Outage Performance of Uplink User-Assisted Relaying in 5G Cellular Networks

Hussain Elkotby and Mai Vu
Department of Electrical and Computer Engineering, Tufts University, Medford, MA, USA
Emails: Hussain.Elkotby@Tufts.edu, Mai.Vu@Tufts.edu

***Abstract*—We use stochastic geometry to analyze the performance of a user-assisted decode-and-forward (DF) relaying scheme where an active user relays data through another idle user in uplink cellular communication. We propose a new geometric policy based on the random selection of an idle user within a certain area mid-way between the active user and base station, and compare this policy to the common nearest neighbor geometric policy. These probabilities are further used in the analytical derivation of the moments of inter-cell interference power caused by system-wide deployment of this user-assisted DF relaying. We then numerically evaluate the outage probability performance and show that user-assisted relaying can significantly improve reliability for active users near the cell edge. We also show that the proposed mid-way policy significantly improves outage performance over the nearest neighbor policy even when the nearest neighbor cooperation probability is higher. This result suggests that mid-way relay selection is a highly effective cooperation policy.**

I. Introduction

Mobile operators driven by the increasing number of subscribers every year and customer demand for new and better services place pressing requirements on the underlying wireless technologies. Those requirements push the cellular networks to become more heterogeneous in architecture and denser in deployment. Additionally, cellular networks can leverage the high density of cellular devices induced by the increase in the number of subscribers to enable D2D communications as a mode for relaying through other users. This mode of cooperation is expected to improve the network data transmission rates and reliability.

User-assisted relaying through D2D communications is expected to play an important role in future generations wireless networks. D2D communications enable two proximity users to transmit signal directly without going through the base station. User-assisted relaying provides more flexibility than fixed relaying in expanding the base station (BS) coverage into obscured areas, especially where there is high density of idle UEs [1], [2]. This user-assisted relaying mode can be supported by novel pricing models to tempt devices to participate in this type of communication [3] and emerging energy harvesting techniques to combat the battery power drainage problem [4]. In this paper, we analyze the performance of user-assisted relaying when deploying system-wide in a cellular network and study its effect on system performance in terms of reliability.

For cellular network analysis, stochastic geometry has been shown to be analytically tractable and capture some of the main performance trends. Stochastic geometry is used to develop a tractable model for downlink heterogeneous cellular networks in [5]. This model is further used to analyze downlink coordinated multipoint beamforming [6]. Stochastic geometry is also used to analyze the performance of decode-and-forward relaying techniques in uplink cellular networks under the specific setting of a fixed number of relays deployed at a fixed distance from the BS with equal angular separation in each cell [7]. Stochastic geometry is recently used to study the average rate performance of user-assisted relaying in uplink cellular networks in which transmitters may be aided by nearby idle users using a half-duplex partial decode-and-forward (pDF) scheme [2]. For large decentralized wireless network, outage performance is also analyzed for a full-duplex decode-and-forward (DF) scheme [8]. In these works, the transmitter chooses the relaying user as the nearest neighbor satisfying certain criteria [2], [6], [8].

In this paper, we study the performance of a DF user-assisted relaying scheme in uplink cellular networks. To the best of our knowledge, this paper and our previous work in [2] are the first to analyze user-assisted relaying in a network-wide cellular uplink context. We use stochastic geometry as a tool to model and analyze the interference in the whole network as well as cooperation policies governing the selection of an idle user to act as a relay. We propose a new practical cooperation policy based on the random selection of an idle user within a certain area mid-way between the active user and base station. We also compare this policy to the commonly considered nearest neighbor geometric policy, in which each active user selects the closest idle user to act as a relay [2]. As a basis for analysis, we assume all nodes are equipped with a single antenna and leave the case of multiple antenna nodes to a future work.

II. Relaying Scheme and Network Model

A. Network Stochastic Geometry Model

We consider a cellular system which consists of multiple cells, each cell has a single base station (BS) and each base station serves multiple user equipments (UEs). Each UE uses a distinct resource block in each cell and may suffer out-of-cell interference due to frequency reuse in all other cells. We assume that each UE is served by the single BS closest to that user. Within this system, we study the impact on performance of the cooperation technique in which an active UE (AUE) can relay its message to the BS through another UE that is in an idle state (IUE), in addition to its direct link, using the DF relaying scheme described in Section II-B.

For the geometric model, we employ stochastic geometry to describe the uplink cellular network. We assume that the AUEs in different cells that will contend for the same resource block and cause interference to each other are distributed on a two-dimensional plane according to a homogeneous and stationary Poisson point process (p.p.p.) Φ_1 with intensity λ_1. We also assume that Φ_1 is independent of another p.p.p. Φ_2 with intensity λ_2 that represents the distribution of the set of IUEs which can participate in relaying the messages transmitted by UEs in Φ_1. Furthermore, under the assumption that each BS serves a single mobile in a given resource block, we follow the same approach in describing BSs distribution as proposed in [9], where each BS is uniformly distributed in the Voronoi cell of its served UE.

