A Structural Econometric Analysis of Entrepreneurs with Multiple Occupations – Evidence from Urban Thai Data

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Abstract

This paper structurally estimates a model in which risk neutral agents maximize total income by optimally allocating capital and labor into entrepreneurship, subject to credit and time constraints. The model predicts more than one occupation for individuals whose equilibrium entrepreneurial labor allocation does not exhaust the time-constraint. This can arise in two scenarios: (a) the first-best scale of the business does not exhaust the time-constraint or (b) the credit-constrained scale of the business does not exhaust the time-constraint. The magnitude of each type of entrepreneurs is estimated via GMM on a sample of business owners from semi-urban Thai villages, where 20% of business owners report a second occupation. The results imply that 85% of entrepreneurs hold a second job because they are skill-constrained, and therefore would not specialize in business ownership if the credit constraint is relaxed. These two groups of multiple occupation holders are also predicted to be considerably heterogeneous in terms of initial wealth, schooling and entrepreneurial talent.

1 Introduction

There are at least three economic arguments for why the holding of multiple occupations is an indication of incomplete markets. First, imperfect insurance in consumption could lead to individuals diversifying income sources. Second, imperfect credit markets could lead to individuals not being able to "raise the capital they would need to run a business that would occupy them fully" (Banerjee and Duflo, 2007, p162). Third, and somewhat related is the idea that apart from credit constraints, underdeveloped labor markets could lead to individuals taking up a low productivity side activity to occupy what would otherwise be idle time.

I explore the latter two arguments. The primary goal of this paper is to understand why we observe multiple occupations, and more specifically, to quantify the importance of the financial constraint channel as opposed to lack of entrepreneurial skill channel in explaining the phenomenon. This is necessary to characterize whether multiple occupations is a sign of economic inefficiency (and therefore whether we should aim to discourage it), and whether and what kind of policies are likely to increase household welfare in this context. For example, if multiple occupations is a result mainly of lack of access to credit, policies that improve this market will

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enhance welfare by allowing households to maximize income. However, if it is mainly a result of underdeveloped labor markets in conjunction with being particularly ill-suited for entrepreneurship (leading to a lack of specialization in either occupation), encouraging further specialization in entrepreneurship will be ineffective in raising incomes, or in encouraging specialization in the first place. Therefore, the magnitude of each type has implications for what kind of policies might be the most effective in increasing incomes.

I develop and structurally estimate a model in which risk neutral agents optimally allocate capital and labor based on their entrepreneurial ability and employability elsewhere to maximize their total income, subject to a collateral constraint and a time constraint. In much of the previous literature that applies classical occupational choice models to study entrepreneurship in developing countries, agents are able to choose only one occupation, and consequently do not distinguish between those who specialize solely in business ownership and those that take on another occupation in addition.

I relax the assumption that occupational choice is a binary and a mutually exclusive choice. Each agent in the model has access to two technologies. The entrepreneurial technology allows the agent to combine capital and labor, and income is assumed to be zero if either of the inputs are zero.[1] The second technology only uses labor as an input. I call this technology wage-work for simplicity, although it could also represent subsistence occupations where capital plays no role. Agents maximize expected *total* income from the two technologies by choosing capital, entrepreneurial labor, and wage-work labor. Following the existing occupational literature in development, I also allow a collateral constraint for capital investment. In a nutshell, the model in this paper is an extension of Evans and Jovanovic (1989) – similar to their model, agents are heterogeneous in their entrepreneurial talent, initial wealth and wage, are expected income maximizers, and can only afford a positive multiple of their collateralizable wealth as capital investment. However, in addition to this basic framework, I allow households to allocate a fixed time endowment between entrepreneurship and wage-work.

The model assumes a Cobb-Douglas production for entrepreneurship with labor and capital as variable inputs that are gross complements. In addition, it assumes that the marginal products of capital and labor with respect to entrepreneurial output are higher (everywhere) for agents with higher entrepreneurial talent. The model predicts multiple occupations for individuals whose equilibrium entrepreneurial labor allocation does not exhaust the time constraint. This can arise both for individuals who are credit-constrained, and for those who are not credit-constrained. In the former case, capital investment in business is lower than it would be in the first-best scenario. Due to the complementarity in the two inputs, entrepreneurial labor is also suboptimal *and* does not exhaust the time-constraint. I call these multiple occupation holders "credit-constrained". The second type of multiple job holders are those who are predicted to be unable to occupy their time endowment fully with entrepreneurship, even though they are not credit-constrained at their level of entrepreneurial talent. In that sense, I call them "skill-constrained".

I estimate the model on data from a 2005 survey of urban households in Thailand (Townsend

[1] I use a Cobb-Douglas production technology, that has decreasing returns to scale in capital and labor. An important implication is that optimal entrepreneurial labor and capital is never zero, unless a constraint on capital makes it zero. As a result, the model predicts that everyone is an entrepreneur. I focus only on business owners when I take the model to the data. Assuming a fixed cost of operating a business would produce exclusive wage-workers. However, estimation of such a model on the entire sample of business owners and wage-workers produces extremely poor model fit. I leave the incorporation of exclusive wage-workers through more suitable extensions of the model for future work.

Thai Project), restricting the sample to the subset of business households.[2] About 61% of households report owning at least one business. Among business owning households, I identify the individual business owner, and find that 20.4% of them also report a secondary occupation. On average, business owners with multiple occupations in the data are richer (in terms of household asset five years prior to the survey) and have higher years of schooling, compared to business owners that only have the one occupation. However, multiple occupation holders have lower gross business income on average compared to those with single occupation.

I structurally estimate the model via GMM to match the proportion of multiple occupation holders (overall and among different subsamples), and the average business incomes of multiple and single occupation holders respectively. I find that 85% of the business owners with multiple occupations (or 18% of the sample) are skill-constrained. That is, they are predicted to hold two jobs because given their level of entrepreneurial talent, the optimal entrepreneurial labor does not fully occupy them even though they are predicted to invest the first-best level of capital. The remaining 15% of multiple occupation holders (or 3.2% of the sample) are credit-constrained. The model predicts that on average, individuals with multiple jobs are less entrepreneurially skilled, and as observed in the data, have higher starting wealth and years of schooling, compared to those with a single occupation. Within the multiple occupations group, the skill-constrained are predicted to be less entrepreneurially talented, considerably richer, and have higher years of schooling compared to those who are predicted to be credit-constrained.

Consistent with the relative importance of lack of entrepreneurial skill as a cause for multiple occupations, a key implication of the model (at the GMM estimates) is that improving access to credit alone might not result in a significant decrease in the holding of multiple occupations. In particular, I estimate that eliminating the credit-constraint enables 2.1% of the sample to specialize in business ownership, and decreases the proportion of multiple occupation holders from 21% to 18.9%. It does however lead to an increase in the incomes of the poorest households (that are most likely to be credit-constrained). While the model implies that credit market inefficiency is not an important cause of multiple occupations among business owners in rural Thailand, the existence of skill-constrained multiple occupations could still be a symptom of underdevelopment, for example, of the wage-labor market.

Related literature

One of the first papers to theoretically explore individuals holding more than one job is Shishko and Rostker (1976). In their model, multiple job holding occurs due to a time constraint on the main job, as long as the second job pays a wage above the marginal rate of substitution of income for leisure at the intersection of the primary wage line and the allowable hours on the first job. The paper estimates elasticities of *moonlighting*[3] on US data (Income Dynamics Panel, 1968-70), and concludes that it decreases with wages and hours worked in the primary job, and increases with the wage on the second job. Similar to Shishko and Rostker (1976), the cause of moonlighting in Krishnan (1990) is the assumption that hours of work are fixed (institutionally) in the primary job, and examines married men's decision to take a second job using the Survey of Income and Program Participation. The paper finds that those who held a second job (5% of married men) worked fewer hours on the first job, longer hours in total, were younger and had

[2]More detail on the Townsend Thai Project can be found at http://cier.uchicago.edu/.

[3]"Moonlighting" is a term used frequently in the literature to refer to dual or multiple job holding.

larger families on average.[4] In my model, hours-constraint due to fixed employment contracts is not an explicit cause of multiple occupations. However, constraints in the market for wage employment, including the inability to find full-time jobs, could lead to multiple occupations for individuals who are not talented enough as entrepreneurs to exhaust their time endowment in business ownership. In that sense, skill-constrained multiple occupation holders (as defined in my model) could answer that they would like to work more in their non-business jobs. Similarly, the credit-constrained business owners (as defined in my model), who would like to employ both more labor and capital in their business could report that they would like to work more hours in their business. However given the credit constraint, they find it optimal to allocate their hours in a second job.

