermore, for the Gaussian and fading channels, power allocation is an important problem encountered by the practical designers. In [10], the optimal power allocation that minimizes the outage probability for multi-hop transmission over Rayleigh channels has been studied. The authors in [11] used the relationship between the mutual information of Gaussian channels and nonlinear minimum mean-square error (MMSE) to find the optimal power allocation that maximizes the mutual information of independent parallel Gaussian channels with arbitrary input distributions. The optimal power allocations that maximize the ergodic capacity for fading broadcast channels and minimize the outage capacity for the fading MAC have been studied in [12], [13], respectively.

In this paper, we propose a new half-duplex cooperative scheme between two users communicating with one destination. Each user has its own information to send to the destination. The transmission is carried out in independent blocks where each block is divided into 3 time slots. During the first time slot, the first user transmits its information to the second user and the destination. The second user decodes this information. Similarly, during the second time slot, the second user transmits and the first user decodes. Finally, both users cooperatively transmit to the destination during the third time slot. We establish the achievable rate region for this scheme by using rate splitting and superposition techniques [14] for encoding and joint maximum likelihood (ML) decoding [15] at the destination at the end of each block. We then analyze the optimal power allocation and time duration that maximize the sum rate using the Lagrangian method [16]. In this paper, we restrict our optimization to symmetric AWGN channels for simplicity, but the analysis can be extended to asymmetric and fading channels when the channel coefficients are known at the transmitter. The optimal power and time duration values allow the system designers to optimally implement this communication system in practice.

The remainder of this paper is organized as follows. Section II presents the channel model. Section III describes the cooperative scheme and provides the analysis of the achievable rate region. Section IV provides the analysis of the optimal power allocations for the sum rate. Section V, presents some numerical results and Section VI concludes the paper.

II. Channel Model

Figure 1 illustrates the channels model for our cooperative communication scheme. It consists of two users working in half duplex mode and one destination. Each link between these terminals is affected by a channel gain and AWGN. These two users have messages (w_1, w_2) to be sent to the destination and they wish to cooperate in order to increase their throughput. In this paper, we will consider the throughput as the sum rate from the two users to the destination. The discrete-time

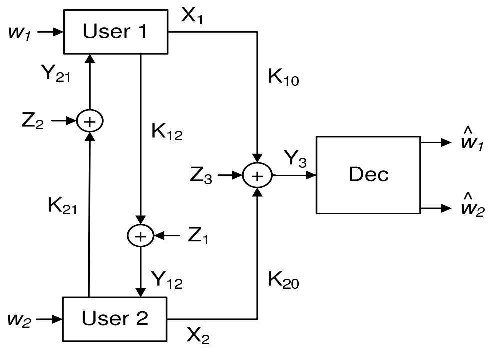

Fig. 1. The channel model for cooperative communication.

mathematical formulation of this model can be expressed as

$$\begin{aligned} Y_{12} &= K_{12}X_1 + Z_1 \\ Y_{21} &= K_{21}X_2 + Z_2 \\ Y_3 &= K_{10}X_1 + K_{20}X_2 + Z_3 \end{aligned} \tag{1}$$

where X_1 and X_2 are the transmitted signals from the first and the second user, respectively. Y_3, Y_{12}, and Y_{21} are the signals received by the destination, the second user, and the first user, respectively. K_{12}, and K_{21} are the inter-terminal channels coefficients. K_{10}, and K_{20} are the channels coefficients between each terminal and the destination. $Z_1 \sim N(0, N_1), Z_2 \sim N(0, N_2)$, and $Z_3 \sim N(0, N_0)$. Here the channel gain K_{ij} are assumed to be real value, but the analysis can be applied to complex-value channels as long as the channel phase is known and can be compensated for at either the receiver or the transmitter.

The half-duplex constraint is satisfied by the requirement that no two channels in (1) occur at the same time. Although TDMA satisfies this constraint, it leads to a rate region smaller than that of MAC even with power control [14] because of no cooperation. In this paper, we aim to cooperation to achieve a larger rate region than the classical MAC and also satisfy the half-duplex constraint.