B. DF Relaying Transmission Scheme

We employ full DF relaying in the half duplex mode [10] where an AUE has the option to relay through an IUE or to perform direct transmission. The IUE chosen to perform relaying is referred to as relay UE (RUE).

In the relaying case, we model the received signal at the i^{th} relay and the destination during 1^{st} and 2^{nd} phases as

$$\text{Phase 1:} \quad Y_{r,i}^b = h_{sr}^{(i)} x_{s,i}^b + I_{r,i}^b + Z_{r,i}^b \tag{1}$$

$$Y_{d,i}^b = h_{sd}^{(i)} x_{s,i}^b + I_{d,i}^b + Z_{d,i}^b \tag{2}$$

$$\text{Phase 2:} \quad Y_{d,i}^m = h_{sd}^{(i)} x_{s,i}^m + h_{rd}^{(i)} x_{r,i}^m + I_{d,i}^m + Z_{d,i}^m \tag{3}$$

where b stands for broadcast transmission in which AUE broadcasts to both the RUE and BS; m denotes multiple access transmission in which both AUE and RUE send information to BS; the signals x_s^b and x_s^m are the transmitted codewords from AUE in the 1^{st} and 2^{nd} phases; the signal x_r^m is the transmitted codeword from RUE in the 2^{nd} phase; Z_r^b, Z_d^b, and Z_d^m are $i.i.d$ $\mathcal{CN}(0, \sigma^2)$ that represent the noise at RUE and BS in the 1^{st} and 2^{nd} phases; h_{rd}, h_{sr} and h_{sd} are the RUE-to-BS, AUE-to-RUE and AUE-to-BS channels; and $I_{r,i}^b$, $I_{d,i}^b$ and $I_{d,i}^m$ represents the interference received at the i^{th} relay and BS in 1^{st} and 2^{nd} phases.

For DF relaying, the transmit signals from the active and relay users in the two phases can be described as

$$\text{Phase 1:} \quad x_s^b = \sqrt{P_s^b} U_s^b, \tag{4}$$

$$\text{Phase 2:} \quad x_r^m = \sqrt{P_r^m} U_s^m, \quad x_s^m = \sqrt{P_s^m} U_s^m. \tag{5}$$

where the codewords U_s^b and U_s^m are independent standard Gaussian with zero mean and unit variances. The transmit power of AUE and RUE need to satisfy the power constraints:

$$\alpha_1 P_s^b + \alpha_2 P_s^m = P_s, \quad \alpha_2 P_r^m = P_r, \tag{6}$$

where P_s represents the total power allocated to the AUE within a single transmission period; P_r represents the total power a RUE is willing to use to relay other users' messages; α_1 and $\alpha_2 = 1 - \alpha_1$ represent the portions of the transmission time allocated to the first and second phases, respectively.

At the end of the 2^{nd} phase, BS utilizes both received signals, Y_d^b in eq. (2) and Y_d^m in eq. (3), to decode the transmitted message using maximum likelihood (ML) decoding rule.

In the direct transmission case, we model the received signal at the i^{th} BS as

$$\text{Both phases:} \quad Y_{d,i} = h_{sd}^{(i)} x_{s,i} + I_{d,i} + Z_{d,i} \tag{7}$$

where x_s is the transmitted codeword from AUE in direct transmission; and $Z_d \sim \mathcal{CN}(0, \sigma^2)$ represents the noise; and $I_{d,i}$ represents the average interference received at the i^{th} BS during the whole two-phase transmission period.

All the channels $h_{xy}, xy \in \{sr, sd, rd\}$, are complex values with uniformly distributed phases that capture both the small and large scale fading and can be written in the form $h_{xy} = e^{j\theta_l} |h_{xy}|$, where $\theta_l \sim \mathcal{U}[0, 2\pi], l \in \{1, 2, r\}$.

C. Out-of-Cell Interference

User-assisted relaying actually increases the amount of out-of-cell interference in the network as some idle UEs are now transmitting when relaying information of AUEs. It is therefore necessary to understand this out-of-cell interference power, particularly its distribution, in order to assess the overall impact of user-assisted relaying on system performance. In the development of the interference model, we consider the cell under study to be of a fixed radius, R_c, that is typically proportional to the AUEs density as $R_c = 1/(2\sqrt{\lambda_1})$.