More recently, Choe et al. (2015) have modeled multiple occupations as a result of both the "hours-constraint" and a preference for job differentiation by using a Stone-Geary utility function. They motivate this by noting that about 60% of dual-job holding episodes among male workers in the British Household Panel Survey are self-reportedly unconstrained by hours on the main job. The preference for job differentiation could be a result of preference for heterogeneous jobs (Conway and Kimmel, 2001), hedging against unemployment (Bell et al., 1997) or to transition to a new primary job (Panos et al., 2011). These reasons could encapsulate both skill-constrained multiple occupation holders (as I define it in this paper) and risk-diversification motives.

The practice of multiple occupations is especially prevalent in developing countries. In their review of poor urban households, Banerjee and Duflo (2007) report that 47% in Cote d'Ivoire and Indonesia, 36% in Pakistan, 24% in Mexico and 20% in Peru derive their income from more than one source. They argue that individuals might take up a second job to occupy what would otherwise be wasted time, and that credit constraint and lack of skill could be the two main reasons for having left-over time. I model occupational choice and multiple occupations exactly along these lines. In particular, unlike much of the prior work, the model in this paper explicitly generates the holding of multiple occupations due to lack of entrepreneurial skill (or a wage-job that employs them full time). Banerjee and Dulfo (2007) also notes that while risk diversification is a possible reason, most households in their surveys tend to employ themselves in relatively safe jobs. Similar to the urban survey used in this paper, the data in their paper come from urban settings.

Nonetheless, risk diversification is possibly an important reason for occupational diversification, and has been the focus of much of the empirical work behind occupational diversification in developing countries – especially for regions and households that deal with considerable income risk. Bandyopadhyay and Skoufias (2015) study how households in flood-prone areas in rural Bangladesh cope with risk, and find that households in areas of high rainfall variability have more occupational diversity.[5] Shenoy (2015) uses the rural surveys of the Townsend Thai data (I use the urban surveys from the same project), and estimates that Thai rice farmers expecting

[4]Some statistics on the rate of multiple occupations from various studies: the weekly rate of dual job holding at about 9% in the UK between 1994 and 2002, and about 5% in the US between 2000 and 2010; the rate of dual-job holding Russian males doubled from the early to the mid 90's and stayed around 12% for the decade; a survey of Tanzanian workers with a regular formal sector job found that more than half of them also held a job in the informal sector. (Choe et al., 2015).

[5]Higher rainfall variability is also associated with lower consumption; the use of credit (household took a loan in the past 12 months) or safety nets (a household member availed of safety-net programs in the last 12 months) are found not to completely mitigate the negative effects of rainfall variability on consumption, whereas access to markets does seem to offset them.

a harvest in the next three months take on one extra activity when the volatility in international rice prices rises by 21%. In addition, the paper finds no evidence that occupational diversity decreased following the introduction of the Million Baht Program in 2001, a credit injection that gave one million baht for public lending in every village. The latter result is consistent with the finding in my paper that credit-constraint is not a major cause of multiple occupations in the Townsend data. In a related finding, Bianchi and Bobba (2012) report that current occupational choices are more responsive to future cash transfers compared to current cash transfers in Mexico, and speculate that the program created an increase in business ownership by enhancing the willingness to bear risk as opposed to simply relaxing current liquidity constraints. The model in this paper assumes risk neutral agents, and therefore does not account for risk diversification as a motive behind holding multiple occupations. It however does provide a framework that can be extended to include this motive, for example, by assuming risk aversion or by allowing variance in total income to enter the objective function negatively.[6]

The paper is organized as follows. Section 2 presents the model, defining each type of multiple occupation holding, followed by descriptive summaries of multiple occupations in the Townsend Thai Data's urban survey in Section 3. I then proceed to structurally estimate the model, present the main results and analysis in Section 4, and finally conclude in Section 3.5.

2 Model

I start with the assumption that agents are endowed with initial wealth z, years of schooling x and entrepreneurial talent θ. They can derive total income from an entrepreneurial production function and a non-entrepreneurial wage function.

$$y(z, x, \theta) = y^E(z, x, \theta) + y^W(x).$$

Additionally, total income from entrepreneurship $y^E(z, \theta)$ depends on talent and initial wealth in the following manner:

$$y^E(z, \theta) = \theta k^\alpha h^\beta + r(z - k), \qquad \alpha > 0,\ \beta > 0,\ \alpha + \beta < 1,$$

where α and β are elasticities of entrepreneurial income with respect to capital and labor respectively, and r is the gross rate of interest at which agents can borrow and lend. The restriction that $\alpha + \beta < 1$ implies that the production function has decreasing returns to scale. A credit constraint is modeled in the form of a ceiling for capital investment:

$$k \leq \lambda z, \quad \lambda > 0.$$

This form of credit constraint can be micro-founded from a market failure of limited enforcement

[6] A behavioral model is used in understanding the motives behind multiple occupations in Hlouskova et al. (2015). The paper considers how loss aversion, the value of the reference level of income, and the expected return to risk affect the decision to hold multiple jobs. Their model predicts that a worker will not seek a risky job if she has an income reference level equal to what she can earn from a safe job. At any other reference level, the worker seeks new ventures provided she is compensated with a higher expected wage and is sufficiently loss averse.

leading to collateralized loans. Additionally, labor allocation h is subject to a time constraint:

$$h \leq T,$$

where T represents the maximum time available for work. I assume that T is not endogenous, and that it is fixed for all agents. This ignores the possibility of a market for hiring paid workers (more talented entrepreneurs might want to endogenously hire additional workers). It also ignores the fact that some business households employ unpaid family workers.[7] I also abstract away from differences in preference for leisure. [8]

The second source of income $y^W(x)$ is assumed to only depend on years of schooling, and is assumed to be linear in the time allocated to the activity, $(T-h)$:

$$y^W(x) = w(T-h),$$

with a Mincerian wage function, $w = \mu(1+x)^\gamma$. The parameter μ represents the wage for individuals with zero years of schooling, and γ represents the elasticity of wage income with respect to schooling.

The agent faces the following optimization problem.

$$\max_{k,h} w(T-h) + \theta k^\alpha h^\beta + r(z-k) \quad \text{s.t.} \quad h \in [0,T],\ k \in [0, \lambda z] \tag{1}$$

There are four threshold values of entrepreneurial talent. Suppose a household is able to optimally set k and h at their first-best levels, k_u and h_u. Ignoring the constraints for now, the first-order conditions imply that the first-best levels of entrepreneurial capital and labor are

$$k_u \equiv (\frac{\theta\alpha}{r})^{\frac{1-\beta}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}, \quad h_u \equiv (\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{1-\alpha}{1-\alpha-\beta}} \tag{2}$$

The model predicts that entrepreneurial talent θ is positively associated with the level of inputs in the first-best case, while wage (or equivalently x) is negatively related to the two. Therefore, individuals with higher θ, for a given level of wealth and schooling, are more likely to be credit-constrained and time-constrained. Individuals with higher w, for a given level of wealth and talent, are less likely to be credit-constrained and time-constrained.