III. Cooperative Scheme and Rate Region

A. Cooperative Scheme

To satisfy the half duplex constraint, the transmission is done in independent blocks of fixed length (consisting of n symbols), and each block is divided into three time slots. The lengths of the 1st, 2nd, and the 3rd time slots are α_1, α_2 and $(1-\alpha_1-\alpha_2)$, respectively. Let w_1 be the message of the first user to be sent during a specific block. The first user divides its message into three parts (w_{10}, w_{12}, w_{13}). Similarly, the second user divides its message w_2 into (w_{20}, w_{21}, w_{23}). During the 1st time slot, the first user sends (w_{10}, w_{12}) and the second user decodes both parts. Similarly, during the 2nd time slot, the second user sends (w_{20}, w_{21}) and the first user decodes them. Finally, during the 3rd time slot, the first user sends (w_{13}, w_{12}, w_{21}) and the second user sends (w_{23}, w_{21}, w_{12}) while the destination utilizes what it receives during all three time slots to decode (w_1, w_2) by joint ML decoding [15].

The encoding and decoding of our scheme can be explained with help from Table I. In this coding scheme, half-duplex applies because the user either transmits or receives during any time slot. Furthermore, partial decode-forward [17] (Lecture 17) applies since each user decodes what it receives from the other user in one of the first 2 time slots and then partially forwards the decoded information along with new information during the 3rd time slot.

The first user constructs its transmitted signals during the 1st and the 3rd time slots (X_{10}, X_{13}) as follows.

$$\begin{aligned} X_{10} &= \sqrt{P_{10}}\check{X}_{10}(w_{10}) + \sqrt{P_U}U(w_{12}) \\ X_{13} &= \sqrt{P_{13}}\check{X}_{13}(w_{13}) + \sqrt{c_2P_U}U(w_{12}) + \sqrt{c_3P_V}V(w_{21}) \end{aligned}$$

Similarly, the second user constructs its transmitted signals during the 2nd and the 3rd time slots (X_{20}, X_{23}) as

$$\begin{aligned} X_{20} &= \sqrt{P_{20}}\check{X}_{20}(w_{20}) + \sqrt{P_V}V(w_{21}) \\ X_{23} &= \sqrt{P_{13}}\check{X}_{23}(w_{23}) + \sqrt{d_2P_V}V(w_{21}) + \sqrt{d_3P_U}U(w_{12}) \end{aligned}$$

where $\check{X}_{10}, \check{X}_{20}, \check{X}_{13}, \check{X}_{23}, U,$ and V are independent and identically distributed according to $N(0, 1)$.

The power constraints for the two transmitters are

$$\begin{aligned} \alpha_1(P_{10} + P_U) + (1-\alpha_1-\alpha_2)(P_{13} + c_2P_U + c_3P_V) = P_1 \\ \alpha_2(P_{20} + P_V) + (1-\alpha_1-\alpha_2)(P_{23} + d_3P_U + d_2P_V) = P_2 \end{aligned}$$

where (c_2, c_3, d_2, d_3) are constants specifying the amount of power, relative to P_U and P_V, used to transmit the cooperative information (w_{12}, w_{21}) during the 3rd time slot. These power constraints ensure that the total energy each user spends in each block is constant.

From Figure 1 and Table I, the specific signaling for our scheme over the AWGN channel can be expressed as

$$\begin{aligned} Y_{12} &= K_{12}X_{10} + Z_1 \\ Y_{21} &= K_{21}X_{20} + Z_2 \\ Y_1 &= K_{10}X_{10} + Z_{31} \\ Y_2 &= K_{20}X_{20} + Z_{32} \\ Y_3 &= K_{10}X_{13} + K_{20}X_{23} + Z_{33} \end{aligned}$$

where Y_{12} is the signal received by the second user during the 1st time slot, and Y_{21} is the signal received by the first user during the 2nd time slot. Y_1, Y_2, and Y_3 are the signals received by the destination during the 1st, 2nd, and the 3rd time slots, respectively. $Z_{3i} \sim N(0, N_0),\ i = 1, 2, 3$.