Interference can be expressed at the BS, during the 1^{st} and 2^{nd} phase, and at the RUE during the 1^{st} phase as

$$\begin{aligned}
I_{d,i}^b &= \sum_{k \neq i} B_k h_{sd}^{(k,i)} x_{s,k}^b + (1 - B_k) h_{sd}^{(k,i)} x_{s,k}, \\
I_{d,i}^m &= \sum_{k \neq i} B_k \left(h_{sd}^{(k,i)} x_{s,k}^m + h_{rd}^{(k,i)} x_{r,k}^m \right) + (1 - B_k) h_{sd}^{(k,i)} x_{s,k}, \\
I_{r,i}^b &= \sum_{k \neq i} B_k h_{sr}^{(k,i)} x_{s,k}^b + (1 - B_k) h_{sr}^{(k,i)} x_{s,k},
\end{aligned} \tag{8}$$

where the summation is over all AUEs. Here, $h_{sd}^{(k,i)}$ and $h_{rd}^{(k,i)}$, respectively, are the channel fading from the k^{th} AUE in Φ_1 and the associated RUE in Φ_2 to the BS associated with the i^{th} AUE in Φ_1; and $h_{sr}^{(k,i)}$ is the channel fading from the k^{th} AUE in Φ_1 to the RUE associated with the i^{th} AUE in Φ_1. In the direct transmission case, the interference term $I_{d,i}$ in eq. (7) is equivalent to $I_{d,i}^b$ during the first phase and $I_{d,i}^m$ during the second phase.

The Bernoulli random variable $B_k \sim$ Bern (ρ) captures the transmission strategy of the k^{th} UE in Φ_1 with success probability ρ, where $B_k = 1$ is used to indicate the k^{th} AUE decision to exploit the help of another IUE and apply the relaying transmission strategy, and $B_k = 0$ indicates direct transmission. A Bernoulli random variable can represent the transmission strategy with a certain probability, ρ, because, as we show in Section III, the developed cooperation policies will be independent for each AUE.

For each realization of the UE and BS locations, the out-of-cell interference in Eq. (8) can be modeled as a complex Gaussian random variable with zero mean and variance $\mathcal{Q}_i$ that represent the interference power. This interference power $\mathcal{Q}_i$, however, varies with the UE and BS locations and therefore is random. We model the interference power to the cell under study, $\mathcal{Q}_i$, as Gamma random variable γ_i by fitting the mean and variance analytically obtained as in [2], which we restate below

Lemma 1 (Interference Power Statistics [2]). *For network-wide deployment of user-assisted relaying, the out-of-cell interference generated at the destination BS and the RUE have the following statistics:*

(i) *The first two moments of interference power at the destination BS during the* 1^{st} *and* 2^{nd} *phase, respectively, are*

$$\mathbb{E}\left[\mathcal{Q}_{d,i}^{b}\right]=\frac{2\pi\lambda_1\zeta_1}{\alpha-2}R_c^{2-\alpha}, \qquad \mathbb{E}\left[\mathcal{Q}_{d,i}^{m}\right]=\frac{2\pi\lambda_1\zeta_3}{\alpha-2}R_c^{2-\alpha}, \quad (9)$$

$$var\left[\mathcal{Q}_{d,i}^{b}\right]=\frac{\pi\lambda_1\zeta_2}{\alpha-1}R_c^{2(1-\alpha)}, \quad var\left[\mathcal{Q}_{d,i}^{m}\right]=\frac{\pi\lambda_1\zeta_4}{\alpha-1}R_c^{2(1-\alpha)}. (10)$$

(ii) *The first two moments of interference power at the idle UE associated as a relay with the* i^{th} *AUE are*

$$\mathbb{E}[\mathcal{Q}_{r,i}]=\lambda_1\zeta_1\int_0^{2\pi}\int_{R_c}^{\infty}\left(r^2+D^2-2rD\cos(\theta)\right)^{-\frac{\alpha}{2}}rdrd\theta, \quad (11)$$

$$var[\mathcal{Q}_{r,i}]=\lambda_1\zeta_2\int_0^{2\pi}\int_{R_c}^{\infty}\left(r^2+D^2-2rD\cos(\theta)\right)^{-\alpha}rdrd\theta, \quad (12)$$

where $\zeta_1=\rho P_{s,k}^b+(1-\rho)P_{s,k}$,

$$\zeta_2=2\left[\rho(P_{s,k}^b)^2+(1-\rho)P_{s,k}^2\right],$$
$$\zeta_3=\rho(P_{s,k}^m+P_{r,k}^m)+(1-\rho)P_{s,k},$$
$$\zeta_4=2\big[\rho(P_{s,k}^m+P_{r,k}^m)^2+(1-\rho)P_{s,k}^2-\rho P_{s,k}^m P_{r,k}^m\big]. \quad (13)$$

III. Cooperation Policies and Probabilities

In this section, we introduce a new cooperation policy, Mid-way (MW) relay selection, based on the random selection of an idle user within a certain area mid-way between the AUE and the BS. We define this policy based on an observation in our previous work [2], which suggests that relay cooperation is most beneficial when the RUE is about midway between the AUE and BS. Further, we contrast the performance under this policy with the geometric nearest neighbor (NN) policy in [2].