Define $A(x)$ as the minimum level of θ at which an individual with x years of schooling has $h_u > 1$.

$$\text{i.e.} \quad h_u > T \Leftrightarrow \theta > T^{1-\alpha-\beta}(\frac{r}{\alpha})^\alpha(\frac{w}{\beta})^{1-\alpha} \equiv A(x) \tag{3}$$

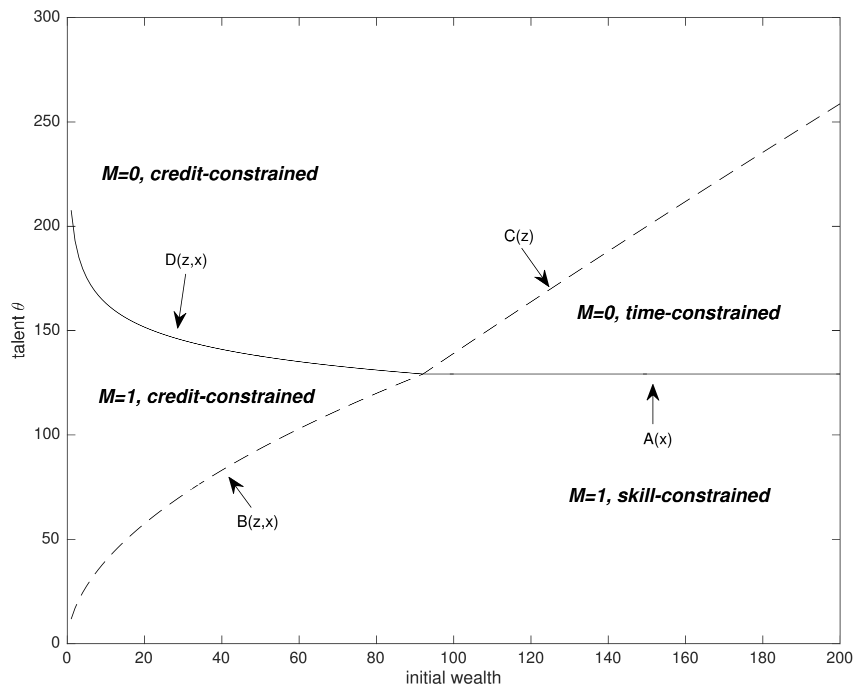
Similarly, define $B(z,x)$ as the minimum level of θ at which an individual with characteristics (z,x) finds $k_u > \lambda z$.

$$\text{i.e.} \quad k_u > \lambda z \Leftrightarrow \theta > (\lambda z)^{1-\alpha-\beta}(\frac{r}{\alpha})^{1-\beta}(\frac{w}{\beta})^\beta \equiv B(z,x) \tag{4}$$

[7]The majority of businesses observed in the data do not hire paid employees, which could reflect low productivity or a labor market failure (for example, moral hazard concerns).

[8]The model can be extended, at the cost of additional parameters, to allow the time endowment T to depend on variables that could potentially influence preference for leisure, such as number of children and household size.

Figure 1: Occupational map in (z, θ) plane for a given level of x



Note: $M = 1$ indicates multiple occupations, and $M = 0$ indicates single occupation.

Suppose k is set at λz. The optimal entrepreneurial labor choice can be found to be equal to

$$\hat{h} \equiv \Big(\frac{\theta(\lambda z)^{\alpha}\beta}{w}\Big)^{\frac{1}{1-\beta}} \tag{5}$$

and similarly, if h is set at T, the optimal capital input can be found to be equal to

$$\hat{k} = \Big(\frac{\theta\alpha T^{\beta}}{r}\Big)^{\frac{1}{1-\alpha}} \tag{6}$$

We again need to ensure that $\hat{k}$ and $\hat{h}$ do not violate the constraints. Define $C(z)$ as the level of talent above which $\hat{k} > \lambda z$.

$$\hat{k} > \lambda z \Leftrightarrow \theta > (\lambda z)^{1-\alpha}\frac{r}{\alpha T^{\beta}} \equiv C(z) \tag{7}$$

Finally, define $D(z, x)$ as the level of talent above which $\hat{h} > 1$.

$$\hat{h} > T \Leftrightarrow \theta > T^{1-\beta}\frac{w}{(\lambda z)^{\alpha}\beta} \equiv D(z, x) \tag{8}$$

Figure 1 plots the occupational map for different values of initial wealth z and talent θ, for a given value of schooling, x, at plausible parameters.[9] The solid line in the figure divides the plane into multiple occupation holders ($M = 1$) and single occupation holders ($M = 0$), equal to

[9] The parameters are set at the GMM estimates presented later in the paper.

$D(z,x)$ if the agent is credit-constrained, and equal to $A(x)$ if the agent is not credit-constrained. The dashed line in the figure determines whether an agent is credit-constrained or not, and is equal to $B(z,x)$ if the agent holds multiple occupations and $C(z)$ if the agent specializes in business ownership. Consequently, the model predicts four types of business owners.

As a solution to the optimization problem defined in 1, the first-best levels of capital and entrepreneurial labor are allocated if both are simultaneously affordable. That is,

$$\begin{aligned} (k^*, h^*) &= (k_u(\theta), h_u(\theta)) \quad \text{if} \quad k_u < \lambda z,\ h_u < T \\ &= (k_u(\theta), h_u(\theta)) \quad \text{if} \quad \theta < A(x),\ \theta < B(z,x) \end{aligned} .$$

The first type of multiple occupations therefore arises when an agent is of type (θ, z, x) such that $\theta < \min\{A(x), B(z,x)\}$ - see the area labeled as *M=1, skill-constrained* in Figure 1. Given the level of initial wealth z and schooling x, the level of entrepreneurial talent is sufficiently low to require capital and entrepreneurial labor allocations that do not exhaust the time constraint. As a result, they take a second job to maximize total income.[10] In Figure 1, it can be seen that individuals with relatively high wealth but low talent are likely to be of this type.

A second type of multiple occupations is illustrated as *M=1, credit-constrained* in Figure 1. It arises when an agent is of type (θ, z, x) and the following conditions apply:

$$\begin{aligned} (k^*, h^*) &= (\lambda z, \hat{h}(\theta)) \quad \text{if} \quad k_u(\theta) > \lambda z, \hat{h}(\theta) < T \\ &= (\lambda z, \hat{h}(\theta)) \quad \text{if} \quad B(z,x) < \theta < D(z,x) \end{aligned} .$$

In this case, given the level of initial wealth z and wage w, the level of entrepreneurial talent is high enough for the credit constraint to bind (talent θ is above the threshold $B(z,x)$). At the same time, the constrained-optimal level of equilibrium entrepreneurial labor does not exhaust the time constraint (talent θ is below the threshold $D(z,x)$). Consequently, individuals take a second job to maximize total income; those with low wealth and relatively low talent are likely to be in this category. For a given level of talent, whether an increase in wealth leads to specialization in business depends on whether talent θ is above or below the threshold $A(x)$. That is, when $\theta > A(x)$ (take $\theta = 150$ in Figure 1 as an example), an increase in wealth could lead a multiple occupation holder ($M = 1$) to specialize in business ($M = 0$). In contrast, if $\theta < A(x)$, any increase in wealth z does not lead to a switch to specialization. It only makes a credit-constrained multiple occupation holder into a skill-constrained multiple occupation holder.

Similarly, the model predicts two types of single-occupation business owners, depending on whether the business owner can invest the first-best level of capital or not. First, an agent of type (θ, z, x) such that $A(w) < \theta < C(z)$, finds her first-best labor choice in business is higher than the time endowment T while unconstrained by credit (see the area labeled *M=0, time constrained* in Figure 1). Therefore,

$$\begin{aligned} (k^*, h^*) &= (\hat{k}(\theta), T) \quad \text{if} \quad \hat{k}(\theta) < \lambda z, h_u(\theta) > T \\ &= (\hat{k}(\theta), T) \quad \text{if} \quad A(x) < \theta < C(z) \end{aligned} .$$

Finally, an agent with (θ, z, w) such that $\theta > \max\{D(z,w), C(z)\}$, finds her first-best labor choice

[10] Note that the "skill" here refers to entrepreneurial skill. Everything else equal, an individual could be skill-constrained because of a high wage w - higher w decreases the first-best entrepreneurial labor allocation h_u as specified in equation 2.

in business is higher than time endowment, in addition ot being credit-constrained (labeled as *M=0, credit-constrained* in Figure 1). That is,

$$\begin{aligned}(k^*, h^*) &= (\lambda z, T) \quad \text{if} \quad \hat{k}(\theta) > \lambda z, \hat{h}(\theta) > T \\ &= (\lambda z, T) \quad \text{if} \quad \theta > D(z,x), \theta > C(z)\end{aligned}.$$

A formal solution to the optimization problem defined in equation 1 is presented in Appendix 5. For simplicity, assume for now that there is no correlation between talent θ and the observables (z, x); I will relax this assumption when I estimate the model later. For a given level of schooling x, the model has the following implications:

(i). less entrepreneurially talented entrepreneurs are more likely to divide their time between their business and a second occupation

(ii). low-talent-high-asset entrepreneurs are likely to hold multiple occupations because they are skill-constrained

(iii). medium-talent-low-asset entrepreneurs are likely to hold multiple occupations because they are credit-constrained

(iv). high-talent-low-asset entrepreneurs are likely to specialize in business *and* be constrained by credit

(v). high-talent-high-asset entrepreneurs are likely to specialize in business *and* be constrained by the time endowment

Predicted probability of multiple occupations

In order to derive probabilities of multiple occupations predicted by the model, I assume a distribution for the unobserved heterogeneity in talent θ. Specifically, I follow the literature on structural estimation of occupational choice models (Paulson et al., 2006; Karaivanov, 2012 for example), and assume that talent is possibly correlated with initial wealth z and years of schooling x and that the error term is log-normally distributed.

$$\ln\theta = \delta_0 + \delta_1 \ln z + \delta_2 \ln(1+x) + \epsilon \tag{9}$$

$$\epsilon \sim N(0, \sigma)$$

For an agent with type (z, x), the probability of holding multiple occupations is equal to the probability that the equilibrium entrepreneurial labor allocation h^* is less than the time endowment T, which is, as discussed previously, the sum of the probability of being in skill-constrained multiple occupations and the probability of being in credit-constrained multiple occupations. The following proposition derives the probability of multiple occupations formally.