B. Achievable Rate Region

Using the above encoding and decoding scheme, we can establish the achievable rate region of this scheme. Because of limited space, we omit the proof here, However, the proof for the general discrete-memoryless channel is available in [18]. The achievable rate region can be expressed as

$$\begin{aligned} R_1 &\le I_2 + I_5 \triangleq J_1 \\ R_2 &\le I_4 + I_6 \triangleq J_2 \\ R_1 + R_2 &\le I_7 + I_2 + I_4 \triangleq S_1 \\ R_1 + R_2 &\le I_2 + I_9 \triangleq S_2 \\ R_1 + R_2 &\le I_4 + I_8 \triangleq S_3 \\ R_1 + R_2 &\le I_{10} \triangleq S_4 \end{aligned} \tag{2}$$

	1st slot with length $\alpha_1 n$	2nd slot with length $\alpha_2 n$	3rd slot with length $(1-\alpha_1-\alpha_2)n$
First user Tx	$x_{10}^{\alpha_1 n}(w_{10}, w_{12})$	$--$	$x_{13,(\alpha_1+\alpha_2)n+1}^{n}(w_{13}, w_{12}, \tilde{w}_{21})$
Second user Tx	$--$	$x_{20}^{\alpha_2 n}(w_{20}, w_{12})$	$x_{23,(\alpha_1+\alpha_2)n+1}^{n}(w_{23}, \tilde{w}_{12}, w_{21})$
Y_{21}	$--$	$(\tilde{w}_{20}, \tilde{w}_{21})$	$--$
Y_{12}	$(\tilde{w}_{10}, \tilde{w}_{12})$	$--$	$--$
Y	Y_1	Y_2	Y_3
	$(\hat{w}_{12}, \hat{w}_{21}, \hat{w}_{10}, \hat{w}_{20}, \hat{w}_{13}, \hat{w}_{23})$		

Table I: The encoding and decoding schemes for half duplex cooperative scheme.

where

$$I_2 = \alpha_1 C\left(\frac{K_{12}^2\left(P_U + P_{10}\right)}{N_1}\right)$$
$$I_4 = \alpha_2 C\left(\frac{K_{21}^2\left(P_V + P_{20}\right)}{N_2}\right)$$
$$I_5 = (1-\alpha_1-\alpha_2) C\left(\frac{K_{10}^2 P_{13}}{N_0}\right)$$
$$I_6 = (1-\alpha_1-\alpha_2) C\left(\frac{K_{20}^2 P_{23}}{N_0}\right)$$
$$I_7 = (1-\alpha_1-\alpha_2) C\left(\frac{K_{10}^2 P_{13} + K_{20}^2 P_{23}}{N_0}\right)$$

and I_8, I_9, and I_{10} are given in (3).

IV. Optimum Power Allocation and Time Duration

In this section, we focus on solving for the optimal values of $(\alpha_1, \alpha_2, P_{10}, P_U, c_2, c_3, P_{20}, P_V, d_2, d_3)$ that give the maximum sum rate. To simplify the analysis in order to get closed-form expressions and insights to this optimization, we will consider a symmetric case $P_1 = P_2 = P$, $N_0 = N_{12} = N_{21} = N$, $K_{10} = K_{20}$, and $K_{12} = K_{21}$. The optimization, however, can be generalized to the asymmetric case. Because of our symmetric assumption, it can be easily noted that to maximize the sum rate, the power allocations and time durations for both users will be the same and hence $\alpha_1 = \alpha_2 = \alpha$, $P_{10} = P_{20}$, $P_{13} = P_{23}$, $P_U = P_V$, $c_2 = d_2$, and $c_3 = d_3$. As a consequence, S_2 and S_3 are equal. We start with special cases when $\alpha = 0$, or 0.5. Then we move to the case when $0 < \alpha < 0.5$, for which case the optimal values depend on whether $K_{12} > K_{10}$ or $K_{12} < K_{10}$.

From the coding of our scheme, it can be easily seen that when $\alpha = 0$ our scheme becomes the classical MAC with the same individual and sum rates. On the other hand, when $\alpha = 0.5$, our scheme goes to the classical TDMA with power control scheme given in [14].

For $0 < \alpha < 0.5$: We will use the Lagrangian method to find the optimal power allocations and time durations that maximize the sum rate. Therefore, to maximize the sum rate, we need to solve the following problem:

$$\begin{aligned} &\text{maxmin} \quad (S_1, S_2, S_4) \qquad (4)\\ &\quad \text{s.t.} \quad P = \alpha(P_{10} + P_U) + (1-2\alpha)\left(P_{10} + P_U(c_2 + c_3)\right) \end{aligned}$$

Again we consider two cases depending on the relative value of K_{12} compared to K_{10}.