A. Cooperation Policies

1) Mid-Way Policy: The cooperation policy, E_{MW}, is the event that there is at least one IUE in an area A mid-way between the AUE and the BS. This area can contain a set of candidate IUEs, from which the RUE can be chosen randomly. Event E_{MW} is formally defined as

$$E_{\text{MW}} \quad = \left\{\{y_k : y_k \in A\} \neq \phi\right\}, \quad (14)$$

where area A is defined in terms of the distance, r_1, between an AUE and its BS and a cone angle $2\psi_0$ centered at the AUE location and bisected by the line connecting the AUE and the BS. This area A is shown in Fig. 1 in green. We choose the cone angle to ensure that the distance D between the RUE and the BS is no more than the distance r_1 between the AUE and BS, that is

$$r_1^2 \quad \geq \quad D^2=b^2r_1^2+r_1^2-2br_1^2\cos\psi_0$$

We then have $\cos\psi_0 \leq b/2$, from which we select ψ_0 to satisfy this condition with equality in order to maximize the cooperation probability in this policy.

2) Nearest Neighbor Policy [2]: The NN geometric policy E_{NN} selects the RUE as the closest IUE in between the AUE and the BS and is defined as

$$E_{\text{NN}}=\left\{r_2 \leq r_1, D \leq r_1\right\}. \quad (15)$$

Here r_1 and r_2 respectively denote the direct distance between the AUE and its BS and the cooperation distance between the

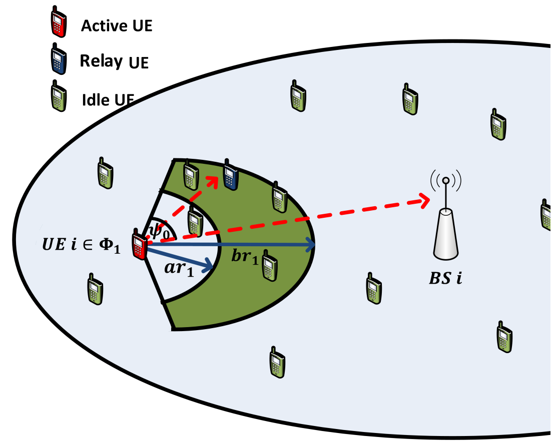

Fig. 1: Mid-way policy relay selection criteria.

AUE and RUE. The extra condition $D \leq r_1$ ensures that the RUE is closer to the base station than the AUE, effectively eliminating all cases that can result in an infinite interference at the relay.

Both policies E_{MW} and E_{NN} are practical policies in the sense that they do not require full knowledge of either the channel fading or the interference at the decision making node. Instead, each policy only requires the decision making nodes to know the distances from the AUE to the idle user and to the base station.

B. Cooperation Probabilities

Here we analytically derive the cooperation probabilities for the geometric policies E_{MW} and E_{NN} given the direct AUE-to-BS distance, r_1. These cooperation probabilities will be used in the outage analysis in Section IV. We also derive the average cooperation probabilities ρ_{NN} and ρ_{MW}, averaged over r_1, which are used in the out-of-cell interference analysis in Section II-C in place of ρ. First, we note that the distribution of the direct distance r_1 can be shown to be Rayleigh directly from the null probability of a two dimensional p.p.p. distribution [9], i.e.,

$$f_{r_1}(r_1) \quad = 2\pi\lambda_1 r_1 e^{-\lambda_1\pi r_1^2}. \quad (16)$$

Moreover, we can derive the distribution of the cooperation AUE-to-RUE distance r_2 between the i^{th} AUE and its associated RUE as in Lemma 2.

Lemma 2 (Relaying link distance distribution)**.** *The probability density function of the AUE-to-RUE distance are*
(i) For policy E_{MW}*:*

$$f_{r_2}(r) \quad = \frac{2r}{(b^2-a^2)r_1^2}, \quad ar_1 \leq r \leq br_1. \quad (17)$$

(ii) For policy E_{NN}*:*

$$f_{r_2}(r) \quad = 2\pi\lambda_2 r e^{-\lambda_2\pi r^2}. \quad (18)$$

Proof: (i) For policy E_{MW}: In this cooperation policy, the AUE selects a random idle user within an area A that is defined as the partial annulus with radii ar_1 and br_1, $a < b \in [0,1]$ and limited by the $2\psi_0$ angular cone bisected by the line connecting the AUE and BS. Since the randomly chosen idle user belongs to the p.p.p. Φ_2, then it is uniformly distributed within the defined area A.

Since the probability contained in a differential area must be invariant under change of variables, we can write

$$\begin{aligned} |f_A(A)dA| &= |f_{r_2,\Theta}(r,\theta)drd\theta| \\ \Rightarrow f_{r_2,\Theta}(r,\theta) &= f_A(A)\frac{dA}{drd\theta} = \frac{2r}{(b^2-a^2)r_1^2}.\frac{1}{2\psi_0} \end{aligned}$$

Then, it follows that $f_{r_2}(r)$ is as in (17) and $f_\Theta(\theta) = \frac{1}{2\psi_0}$ due to the independence of r_2 and Θ.