Proposition 2

Define M as the indicator for holding two occupations. The model predicts that the probability of holding two occupations, conditional on characteristics (z, x) and model parameters ψ is

$$P(M=1) \quad = \quad 1_{A>B}\Phi(b) + (1_{A<B})\Phi(a) + 1_{B<D}\{\Phi(d) - \Phi(b)\}$$

where $a \equiv (\ln A(x) - \bar{\theta})/\sigma$, $b \equiv (\ln B(z,x) - \bar{\theta})/\sigma$, $d \equiv (\ln D(z,x) - \bar{\theta})/\sigma$, and $\bar{\theta} = \delta_0 + \delta_1 \ln z + \delta_2 \ln(1+x)$. $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

Proof

$$\begin{aligned} P(M=1|z,x,\psi) &= P(h^* < T|z,x,\psi) \\ &= P(\theta < B(z,x), \theta < A(x)) + P(\theta > B(z,x), \theta < D(z,x)) \\ &= P(\theta < \min[(B(z,x), A(x)]) + P(B(z,x) < \theta < D(z,x)) \\ &= 1_{A>B}\Phi(b) + (1_{A<B})\Phi(a) + 1_{B<D}[\Phi(d) - \Phi(b)] \end{aligned}$$

□

Note that the probability that talent θ is jointly below the thresholds $B(z,x)$ and $A(x)$ gives the probability of skill-constrained multiple occupations, and the probability that θ is between the thresholds $B(z,x)$ and $D(z,x)$ gives the probability of credit-constrained multiple occupations. Appendix 5 derives model predictions for expected gross business income, conditional on multiple occupations and single occupation respectively; these will constitute additional moments that I use to structurally estimate the parameters of the model in Section 4.

3 Data

I use data from the Townsend Thai Project Initial Household Survey 2005 (Urban Area), a household survey of six provinces of urban Thailand.[11]

For simplicity, I focus on households that own a single business, and evaluate the occupational choice of the business owner. The indicator variable for multiple occupations, M, is equal to one if the individual has a secondary occupation. The proxy for z is total asset of the household five years prior to the survey year, the proxy for x is years of schooling of the business owner, and $q^E = \theta k^\alpha$ is measured by business income. I exclude households in the top percentile of wealth and business income. Key variables are summarized in Table 1. About 20% of business owners in the sample hold a secondary occupation, and are therefore defined as multiple occupation holders. They have significantly higher initial wealth z and higher years of schooling on average x, but have lower average business income (and average total gross income) than those with a single occupation.[12]

Table 2 reports the coefficient estimates respectively from probit and linear probability regressions of M (indicator for holding multiple occupations) on initial wealth z and its square, years of schooling x of the business owner and its square, the business owner's gender and age, and household size. The estimates indicate that initial wealth increases the probability of holding multiple occupations (although at a decreasing rate). The effect of initial wealth z on the probability of multiple occupations is theoretically ambiguous according to the model in Section 2. For a given level of talent θ and schooling x, the probability of credit-constrained multiple occupations is higher for relatively poorer households, whereas for the same level of talent θ and schooling x, the probability of skill-constrained multiple occupations is higher for relatively richer households (see Figure 1). With the observed positive correlation between multiple occu-

[11] The survey picked 16 communities in municipal areas of each *amphoe* or county under the ongoing Townsend's project (Rural Survey), totaling 96 communities overall. From each community, the survey randomly selected households who are present in the community-fund list (those that applied to Government Housing Bank or Bank of Agriculture and Agricultural Cooperatives). Each community fund must have no less than 95% of all households in their lists. More detail on the Townsend Thai Project can be found at http://cier.uchicago.edu/.

[12] Multiple occupation business owners are more likely to be male, but there is no statistically significant difference in average age of the principle earner and average household size between the two groups.

Table 1: Summary statistics

	Multiple occ.	Single occ.
wealth 5 years ago (000's)	723.0*	554.5*
	(722.5)	(735.8)
	[499.9]	[306.7]
business income (gross)	208.8*	317.1*
	(298.9)	(437.7)
	[108.0]	[156.6]
total earned income (gross)	320.9*	399.0*
	(334.2)	(439.9)
	[216.0]	[261.8]
years of schooling	8.0*	7.2*
	(4.4)	(4.0)
household size	4.3	4.3
	(1.6)	(2.0)
age	48.3	49.4
	(11.0)	(11.0)
gender (male)	0.6*	0.4*
	(0.5)	(0.5)
sample size	145	566
proportion	79.6%	20.4%

Mean, standard deviation (in parentheses) median (in brackets).

* indicates difference-in-means test is significant at 5% level.

Table 2: Reduced-form regressions of multiple occupations
Dependent variable: indicator variable for multiple occ. (M)

	Probit	Linear prob.
wealth 5 years ago (mln)	0.741***	0.207***
	(0.199)	(0.053)
wealth squared (mln)	-0.190***	-0.052***
	(0.066)	(0.016)
years of schooling	-0.118***	-0.033***
	(0.058)	(0.017)
schooling squared	0.007**	0.002***
	(0.003)	(0.001)
gender (male)	0.392***	0.108***
	(0.109)	(0.030)
age	-0.010*	-0.003*
	(0.006)	(0.002)
household size	-0.007	-0.002
	(0.027)	(0.007)
intercept	-0.432	0.313***
	(0.422)	(0.116)
sample size	711	711

$*p < 0.10, **p < 0.05, ***, p < 0.01$

Table 3: Types of businesses among multiple occupation holders

	trader	service	producer	livestock	overall
% of sample	59.9%	32.5%	3.4%	4.2%	100%
% with multiple occupation	18.8%	14.3%	41.7%	73.3%	20.4%
wealth 5 years ago (000s)	552.7	627.5	510.5	866.8	588.8
gross business income (000s)	366.1	197.2	339.9	122.1	300.0
years of schooling	7.1	7.6	9.6	7.3	7.4
gender (male indicator)	0.4	0.5	0.8	0.8	0.4
age	49.6	48.2	47.1	53.5	49.2
household size	4.4	4.0	4.2	4.4	4.3

Table 4: Types of non-business occupations among multiple occupation holders

Second occupation	farmer	wage - profes.	wage - other	other	*overall*	$M = 0$
% of $M = 1$	55.9%	11.7%	18.6%	13.8%	*100%*	-
wealth 5 years ago (000s)	719.3	1263.8	397.2	717.9	*723.0*	*544.5*
business income (000s)	198.9	272.8	143.6	282.3	*208.8*	*317.1*
years of schooling	6.4	13.5	7.7	10.2	*8.0*	*7.2*
gender (male indicator)	0.6	0.6	0.5	0.4	*48.3*	*49.4*
age	50.6	50.4	43.9	43.3	*0.6*	*0.4*
household size	4.5	4.2	4.0	4.0	*4.3*	*4.3*

pations and initial wealth, the model will likely map the multiple occupation holders into the skill-constrained type.

The regression results also indicate that years of schooling x is associated with lower probability of multiple occupations for lower years of schooling, and higher probability for higher years of schooling.[13] According to the model in Section 2, the effect of x on the probability of multiple occupations is positive for a given level of initial wealth z and talent θ; the opportunity cost of allocating an additional hour in business is higher for individuals with more years of schooling. However, this relationship could become ambiguous when we allow for a correlation between entrepreneurial talent and schooling.