A. For $K_{12} < K_{10}$

In this case, it can be shown that S_1 is the minimum among (S_1, S_2, S_4) as follows. Since S_1 and S_2 have I_2 as a common part, we just need to show that $I_9 > I_4 + I_7$ which, after some mathematical manipulations, can be expressed as

$$\log\left(1 + \frac{\left(K_{10}\sqrt{d_3 P_V} + K_{20}\sqrt{d_2 P_V}\right)^2}{N + K_{10}^2 P_{13} + K_{20}^2 P_{23}}\right)^{1-2\alpha} >$$
$$\log\left(1 + \frac{(K_{21}^2 - K_{20}^2)(P_{20} + P_V)}{N + K_{20}^2 (P_{20} + P_V)}\right)^{\alpha} \qquad (5)$$

If $K_{20} > K_{21}$, then (5) is correct because the right-hand-side expression is negative while the left-hand-side one is positive. Similarly, it can be shown that $S_1 < S_4$. Hence, we just need to maximize S_1 and by using the Lagrangian method, we get the following optimal values: $\alpha = 0$ and $P_{13} = P$, which lead to the following throughput:

$$S_1^{\star} = 0.5\log\left(1 + \frac{2K_{10}^2}{N} P\right) \qquad (6)$$

We can see that this sum rate is the same as that of the classical MAC.

B. For $K_{12} > K_{10}$

In this case, it is not obvious which expression among (S_1, S_2, S_4) is the minimum. However, for the maxmin problem in (4), the optimal power and time allocation will be either the optimal for one of these expressions alone, or at the intersection of two expressions. Thus, we first fix α and find the optimal power allocations for each expression subject to the power constraint. We also find the optimal power allocations for each expression subject to the power constraint and that it is equal to one other expression. Then, we will vary α to find the optimal value which maximizes the sum rate.

1) Maximizing each expression in (S_1, S_2, S_4) s.t. the power constraint:

Maximizing S_1: To maximize S_1 $s.t.$ power constraint, using the Lagrangian method, we can show that the optimal power allocations at a fixed α are

$$P_{10}^{\star} + P_U^{\star} = 2P + (1-2\alpha)\left(\frac{N}{K_{10}^2} - \frac{N}{K_{12}^2}\right)$$
$$P_{13}^{\star} = P - \alpha\left(\frac{N}{K_{10}^2} - \frac{N}{K_{12}^2}\right)$$

By substituting these optimum values in S_1, we get

$$S_1^{\star}(\alpha) = \alpha\log\left(\frac{K_{12}^2}{K_{10}^2}\right) + \frac{1}{2}\log\left(1 + \frac{2K_{10}^2}{N}P - 2\alpha\left(1 - \frac{K_{10}^2}{K_{12}^2}\right)\right)$$

$$I_8 = \alpha_1 C\left(\frac{K_{10}^2(P_{10}+P_U)}{N_0}\right) + (1-\alpha_1-\alpha_2)C\left(\frac{K_{10}^2(P_{13}+c_2P_U)+K_{20}^2(P_{23}+d_3P_U)+2K_{10}K_{20}\sqrt{(c_2P_U)(d_3P_U)}}{N_0}\right)$$

$$I_9 = \alpha_2 C\left(\frac{K_{20}^2(P_{20}+P_V)}{N_0}\right) + (1-\alpha_1-\alpha_2)C\left(\frac{K_{10}^2(P_{13}+c_3P_V)+K_{20}^2(P_{23}+d_2P_V)+2K_{10}K_{20}\sqrt{(d_2P_V)(c_3P_V)}}{N_0}\right)$$

$$\begin{aligned} I_{10} = &\alpha_1 C\left(\frac{K_{10}^2(P_{10}+P_U)}{N_0}\right) + \alpha_2 C\left(\frac{K_{20}^2(P_{20}+P_V)}{N_0}\right) + (1-\alpha_1-\alpha_2)\cdot \\ &C\left(\frac{K_{10}^2(P_{13}+c_2P_U+c_3P_V)+K_{20}^2(P_{23}+d_2P_V+d_3P_U)+2K_{10}K_{20}(\sqrt{(c_2P_U)(d_3P_U)}+\sqrt{(d_2P_V)(c_3P_V)})}{N_0}\right) \end{aligned} \quad (3)$$