(ii) For policy E_{NN}: In this policy, each UE in Φ_1 chooses the nearest UE in Φ_2 to assist it in relaying its message to the serving BS. Then, it follows directly from the null probability of a two dimensional p.p.p. that the distribution of the AUE-to-RUE distance r_2 is Rayleigh distributed [2], [9]. ■

The probabilities of cooperation policies E_{MW} and E_{NN} defined in (14) and (15), respectively, are derived in closed form as in Theorem 1 next.

Theorem 1 (Cooperation probabilities)**.** *The probability of deploying user-assisted relaying for a randomly located AUE within a cell can be evaluated as follows:*

(i) For policy E_{MW}

$$\mathbb{P}\{E_{MW}|r_1\} = 1 - e^{-\lambda_2 A} \quad (19)$$

$$\rho_{MW} = 1 - \frac{2\pi\lambda_1}{2[\pi\lambda_1 + \lambda_2\psi_0(b^2-a^2)]}. \quad (20)$$

(ii) For policy E_{NN}

$$\mathbb{P}\{E_{NN}|r_1\} = \frac{1}{3}\left(1 - e^{-\pi\lambda_2 r_1^2}\right) + \int_{\frac{-\pi}{2}}^{\frac{-\pi}{3}} \mathcal{E}_1 d\psi_0 + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \mathcal{E}_1 d\psi_0 \quad (21)$$

$$\rho_{NN} = \frac{\lambda_2}{3(\lambda_1+\lambda_2)} + \int_{\frac{-\pi}{2}}^{\frac{-\pi}{3}} \mathcal{E}_2 d\psi_0 + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \mathcal{E}_2 d\psi_0. \quad (22)$$

where $\mathcal{E}_1$ and $\mathcal{E}_2$ are defined as in Eq. (31) *and Eq.* (32)*.*

Proof: See Appendix A for details. ■

We use numerical integration to compare between these probabilities in Section V-A. We note that both probabilities in (20) and (22) are proportional to the IUEs to AUEs density ratio λ_2/λ_1 and achieve their maximum when this ratio approaches infinity, i.e. as the density of the idle users increases. We can evaluate these maximum probabilities as

$$\begin{aligned} \rho_{\text{MW}}^{max} &= \lim_{\lambda_2\to\infty} \rho_{\text{MW}}(\lambda_1,\lambda_2) = 1, \\ \rho_{\text{NN}}^{max} &= \lim_{\lambda_2\to\infty} \rho_{\text{NN}}(\lambda_1,\lambda_2) = 0.5. \end{aligned} \quad (23)$$

The maximum probability for E_{NN} is 0.5 because of our restriction of the spatial cooperation domain to the idle UEs that are closer to the BS than is the AUE. Effectively, we only consider potential relays approximately in a half circle centered at the AUE and inside the cell under consideration.

IV. Transmission Rate and Outage Performance

A. Achievable Transmission Rate

With the transmit signals in Eqs. (4)−(5) and using joint ML decoding at BS, we obtain the following achievable rate for DF relaying:

$$R_{DF} \le \min(C_1, C_2), \quad (24)$$

where

$$\begin{aligned} C_1 &= \alpha_1 \log\left(1 + \left|\tilde{h}_{sr}^{(i)}\right|^2 P_s^b\right), \\ C_2 &= \alpha_1 \log\left(1 + \left|\tilde{h}_{sd}^{(b,i)}\right|^2 P_s^b\right) \\ &\quad + \alpha_2 \log\left(1 + \left(\left|\tilde{h}_{sd}^{(m,i)}\right|\sqrt{P_s^m} + \left|\tilde{h}_{rd}^{(i)}\right|\sqrt{P_r^m}\right)^2\right) \end{aligned} \quad (25)$$

Here C_1 and C_2 are the rate of information that can be decoded at the RUE and BS, respectively. These rates are achievable provided the standard full channel knowledge at receivers and AUE-RUE coherent phase knowledge [11], [12].

Further, the maximum achievable rate of the direct transmission case can be obtained as

$$C = \alpha_1 \log\left(1 + \left|\tilde{h}_{sd}^{(b,i)}\right|^2 P_{s,i}\right) + \alpha_2 \log\left(1 + \left|\tilde{h}_{sd}^{(m,i)}\right|^2 P_{s,i}\right) \quad (26)$$

where $\tilde{h}_{sr}^{(i)} = \frac{h_{sr}^{(i)}}{\sqrt{\mathcal{Q}_{r,i}+\sigma^2}}, \quad \tilde{h}_{sd}^{(b,i)} = \frac{h_{sd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^b+\sigma^2}},$

$$\tilde{h}_{sd}^{(m,i)} = \frac{h_{sd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^m+\sigma^2}}, \quad \tilde{h}_{rd}^{(i)} = \frac{h_{rd}^{(i)}}{\sqrt{\mathcal{Q}_{d,i}^m+\sigma^2}}, \quad (27)$$

where $\mathcal{Q}_{r,i}, \mathcal{Q}_{d,i}^b$, and $\mathcal{Q}_{d,i}^m$ represent interference power respectively at the relay, the BS during 1^{st} phase and the BS during 2^{nd} phase as modeled in Section II-C.