Other variables that significantly affect the probability of holding multiple jobs include gender (being a male business owner is associated with a higher probability of multiple jobs) and age (older business owners are less likely to have multiple jobs). Household size does not significantly affect the probability of having multiple jobs.

Next, I look at the businesses and second occupations of the business owners in the sample, and summarize the key variables for each group. Table 3 shows that about 60% of the business owners are traders[14], 32.5% are involved in small services (tailor, laundry, restaurant or noodle

[13] For example, from the linear probability regression, the estimates indicate that the marginal effect of an additional year of schooling on the dependent variable is negative for years of schooling lower than 9.5, and positive otherwise. Values for years of schooling range from zero to seventeen.

[14] Traders refer to those who declare themselves as one or are involved in buy-sale of goods under the category "other" as their occupation.

shop, repair shop, rental taxis etc), 3% are small producers (furniture makers etc., including fish farmers) and 4% raise livestock. Given that the majority of business owners are traders, the average business owner is most representative of this group. Those in services are the least likely to hold a second job, while small producers and livestock raisers are the most likely to have two occupations.

Table 4 reports the types of second occupations held by business owners in the sample. The majority of business owners with a second occupation are involved in farming (56%).[15] Among the 30% who are wage-earners, those working in professional occupations (government employee, teacher etc.) are considerably richer and have more years of schooling on average compared to any other group, and particularly compared to non-professional wage-workers.[16]

4 Structural Estimation

4.1 GMM - matched moments and computation

I have a sample of N households, $i = 1, ..., N$ for whom initial wealth z_i, years of schooling x_i, and multiple occupation status M_i are observed in the data. The parameters of the model include the entrepreneurial technology parameters (α, β) and the non-business income parameters (μ, γ), the credit constraint parameter (λ), as well as the distributional parameters of talent θ. I fix the interest rate parameter r at 1.06, which corresponds to the median level of interest charged for loans in the data and the total time endowment T is normalized to 1. Denote by ϕ the set of remaining nine parameters: $\phi = \{\alpha, \beta, \gamma, \mu, \lambda, \delta_0, \delta_1, \delta_2, \sigma\}$.

I estimate ϕ by minimizing the sum of squared percentage deviations between nine model predicted moments and their sample analogs, as listed in Table 5. The first moment is the proportion of multiple occupation overall, with the next six moments matching the proportion of multiple occupation holders for different subsamples (whether z belongs to one of the three quartiles and whether x belongs to one of the three quartiles, defined in Table 5).[17] The remaining two moments are the expected gross income from entrepreneurship, conditional on $M = 1$, and conditional on $M = 0$. The model predictions for these moments are derived in propositions 2, 3 and 4.

Given parameters ϕ, denote the model-predicted moments in the second column of Table 5 by $h_j(z, x, \phi)$, their sample analogs in the third column by h_j^d, and the percentage deviation between the two as $q_j(z, x, \phi) \equiv \frac{h_j(z,x,\phi)-h_j^d}{h_j^d}$, $j = 1, ...J$. Finally, denote by $\mathbf{q}(z, x, \phi)$ the $J \times 1$ vector of $q_j' s$. The GMM estimates are the solution to minimizing $\mathbf{q}(z, x, \phi)'\mathbf{q}(z, x, \phi)$ over ϕ.

Recall that M is measured as the indicator for whether the business owners report a secondary occupation ($M = 1$) or not ($M = 0$). Since I only look at households with a single business, business income q^E is measured as business income reported by the household. Initial wealth z is measured in the data as the total household asset held by the household five years prior

[15] Even if the second job is seasonal, the model in this paper allows for that as long as engaging in it diverts time away from entrepreneurship. Holding seasonal jobs could therefore indicate that the individual is insufficiently skilled to run a business that occupies her full time.

[16] The final group of multiple occupation holder, "other", is made up of individuals engaged in piece-rate work. This includes 4 individuals involved in land rentals as their second occupation.

[17] The proportion of multiple occupation holders for z in the last quartile is a linear combination of the first four moments, and is therefore omitted. The proportion of multiple occupation holders for the fourth quartile of schooling is omitted for the same reason.

Table 5: List of matched moments

Moment	Model	Sample analog
1. average probability of multiple occ	$\frac{1}{N}\sum_{i=1}^{N} P(M_i = 1 \mid z_i, x_i, \phi)$	$\frac{1}{N}\sum_{i=1}^{N} M_i$
2. prob. of multiple occ, $z \leq z_{25}$	$\frac{\sum^N 1_{z_i \leq z_{25}} P(M_i=1 \mid z_i, x_i, \phi)}{\sum^N 1_{z_i \leq z_{25}}}$	$\frac{\sum^N 1_{z_i \leq z_{25}} M_i}{\sum^N 1_{z_i \leq z_{25}}}$
3. prob. of multiple occ, $z \in (z_{25}, z_{50}]$	$\frac{\sum^N 1_{z_i \in (z_{25}, z_{50}]} P(M_i=1 \mid z_i, x_i, \phi)}{\sum^N 1_{z_i \in (z_{25}, z_{50}]}}$	$\frac{\sum^N 1_{z_i \in (z_{25}, z_{50}]} M_i}{\sum^N 1_{z_i \in (z_{25}, z_{50}]}}$
4. prob. of multiple occ, $z \in (z_{50}, z_{75}]$	$\frac{\sum^N 1_{z_i \in (z_{50}, z_{75}]} P(M_i=1 \mid z_i, x_i, \phi)}{\sum^N 1_{z_i \in (z_{50}, z_{75}]}}$	$\frac{\sum^N 1_{z_i \in (z_{50}, z_{75}]} M_i}{\sum^N 1_{z_i \in (z_{50}, z_{75}]}}$
6. prob. of multiple occ, $x \leq x_{25}$	$\frac{\sum^N 1_{x_i \leq x_{25}} P(M_i=1 \mid z_i, x_i, \phi)}{\sum^N 1_{x_i \leq x_{25}}}$	$\frac{\sum^N 1_{x_i \leq x_{25}} M_i}{\sum^N 1_{x_i \leq x_{25}}}$
7. prob. of multiple occ, $x \in (x_{25}, x_{50}]$	$\frac{\sum^N 1_{x_i \in (x_{25}, x_{50}]} P(M_i=1 \mid z_i, x_i, \phi)}{\sum 1_{x_i \in (x_{25}, x_{50}]}}$	$\frac{\sum 1_{x_i \in (x_{25}, x_{50}]} M_i}{\sum 1_{x_i \in (x_{25}, x_{50}]}}$
8. prob. of multiple occ, $x \in (x_{50}, x_{75}]$	$\frac{\sum 1_{x_i \in (x_{25}, x_{50}]} P(M_i=1 \mid z_i, x_i, \phi)}{\sum 1_{x_i \in (x_{25}, x_{50}]}}$	$\frac{\sum 1_{x_i \in (x_{25}, x_{50}]} M_i}{\sum 1_{x_i \in (x_{25}, x_{50}]}}$
9. average entrep. income q^E, $M = 1$	$\frac{\sum_{i=1}^N E(q_i^E \mid M_i=1, z_i, x_i, \phi) P(M_i=1 \mid z_i, x_i, \phi)}{\sum_{i=1}^N P(M_i=1 \mid z_i x_i, \phi)}$	$\frac{\sum_{i=1}^N q_i^E M_i}{\sum_{i=1}^N M_i}$
10. average entrep. income q^E, $M = 0$	$\frac{\sum_{i=1}^N E(q_i^E \mid M_i=0, z_i, x_i, \phi) P(M_i=0 \mid z_i, x_i, \phi)}{\sum_{i=1}^N P(M_i=0 \mid z_i x_i, \phi)}$	$\frac{\sum_{i=1}^N q_i^E (1-M_i)}{\sum_{i=1}^N (1-M_i)}$

M is the indicator variable for multiple occupations.

z is initial wealth, z_j denotes the j^{th} percentile of z.

x is years of schooling of business owners, x_j denotes the j^{th} percentile of x.

Entrepreneurial income refers to the gross output, $q^E = \theta k^\alpha h^\beta$.

to the survey in the form of land, household durables and agricultural assets. Finally, years of schooling x is measured as the years of schooling of the business owner. The estimation sample excludes the top percentile of households in terms of initial wealth and business income to reduce the impact of outliers.