Maximizing $S_2 = S_3$: Similarly, using the Lagrangian method to maximize S_2 leads to the following throughput:

$$\begin{aligned} S_2^\star(\alpha) =& \frac{\alpha}{2}\log\left(1+\frac{K_{12}^2}{N}P_z^\star\right) \\ &+\frac{\alpha}{2}\log\left(1+\frac{K_{10}^2}{N}P_z^\star\right)+\frac{1-2\alpha}{2}\log\left(1+\frac{2K_{10}^2}{N}P_x^\star\right) \end{aligned}$$

where $P_x^\star = (1-2\alpha)^{-1}(P-\alpha P_z^\star)$ and $P_z^\star = P_{10}^\star + P_U^\star$ is as given in (7).

Maximizing S_4: Finally, using the Lagrangian method to maximize S_4 leads to following optimal values:

$$\begin{aligned} P_{13}^\star =& 0;\ P_{U_1}^\star = \frac{1}{2c}\left(P+\frac{\alpha N}{2K_{10}^2}\right) \\ P_{10}^\star =& \left(2-\frac{1}{2c}\right)\left(P+\frac{\alpha N}{2K_{10}^2}\right)-\frac{N}{2K_{10}^2} \end{aligned}$$

where $c = c_2 = c_3$. Then, with these optimum power allocations and with any $c > 0$, S_4 can be expressed as

$$S_4^\star(\alpha) = \frac{1-2\alpha}{2} + 0.5\log\left(0.5+\alpha+\frac{2K_{10}^2P}{N}\right)$$

2) Maximizing each expression in (S_1, S_2, S_4) s.t. the power constraint and that it is equal to one other expression:

Maximizing S_1 s.t. $S_1 = S_2$: To maximize S_1 s.t. the power constraints given that $S_1 = S_2$, using Lagrangian method, we can get the optimal values at $c_2 = c_3 = c$ and

$$P_z^\star = P_{10}^\star + P_U^\star = 0.5\left(-B+\sqrt{B^2-4C}\right)$$

where

$$\begin{aligned} B =& \frac{N}{K_{12}^2}+\alpha\left(\frac{N}{K_{10}^2}-\frac{N}{K_{12}^2}\right)-2P \\ C =& \frac{1-2\alpha}{2}\frac{N}{K_{10}^2}\left(\frac{N}{K_{12}^2}-\frac{N}{K_{10}^2}\right)-P\left(\frac{N}{K_{10}^2}+\frac{N}{K_{12}^2}\right) \end{aligned}$$

$P_x^\star = P_{13}^\star + 2cP_U^\star$ is obtained from the power constraint as $\left((1-2\alpha)^{-1}(P-\alpha P_z^\star)\right)$. Finally, from the constraint $S_1 = S_2$, $P_{13}^\star$ is obtained as

$$P_{13}^\star = \frac{0.5N}{K_{10}^2}\left(2^{\left(\frac{2}{1-2\alpha}\right)F_1}-1\right)$$

with

$$\begin{aligned} F_1 =& \frac{\alpha}{2}\log\left(1+\frac{K_{10}^2}{N}P_z^\star\right)-\frac{\alpha}{2}\log\left(1+\frac{K_{12}^2}{N}P_z^\star\right) \\ &+\frac{1-2\alpha}{2}\log\left(1+\frac{2K_{10}^2}{N}P_x^\star\right) \end{aligned}$$

The above optimal values can be used as the optimal P_{13} if $P_{13}^\star > 0$. If $P_{13}^\star < 0$, we make it 0 and solve the problem again. Then, we have $P_x^\star = 2cP_U^\star = \left((1-2\alpha)^{-1}(P-\alpha P_z^\star)\right)$ and $P_z^\star$ is the solution of the following equation:

$$\log\left(\frac{N+K_{12}^2P_z^\star}{N+K_{10}^2P_z^\star}\right) = \frac{1-2\alpha}{\alpha}\log\left(1+\frac{2K_{10}^2}{N}P_x^\star\right)$$