B. Outage Probabilities

The achievable rates in Eqs. (25) and Eq. (26) are random quantities due to random users locations (r_1, r_2) and channel fading. In this paper we investigate probability of outage as a performance metric. Given that the DF relaying scheme can be deployed using a cooperation policy, either E_{MW} or E_{NN}, the outage probability depends on the outage of each transmission scenario, either DF relaying transmission or direct transmission, in each policy.

The outage probability for E_{MW} policy for a given AUE location at distance r_1 can be written as

$$\begin{aligned} P_{0,\text{MW}} = &\left(1 - \mathbb{P}\{C_1 > R|E_{\text{MW}}, r_1\}\mathbb{P}\{C_2 > R|E_{\text{MW}}, r_1\}\right) \\ &\mathbb{P}\{E_{\text{MW}}|r_1\} + \mathbb{P}\{C < R|\bar{E}_{\text{MW}}, r_1\}\mathbb{P}\{\bar{E}_{\text{MW}}|r_1\} \end{aligned} \quad (28)$$

where $\bar{E}$ is the complementary event of E with $\mathbb{P}\{\bar{E}|r_1\} = 1 - \mathbb{P}\{E|r_1\}$. This outage probability depends on the distribution of the random r_2 distance as given in Eq. (17) with a support set satisfying the event E_{MW} in (14). This dependence is implicit in their effect on the path loss in the expressions of C_1, C_2 and C.

For E_{NN}, outage probability is similarly defined, provided a more restricted support set of r_2 distribution in Eq. (18) as specified by the event E_{NN} in (15).

$$\begin{aligned} P_{0,\text{NN}} = &\left(1 - \mathbb{P}\{C_1 > R|E_{\text{NN}}, r_1\}\mathbb{P}\{C_2 > R|E_{\text{NN}}, r_1\}\right) \\ &\mathbb{P}\{E_{\text{NN}}|r_1\} + \mathbb{P}\{C < R|\bar{E}_{\text{NN}}, r_1\}\mathbb{P}\{\bar{E}_{\text{NN}}|r_1\}. \end{aligned} \quad (29)$$

Both expressions in Eq. (28) for E_{MW} and Eq. (29) for E_{NN} are not easy to be evaluated analytically, so we use numerical methods to evaluate the outage probability performance.

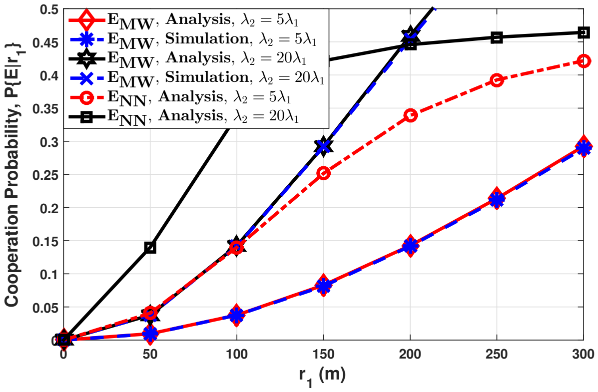

Fig. 2: Cooperation policies probabilities vs. direct link distance, $\lambda_2 = \{5, 20\}\lambda_1$.

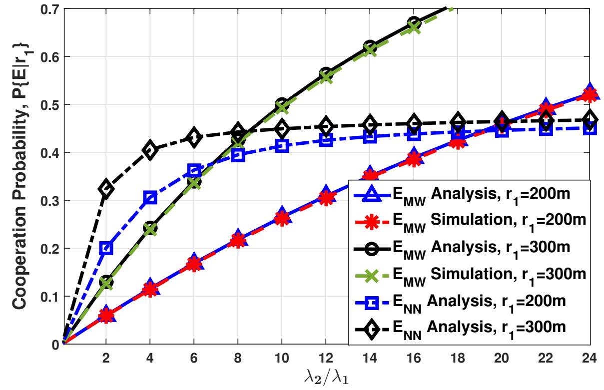

Fig. 3: Cooperation policies probabilities vs. density ratio, ($r_1 = \{200, 300\}$ m, $R_c = 300$ m).