4.2 Estimates and model fit

The structural estimates are reported in Table 6. The estimates of the entrepreneurial income parameters, α and β, imply that a 10% increase in capital k or a 10% increase in entrepreneurial labor h, would respectively lead to approximately 1% and 4% increase in the entrepreneurial income of unconstrained entrepreneurs, all else equal. The credit constraint parameter, λ is estimated to be 0.17, implying that households can invest up to 17% of their initial wealth as capital. At the median wealth, this implies that capital k should be less than 57 thousand baht.

The conditional mean and standard deviation of log entrepreneurial talent is estimated to be 5.6 and 0.2 respectively. Entrepreneurial talent θ is estimated to be negatively correlated with initial wealth z, with an elasticity of -0.08, and positively correlated with schooling x, with an elasticity of 0.1.[18] The estimate of μ implies that for a business owner with zero years of schooling, income from an alternative non-business source would be about 77 thousand baht if all of the labor was allocated to the alternative occupation. The elasticity of this income with respect to schooling, γ, is estimated to be about 0.1.

Table 6 reports the value of the GMM criterion function, following the percentage deviations of each of the nine moments used to estimate the structural parameters. The model is able to reproduce the proportion of multiple occupation business owners closely, within 3% of the observed proportion. It is also able to match the proportion with multiple occupations for the first two quartiles of initial wealth within 5% of the observed analogs. The multiple occupation proportion for the subsample with wealth in the third quintiles are underestimated by 21%, with a 5.6 percentage points difference; this is the largest deviation among all targeted moments. The proportions of multiple occupation for the various subsamples based on quartiles of schooling are estimated within 10% (or within two percentage points difference) of their observed counterparts. The model is able to match very closely the average business incomes of multiple occupation holders, and the average non-business incomes of single occupation holders, within 3% of the observed averages.

Figure 2 plots local polynomial fits of observed and predicted probabilities of multiple occupation at the GMM estimates by percentiles of initial wealth, and the entire range of years of schooling, both of which are not matched directly in estimating the parameters. In both graphs, the predicted fits are within the 95% confidence intervals of the observed fits, except in the case at the very bottom percentiles and the top decile of initial wealth. The figure also shows that evaluated at the GMM estimates, the model predicts that the probability of multiple occupations is decreasing at first and then increasing in initial wealth z. This is because, as Figure 3 shows, the probability of skill-constrained multiple occupations is increasing in initial wealth, however the probability of credit-constrained multiple occupations is decreasing in initial wealth.

[18] Evan and Jovanovic (1989) interprets the parameter δ_1 by stating that "[it] may reflect greater past savings by those high-θ people, who, knowing their θ, expected to become entrepreneurs one day. Or, if we stretch the interpretation a bit, it may reflect lower absolute risk aversion of wealthy people, making them more inclined to become entrepreneurs ..." (p816). They estimate δ_1 to be negative and statistically significant for their sample of male workers in the United States, suggesting that high-asset people tend to be relatively poor entrepreneurs.

Table 6: Structural estimates and model fit
Structural estimates

Parameter		Estimate	s.e.
return to capital in y^E	α	0.1083	(0.0139)
return to labor in y^E	β	0.3817	(0.0410)
credit constraint	λ	0.1701	(0.2527)
talent - intercept	δ_0	5.6110	(0.5639)
talent - correlation with wealth	δ_1	-0.0848	(0.0087)
talent - correlation with schooling	δ_2	0.1261	(0.0128)
talent - standard deviation	σ	0.1861	(0.0444)
alternative income parameter	μ	76.94	(9.81)
return to education in y^W	γ	0.1184	(0.0439)

Standard errors are calculated from 99 bootstrap samples.

Model fit

Moment	Predicted	Observed	% Deviation
probability of multiple occ, $M = 1$	0.2106	0.2039	2.37
prob. of $M = 1$, $z \leq z_{25}$	0.1291	0.1348	-4.28
prob. of $M = 1$, $z \in (z_{25}, z_{50}]$	0.1285	0.1236	3.95
prob. of $M = 1$, $z \in (z_{50}, z_{75}]$	0.2147	0.2712	-20.8
prob. of $M = 1$, $x \leq x_{25}$	0.2062	0.1900	8.51
prob. of $M = 1$, $x \in (x_{25}, x_{50}]$	0.1847	0.1972	-6.32
prob. of $M = 1$, $x \in (x_{50}, x_{75}]$	0.2132	0.1946	9.56
average entrep. income q^E, $M = 1$	209.3	213.0	-1.72
average entrep. income q^E, $M = 0$	332.0	322.3	3.00
sum of squared % deviations (GMM criterion function)			0.0694

M is the indicator variable for multiple occupations.

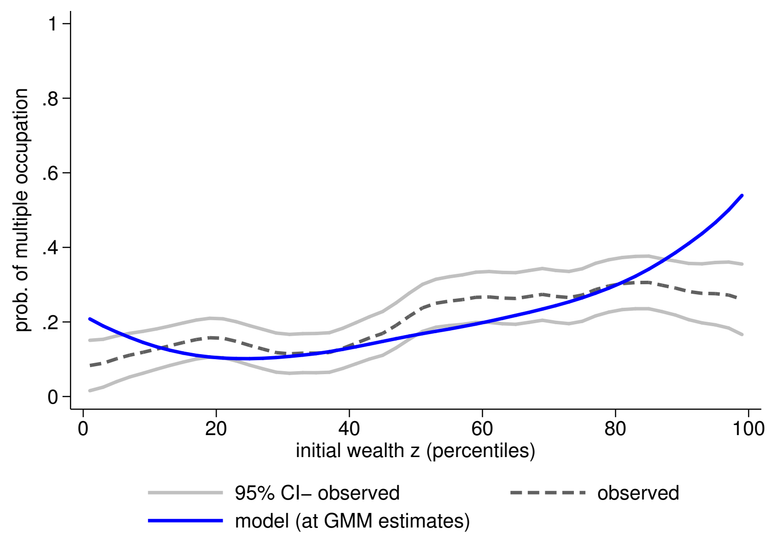
z is initial wealth, z_j denotes the j^{th} percentile of z.

x is years of schooling of business owners, x_j denotes the j^{th} percentile of x.

Entrepreneurial income refers to the gross output, $q^E = \theta k^\alpha h^\beta$.