Maximizing S_1 s.t. $S_1 = S_4$: Following the same steps in the previous optimization, we can obtain the optimal power allocations for S_1 s.t. the power constraint and $S_1 = S_4$. For a fixed α, we get $c_2 = c_3 = c$ and

$$P_z^\star = \frac{0.5}{1+\alpha}\left(-B+\sqrt{B^2-4(1+\alpha)C}\right)$$

where

$$\begin{aligned} B =& -\left(Q+0.25(1-2\alpha)G+3P+1.5\frac{N}{K_{10}^2}\right) \\ C =& 0.5(1-2\alpha)GQ+K\left(2P+\frac{N}{K_{10}^2}\right) \end{aligned}$$

where $G = 8cP_U^\star$ and $Q = \frac{N}{K_{10}^2}-\frac{N}{K_{12}^2}$. Finally, $P_{13}^\star = P_x^\star - 2cP_U^\star = P_x^\star - 0.25G$ is the solution of

$$P_x^\star - 0.25G = \frac{0.5N}{K_{10}^2}\left(2^{\left(\frac{2}{1-2\alpha}\right)F_2}-1\right)$$

with

$$\begin{aligned} F_2 =& \alpha\log\left(1+\frac{K_{10}^2}{N}P_z^\star\right)-\alpha\log\left(1+\frac{K_{12}^2}{N}P_z^\star\right) \\ &+\frac{1-2\alpha}{2}\log\left(1+\frac{K_{10}^2}{N}(2P_x^\star+0.5G)\right) \end{aligned}$$

Again, the above results is used if $P_{13}^\star > 0$. If $P_{13}^\star < 0$, we set it to 0 and solve again to get $P_z^\star$ from the solution of

$$\log\left(\frac{N+K_{12}^2P_z^\star}{N+K_{10}^2P_z^\star}\right) = \frac{1-2\alpha}{2\alpha}\log\left(1+\frac{4K_{10}^2}{N}P_x^\star\right)$$

where $P_x^\star = \left((1-2\alpha)^{-1}(P-\alpha P_z^\star)\right)$.

$$P_z^\star = P - (1-\alpha)\frac{N}{K_{12}^2} - \frac{N}{2K_{10}^2} + \sqrt{\frac{N^2}{2k_{10}^2K_{12}^2} + (1-\alpha)^2\frac{N^2}{K_{12}^4} + \frac{N^2}{K_{10}^4}(0.75-\alpha) + \frac{NP}{K_{12}^2}(2\alpha-1) + P^2} \quad (7)$$

Maximizing S_4 s.t. $S_4 = S_2$: Finally, using the Lagrangian method to maximize S_4 s.t. the power constraints and $S_4 = S_2$ leads to the optimal values of $c_2 = c_3 = c$ and

$$P_z^\star = 0.5\left(B + \sqrt{B^2 + 4C}\right)$$

where

$$B = 2P - \alpha\left(\frac{N}{K_{10}^2} + \frac{N}{K_{12}^2}\right)$$

$$C = P\left(\frac{N}{K_{10}^2} + \frac{N}{K_{12}^2}\right) + \frac{1-2\alpha}{2}\frac{N}{K_{10}^2}Q$$

$P_{13}^\star = P_x^\star - 2cP_U^\star$ is obtained from the solution of

$$P_{13}^\star = \frac{0.5N}{K_{10}^2}\left(2^{\left(\frac{2}{1-2\alpha}\right)F_3} - 1\right)$$

with

$$F_3 = \frac{\alpha}{2}\log\left(1 + \frac{K_{12}^2}{N}P_z^\star\right) - \frac{\alpha}{2}\log\left(1 + \frac{K_{10}^2}{N}P_z^\star\right) + \frac{1-2\alpha}{2}\log\left(1 + \frac{2K_{10}^2}{N}P_x^\star\right)$$

These optimum values are used if $P_{13} > 0$. If $P_{13} < 0$, set $P_{13} = 0$ and solve again to get $P_z^\star$ from the solution of

$$\log\left(\frac{N + K_{12}^2P_z^\star}{N + K_{10}^2P_z^\star}\right) = \frac{1-2\alpha}{\alpha}\log\left(\frac{N + 4K_{10}^2P_x^\star}{N + 2K_{10}^2P_x^\star}\right)$$

where $P_x^\star = (1-2\alpha)^{-1}(P - \alpha P_z^\star)$.