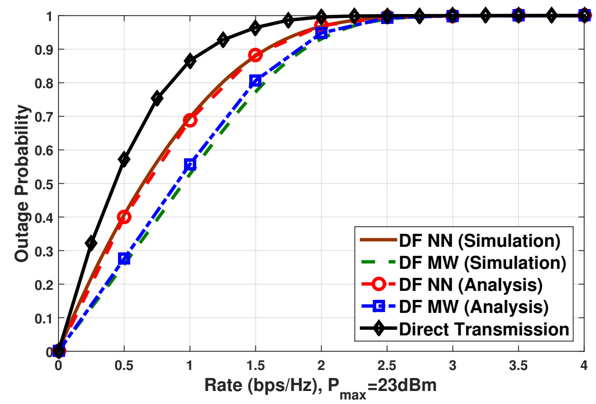

Fig. 4: Rate cumulative distribution function, ($r_1 = 290$ m, $R_c = 300$ m, $\lambda_2 = 20\lambda_1, P_{max} = 23$ dBm).

V. Numerical Performance Results

In this section, we use our geometric network model to numerically analyze performance of user-assisted relaying under the two studied policies. We first discuss and compare the two cooperation policies probabilities. Then, we evaluate and discuss the outage performance of user-assisted relaying in uplink cellular network. In this section, we use the following settings unless otherwise stated: $R_c = 300m$, $a = 0.2, b = 0.5$, $\psi_0 = \cos^{-1} b/2$, $P_{s,i} = P_{r,i}$, $P^b_{s,i} = P^m_{s,i}$, and noise power is assumed to be 74 dBm.

A. Cooperation Probabilities

We compare the cooperation probability of both E_{MW} and E_{NN} versus the AUE-to-BS distance in Fig. 2 and versus IUEs to AUEs density ratio in Fig. 3. It is shown that for low idle users density, i.e. $\lambda_2 \leq 6\lambda_1$, the NN geometric policy leads to higher cooperation probability than the MW policy regardless the distance of the direct link r_1. As the idle users density increases, which policy has higher cooperation probability depends heavily on the location of the AUE. For relatively high IUEs density, the closer the AUE to the cell edge, the more likely the MW geometric policy to have a higher probability of cooperation. For very high idle users density, we find that the cooperation probability of the MW geometric policy is not as limited as the NN geometric policy and that it can approach 1 in contrast to the NN policy approaching only 0.5. Hence, the MW geometric policy appears to be most suitable for very high idle users density, $\lambda_2 \geq 20\lambda_1$. However, despite of non-dominating cooperation probability, we see in the next section that the MW policy outage performance is almost always better than the NN policy outage performance.

B. Performance Analysis of User-Assisted Relaying

In this section, we evaluate the outage performance of DF user-assisted relaying in network deployment, taking into account cooperation probabilities and out-of-cell interference. We perform numerical integrations to compute the average rates in Eqs. (24)$-$(26) based on our developed analytical interference model in Section II-C and cooperation probabilities in Eqs. (20)$-$(22). We then compare these numerical results with system simulation where we carry out detailed simulation of a multi-cell network as discussed in Section II-A.

In Fig. 4, we compare the simulation and numerical results of the transmission rate cumulative distribution function, showing close match between simulation and analysis for both policies. This figure shows the outage probability of an AUE that is close to the cell edge, when $\lambda_2 = 20\lambda_1$. We find that when the AUE is close to the cell edge, the MW policy has much better outage performance than the NN policy. We further note that for cell edge users, either cooperation policy is always better than the deployment of pure direct transmission scheme in the whole network.

In Fig. 5, we study the transmission rate at a fixed outage probability of $P_0 = 0.1$ in the user density range $\lambda_2 \geq 5\lambda_1$, and compare the gain of the NN and MW policies over direct transmission. For the computed range of densities, both NN and MW policies performs better than direct transmission. Note the maximum rate gain of the MW policy reaches 228% at $\lambda_2 = 25\lambda_1$, $r_1 = 0.97R_c$ but is not shown in the figure for better scaling. We also note that at $r_1 = 290$m, the MW policy always performs better than the NN policy regardless of the IUEs density. This result implies that the cooperation probability is not an indicative factor of performance since we can see in Fig. 2 that at $r_1 = 290$m and $\lambda_2 = 5\lambda_1$, the NN policy has a much higher cooperation probability than the MW policy, yet has lower rate gain. Furthermore, the performance of MW policy is still comparable to that of the NN policy even when the gap between their cooperation probabilities is largest at $r_1 = 200$m and $\lambda_2 = 6\lambda_1$ as can be seen in Fig. 3.