Figure 2: Model fit - unmatched moments

Local polynomial fit by initial wealth



Local polynomial fit by schooling

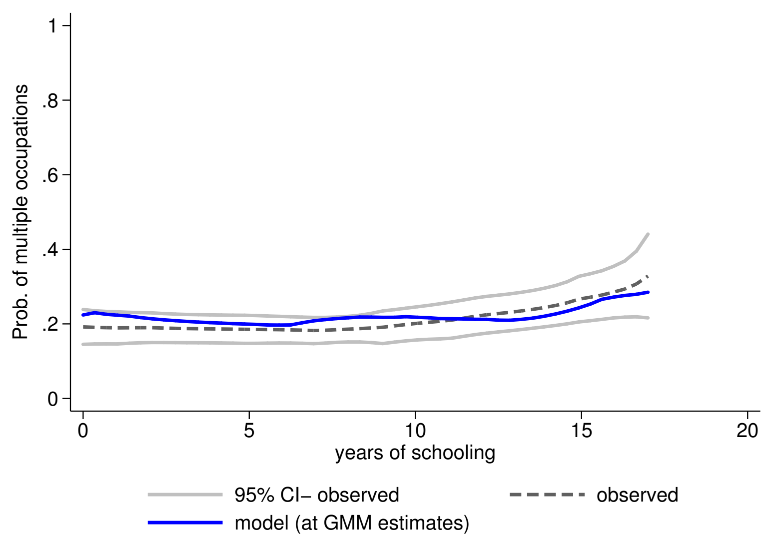
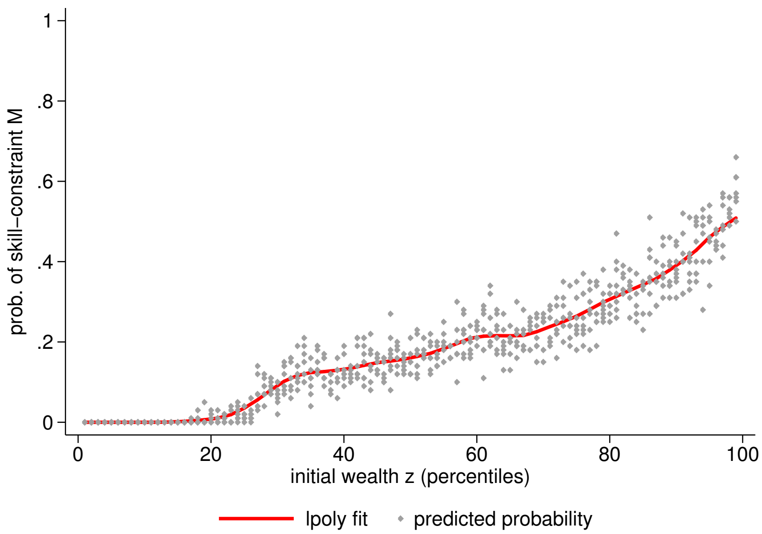
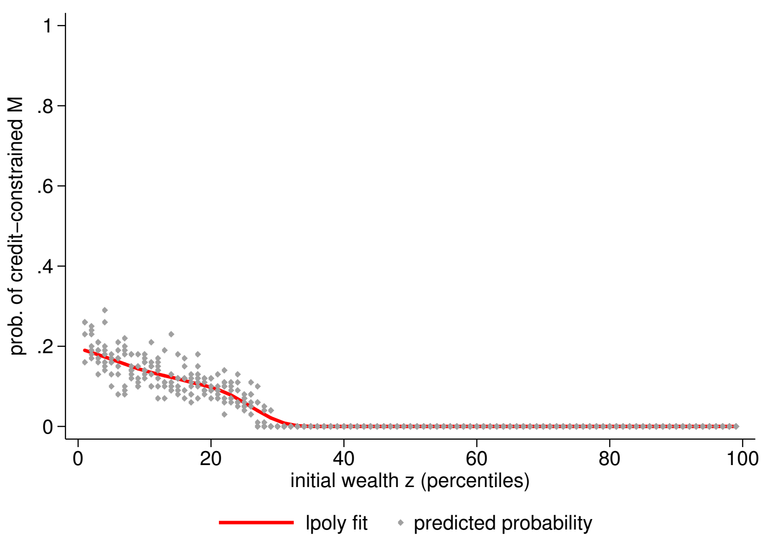


Figure 3: Predicted probability of multiple occupation as function of wealth

Probability of skill-constrained multiple occupations



Probability of credit-constrained multiple occupations



For a given level of entrepreneurial talent θ (and schooling x), an agent with higher initial wealth is more likely to be skill-constrained if talent is below a certain threshold, as shown in Figure 1. In addition, part of the positive correlation between initial wealth and the predicted probability of skill-constrained multiple occupations can be attributed to the negative estimated correlation between θ and initial wealth z (as the estimated value of δ_1). The negative correlation between initial wealth and the predicted probability of credit-constrained multiple occupations is straightforward - poorer households are more likely to have binding credit constraints.

4.3 Analysis

By estimating the magnitude of each type of multiple occupation holding, the goal of the paper is to understand the causes of multiple occupations and analyze the implications for policy. The predicted proportions for various types of entrepreneurs are reported in Table 7. These statistics are calculated by simulating the model at the GMM estimates from Table 6, drawing at random 100 values for the shock ε for each observation $i = 1, ...N$.

The predicted proportion of business owners with multiple occupations is 21%, of which 85% (or 18% of the sample) are able to allocate the first-best levels of capital (k_u) and labor (h_u), as these are respectively lower than the investment limit (λz) and the total time endowment. The remaining 15% (or 3% of the sample) holds a second job while being credit-constrained, and their constrained-optimum labor allocation does not exhaust the time endowment. In total, about 33% of business owners are predicted to be credit-constrained, however, only a few of them engage in multiple occupations.

Most of the sample, 79%, is predicted to specialize in business ownership, of which 38% (or 30% of the sample) specialize in business ownership *and* are credit-constrained. Relaxing the credit constraint therefore will increase their incomes. The remaining 62% (or 49% of the sample) specialize in business because the first-best level of their entrepreneurial labor h_u is greater than the time endowment, and the corresponding constrained-optimal level of capital is lower than the investment limit λz. If they could relax the time constraint to afford h_u (for example by hiring workers), they could potentially find themselves credit-constrained as their desired level of capital would also increase. Otherwise, they would not benefit from a relaxation of the credit limit.

Figure 7 summarizes the characteristics of each type of entrepreneur. Those with multiple occupations are predicted to be less entrepreneurially talented on average, have higher average initial wealth, and higher average years of schooling than those with a single occupation (the latter two points are true in the observed data as well). However, within each group, there is a small fraction that is considerably different from the average. The credit-constrained multiple occupation holders are more talented on average, have much lower median income and lower years of schooling compared to the average multiple occupation entrepreneur. Among single occupation entrepreneurs, those that are credit-constrained are also more talented on average, poorer and have lower years of schooling on average. Note that the differences within multiple occupation holders (or single occupation holders) in terms of average initial wealth and years of schooling, in addition to entrepreneurial talent, are unobserved in the data.

Table 7: Model predictions at GMM estimates

Estimated types of entrepreneurs

Model statistic	Predicted value
Multiple occupation ($M = 1$)	
total	21.0%
skill-constrained, $P(k_u < \lambda z, h_u < T)$	17.8%
credit-constrained, $P(k_u > \lambda z, \hat{h} < T)$	3.2%
Single occupation ($M = 0$)	
total	79.0%
time-constrained, $P(\hat{k} < \lambda z, h_u > T)$	48.8%
credit-constrained, $P(\hat{k} > \lambda z, \hat{h} > T)$	30.2%

Characteristics of entrepreneurs by predicted type

	average θ	average z	average x
Multiple occupation ($M = 1$)			
total	166.2	941.6	7.57
skill-constrained	161.4	1099.3	7.82
credit-constrained	193.6	51.3	6.17
Single occupation ($M = 0$)			
total	239.3	494.9	7.35
time-constrained	223.1	744.9	7.84
credit-constrained	265.4	92.5	6.56

Table 8: Counterfactual analysis

Model predicted averages	baseline	first-best k	$\uparrow \theta$ by 10%
prob. of $M = 1$	21.0%	18.9%	9.9%
$M = 1$, skill-constrained	17.8%	18.9%	8.7%
$M = 1$, credit-constrained	3.2%	0%	1.3%
$M = 0$, time-constrained	48.8%	81.1%	55.1%
$M = 0$, credit-constrained	30.2%	0%	34.9%
average gain in income*			
$z \in$ 1st quintile	-	15.6%	10.1%
$z \in$ 2nd quintile	-	0.7%	10.6%
$z \in$ 3rd quintile	-	0%	10.4%
$z \in$ 4th quintile	-	0%	9.8%
$z \in$ 5th quintile	-	0%	7.9%
whole sample	-	3.3%	9.8%

M= indicator for multiple occupations; z = initial wealth

*Income is calculated as $\arg\max_{k,h} \theta k^\alpha h^\beta - rk + w(T-h) + rz - z$, and income gain is defined as the percentage increase from the baseline.

4.3.1 Counterfactual analysis

I next analyze the effect of two counterfactual scenarios - relaxing the credit constraint and increasing entrepreneurial productivity - on the proportion of multiple occupation holders and income to further illustrate the implications of the estimated model. The second column in Table 8 reports the baseline statistics calculated at the GMM estimates, and the remaining columns report the corresponding statistics under the counterfactuals.

In the first counterfactual policy, I assume that the credit-constraint parameter λ is high enough for everyone to be able to invest the first-best level of capital $k_u(\theta, r)$, such that the credit constraint is virtually eliminated for the given level of entrepreneurial talent and opportunity cost of funds r.[19] An increase in λ can be interpreted as lower collateral requirements due to improvements in contract enforcement.[20] The third column Table 8 shows that when the first-best level of capital is invested, the probability of multiple occupations falls by only about 2 percentage points from 21% to 18.9%. This reflects the fact that relaxing the credit constraint does not affect the 17.8% of skill-constrained multiple occupation holders in the baseline. Some of the previously credit-constrained multiple occupation holders now become skill-constrained, as the first-best labor allocation that complements with the first-best capital does not exhaust the time endowment. By definition, the probability of credit-constrained single occupation goes to zero from 30%.