The above 6 optimizations give six optimal expressions at a fixed alpha. We can then find the optimum value $\alpha^\star$ that provides the largest throughput by plotting these optimal expressions versus α. However, since the optimal point must be for the same power allocations for all expressions, we also need to plot each expression in (S_1, S_2, S_4) at the optimal powers obtained from the optimizations of the other expressions. For example, we need to plot S_1 at the optimal powers of $(\max S_2$ s.t. $S_2 = S_4)$, S_2 at the optimal powers of $(\max S_1$ s.t. $S_1 = S_4)$ and S_4 at the optimal powers of $(\max S_1$ s.t. $S_1 = S_2)$. Then for each group of lines at the same power allocation, identify either its intersection or, in case the lines do not intersect, the optimal point on the lowest line. Among the 6 obtained points (corresponding to the 6 optimizations), the optimal $\alpha^\star$ is the point that gives the largest sum rate.

C. Asymptotic analysis

From (2), we can see that as $K_{12} \to \infty$, S_4 becomes the minimum and $S_4^\star(\alpha)$ is maximized as $\alpha \to 0$. Therefore, compared with the classical MAC, the throughput gain, $G = S_4^\star - S_{MAC}$, approaches

$$G \to 0.5\log\left(1 + \frac{2K_{10}^2P}{N + 2K_{10}^2P}\right) \quad (8)$$

as $K_{12} \to \infty$. Furthermore, we can see that as $P \to \infty$, then $G \to 0.5$.

V. Numerical Results

Figure 2 illustrates the achievable rate region for the proposed scheme compared with the classical MAC. The results are obtained for the symmetric case by using (3) with the following values: $P = 2$, $K_{10} = 1$, $N = 1$, and for different values of K_{12}. This figure shows that cooperation leads to a larger rate region even with half duplex constraint.

Figure 3 shows the optimized sum rates given Section IV. The figure shows no non-trivial intersections among the lines with the same power allocations (plotted as pairs of lines in the same color). However, we can also see that ($S_1^{\max}$ s.t. $S_1 = S_4$) is lower than S_2 at $(\max S_1$ s.t. $S_1 = S_4)$ but is above the other two dashed lines. Furthermore, the maximum of the line ($S_1^{\max}$ s.t. $S_1 = S_4$) is below the lines $S_1^{\max}, S_2^{\max}, S_4^{\max}$. Hence, for this value of $K_{12} = 3$, the optimal sum rate corresponds to the maximum of the line ($S_1^{\max}$ s.t. $S_1 = S_4$) which is at $\alpha = 0.25$. By varying K_{12} and P, we can find the optimum value of α for each case.

In Figure 4, we plot the optimum value of α versus K_{12} for different transmit powers. From this figure, we can see that the optimal value of α decreases as the inter-user channel gain improves. This can be explained as follows. As the channel quality between the two users improves, they can exchange their information in a smaller portion of time and spend a bigger portion to cooperatively transmit their information to the destination. Moreover, as $K_{12} \to \infty$, $\alpha \to 0$ (but not equal to 0), the two users only need an infinitesimally small portion of time to exchange their information and then can fully cooperate during the rest of the time.

Figure 5 illustrates the improvement in throughput of the proposed cooperative scheme compared with the classical MAC. Once K_{12} is bigger than K_{10}, our scheme starts to outperform the classical MAC. As K_{12} increases, the throughput gain increases. This figure suggests that the absolute throughput gain depends little on the transmit power and is more sensitive to the value of K_{12}. Even at a small value of K_{12} (for example at 4 times the direct link strength), this throughput gain is noticeable. As given in (8), the gain eventually saturates to 0.5 bits/sec/Hz.