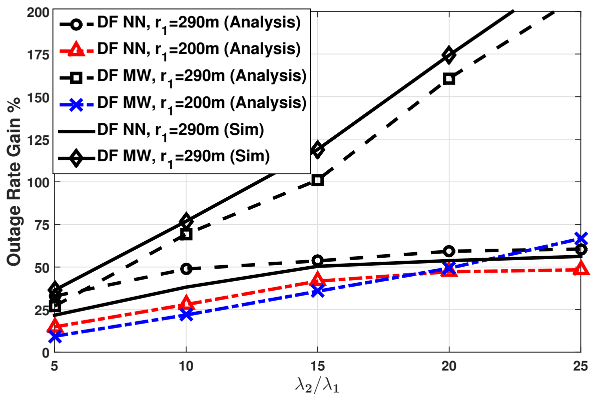

Fig. 5: 10% outage rate versus idle users to AUEs density ratio, ($r_1 = \{200, 290\}$ m, $R_c = 300$ m, $P_{max} = 23$ dBm).

We note from Figs. 4 and 5 that the performance gain of user-assisted relaying over direct transmission increases as the idle users density increases. User-assisted relaying effectiveness increases with higher idle users density, which is expected for future cellular networks with dense subscribers. For the case of single antenna analyzed in this paper, the mid-way selection of the RUE is more effective for DF relaying than the nearest neighbor policy which may result in the selection of a RUE too close to the AUE or the BS.

VI. Conclusion

In this paper, we investigate performance impact of deploying uplink user-assisted decode-and-forward relaying in a cellular network. Using a stochastic geometry model for user and base station locations, we propose a geometric mid-way (MW) cooperation policy that is based on the random selection of an idle user within an area mid-way between the active user and the base station. We contrast this new policy to the commonly considered nearest neighbor (NN) cooperation policy. We show that user-assisted relaying significantly outperforms direct transmission for both policies. We also show that with an appropriate choice of selection area, the MW cooperation policy always has better or at least the same reliability performance as the NN cooperation policy for cell edge users, regardless of the idle user density.

Appendix A: Proof of Theorem 1

In this section, we derive the probability of both the NN cooperation policy, E_{NN}, in (15) and the MW cooperation policy, E_{MW}, in (14). First, we show the derivation of the NN cooperation probability ρ_{NN} as follows

$$\begin{aligned}\mathbb{P}\{E_{\text{NN}}|r_1\} &= \mathbb{P}\big\{r_1 \geq r_2, r_1^2 + r_2^2 - 2r_1r_2\cos\psi_0 \leq r_1^2 | r_1\big\}\\ &= \mathbb{P}\big\{r_1 \geq r_2, r_2 \leq 2r_1\cos\psi_0 | r_1\big\}\\ \rho_{\text{NN}} &= \mathbb{E}_{r_1}\mathbb{P}\{E_{\text{NN}}|r_1\}\\ &= \int_{-\pi/2}^{-\pi/3}\mathcal{E}_2 d\psi_0 + \int_{\pi/3}^{\pi/2}\mathcal{E}_2 d\psi_0 + \int_{-\pi/3}^{\pi/3}\mathcal{E}_3 d\psi_0, \quad (30)\end{aligned}$$

where

$$\mathcal{E}_1 = -\frac{1}{2\pi}\left(e^{-4\pi\lambda_2 r_1^2\cos^2\psi_0} - 1\right) \quad (31)$$

$$\begin{aligned}\mathcal{E}_2 &= 2\pi\lambda_1\lambda_2\int_0^\infty\int_0^{2r_1\cos\psi_0} r_1r_2e^{-\pi(\lambda_1r_1^2+\lambda_2r_2^2)}dr_2dr_1\\ &= \frac{2\lambda_2\cos^2\psi_0}{\pi(\lambda_1+4\lambda_2\cos^2\psi_0)}, \quad (32)\\ \mathcal{E}_3 &= 2\pi\lambda_1\lambda_2\int_0^\infty\int_0^{r_1} r_1r_2e^{-\pi(\lambda_1r_1^2+\lambda_2r_2^2)}dr_2dr_1\\ &= \frac{\lambda_2}{2\pi(\lambda_1+\lambda_2)}, \quad (33)\end{aligned}$$

Substituting Eqs. (32) and (33) into Eq. (30), we obtain Eq.(22) in Theorem 1.

Next, we derive the MW cooperation probability ρ_{MW}. We first note that the probability of having no points in an area A follows from the definition of the p.p.p. and is defined as

$$P(0, A) = e^{-\lambda A}$$

where λ is the density of points in the p.p.p.. Then, the probability of finding at least one point in the area A is

$$P(n > 0, A) = 1 - e^{-\lambda A}. \quad (34)$$

Using (34) and the fact that A is a function of the AUE-to-BS distance which is Rayleigh distributed, we can find ρ_{MW} as

$$\begin{aligned}\rho_{\text{MW}} &= \mathbb{E}_{r_1}\mathbb{P}\{E_{\text{MW}}|r_1\} = \mathbb{E}_{r_1}\big\{P(n>0, A)\big\}\\ &= 1 - 2\pi\lambda_1\int_0^\infty r_1e^{-r_1^2[\lambda_1\pi+\lambda_2\psi_0(b^2-a^2)]}\end{aligned}$$

which leads to Eq. (20).

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