VI. Conclusion

We proposed a novel half-duplex cooperative scheme with partial decode-forward relaying and variable time duration. We then analyze the optimal power allocations and time durations that maximize the sum rate of the symmetric case. The optimization provides closed form expressions for the maximum sum rate. The results showed that if the inter-user channel quality is worse than that between each user and the destination, the performance of our scheme is the same as that of the classical MAC. On the other hand, as the inter-user channel quality increases, our scheme outperforms the classical MAC. It gives a higher throughput and a larger rate region. This result is encouraging for half-duplex cooperative

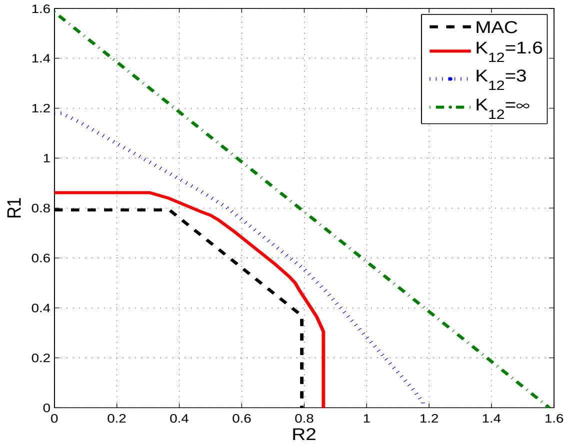

Fig. 2. Achievable rate region for half duplex cooperative scheme compared with classical MAC ($K_{10} = 1, N = 1, P = 2$) and different values of K_{12}.

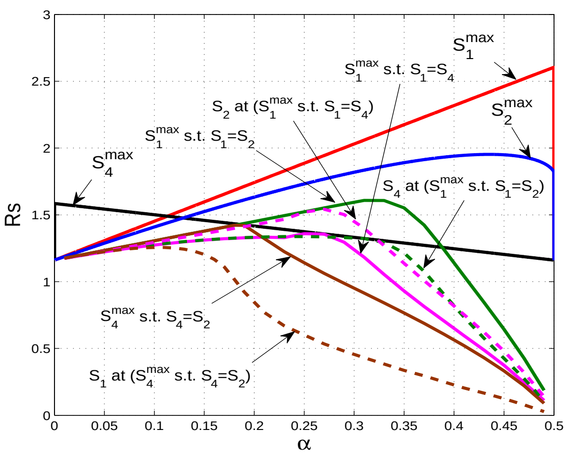

Fig. 3. Optimum $R_1 + R_2$ versus α for ($K_{10} = 1, P = 2$, and $K_{12} = 3$).

communication and calls for further analysis for the fading case.

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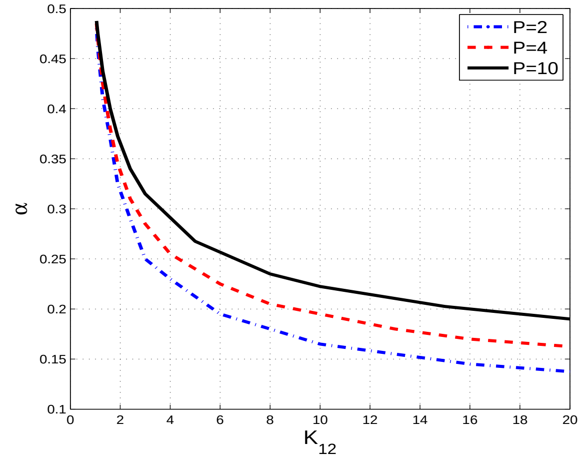

Fig. 4. Optimum α versus K_{12} for ($K_{10} = 1, N = 1$).

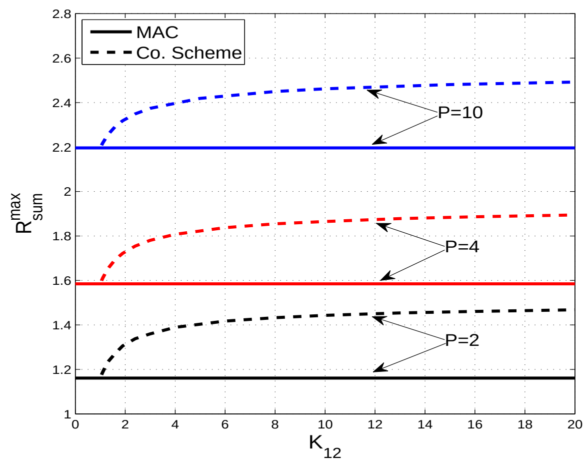

Fig. 5. Maximum sum rate versus K_{12} for MAC and our cooperative scheme with $K_{10} = 1, N = 1$ and different values of P.

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