The gain in income from eliminating the credit constraint is estimated to be about 3% on average.[21] However, Figure 4 shows that there is considerable heterogeneity in how this policy

[19] I achieve this by multiplying the baseline λ by a large number - in this case, by 1000.

[20] Credit injection through microcredit loans for example would have the same effect of increasing equilibrium capital, with the difference being that access to credit is not tied to initial wealth.

[21] Total income is calculated as $\arg\max_{k.h} \theta k^\alpha h^\beta - rk + w(T-h) + rz - z$.

Figure 4: Percentage gain in income - eliminating the credit constraint

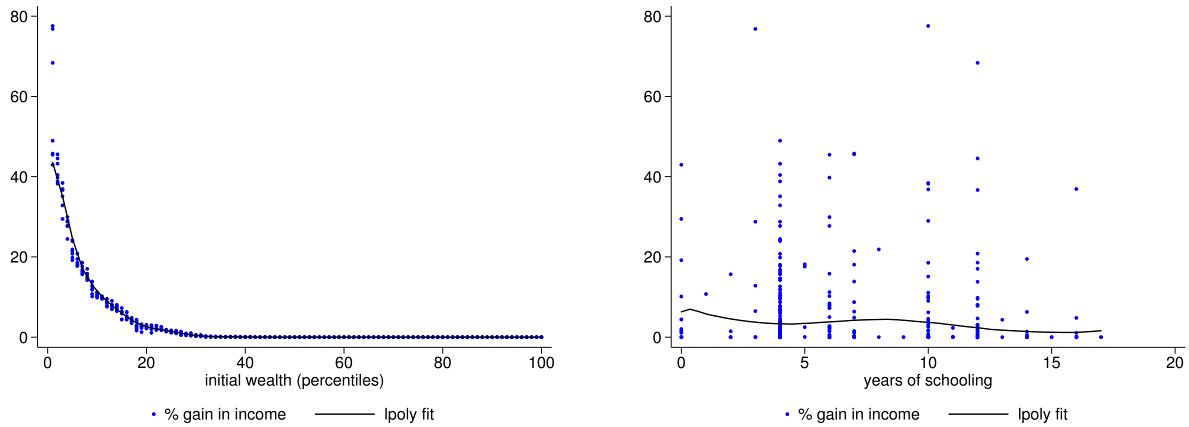
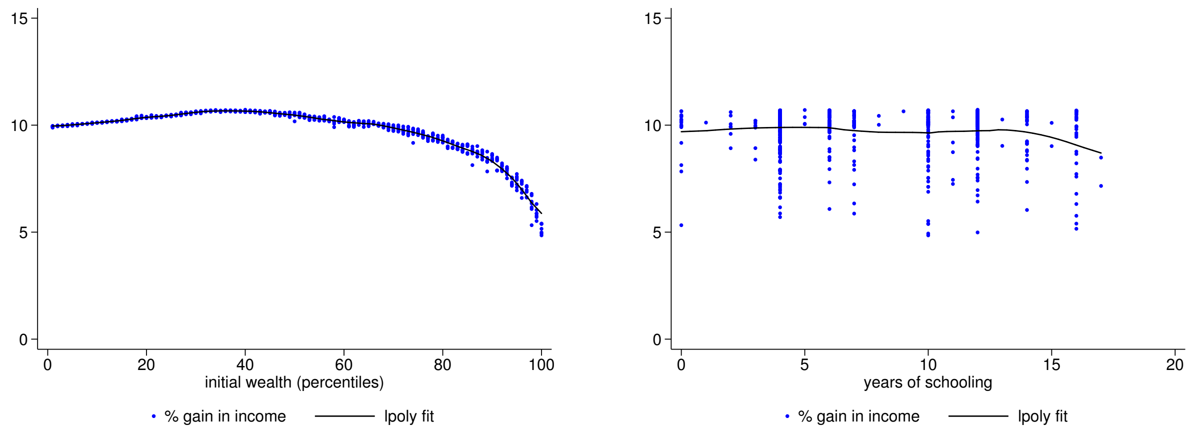


Figure 5: Percentage gain in income - increasing entrepreneurial productivity ($\uparrow \theta$ by 10%)



affects individuals along the wealth distribution. Those in the first quintile of wealth increase their income by 16% on average. However, the effect of the remainder of the sample is predicted to be almost zero. The implication is that only individuals in the first quintile of initial wealth are predicted to be credit-constrained.

Figure 4 shows that the effect of eliminating the credit constraint is slightly lower for those with higher years of schooling. Intuitively, the opportunity cost of entrepreneurial labor is higher for individuals with higher years of schooling (through higher wages w). As a result, the complementary first-best capital k_u is lower, and for a given level of wealth z, the probability that $k_u \leq \lambda z$ is also lower – implying that everything else equal, individuals with higher x are less likely to be credit-constrained.[22] Schooling also affects entrepreneurial talent θ positively through δ_2, making individuals with higher years of schooling more likely to be credit-constrained. However, the direct effect of higher opportunity cost of entrepreneurial labor dominates on average.

The next counterfactual I consider is a sample-wide 10% increase in entrepreneurial talent θ. This policy could be interpreted as a business training program that increases the marginal

[22]This is reflected in the fact that $B(z, x)$ is increasing in x.

productivity of capital and labor inputs. It could also be interpreted as an exercise where the baseline is compared with a more productive economy. The results of a 10% increase in θ are reported in the last column of Table 8. The probability of multiple occupations falls from 21% to 10% - a higher θ increases equilibrium entrepreneurial labor such that specializing in business is now profitable. In the baseline, 33.4% of the sample are credit-constrained among both single and multiple occupation holders. With the 10% increase in entrepreneurial productivity, the proportion that is predicted to be credit-constrained increases slightly to 36.2%.

The average gain in income is estimated to be about 10% for the whole sample. Figure 5 shows that the income gains from increasing θ is relatively homogeneous across wealth and schooling ranging from 5% to 11% (unlike in the case of relaxing the credit constraint where income gains ranged from 0% to 80%). Since everyone in the sample is an entrepreneur, a higher θ affects everyone's income regardless of a switch in occupational choice. Within that, the gain in income for households with higher wealth are smaller as a percentage of their baseline income.

In a nutshell, the GMM estimates suggest that although a significant proportion of business owners are credit-constrained, relaxing the credit constraint would neither decrease the practice of multiple occupations by much, nor increase income in a significant manner for the average household. Completely eliminating the credit constraint is however predicted to increase the incomes of the poorest households by a significant amount. Given that most of the multiple occupation holders are predicted to be skill-constrained, increasing entrepreneurial productivity would be an effective policy to increase income.

5 Conclusions

The lack of specialization is a salient feature of occupational choice in developing countries. This paper examines the role of financial constraints and entrepreneurial skill in explaining the presence of multiple occupations, using survey data on household heads in semi-urban areas in Thailand. Structural estimation of this model is necessary to clarify why we observe this phenomenon, which in turn is crucial when discussing the role for policy. For example, if it turns out that individuals do not specialize in their occupations due to credit constraints, policies that improve access to credit might enhance welfare by allowing households to maximize income. However, if it turns out that individuals do so in response to underdeveloped labor market (leading to irregular, seasonal or contractually limited paid employment), in conjunction with being particularly ill-suited for entrepreneurship, encouraging further specialization in their businesses might be ineffective in raising incomes, or equivalently, in reducing the holding of multiple jobs.

Using data from the Townsend Thai Project's urban survey of 2005, I structurally estimate a model that predicts more than one occupation for individuals whose equilibrium entrepreneurial labor allocation does not exhaust the time constraint. I find that 85% of the business owners with multiple occupations (or 18% of the sample) are skill-constrained. They hold two jobs because given their level of entrepreneurial talent, the optimal entrepreneurial labor does not fully occupy them even though they invest the first-best level of capital. While this type of multiple occupation holding is not a sign of credit market inefficiency, it could still be indicative of underdeveloped labor markets that are unable to employ low-talent entrepreneurs in full-time occupations. The remaining 15% of multiple occupation holders (or 3.2% of the sample) are credit-constrained. A key implication of the estimated magnitudes of the two types of multiple occupation holders is that relaxing the credit constraint would not significantly decrease the practice of multiple occupations. For instance, I estimate that eliminating the credit-constraint only enables 2.1% of the sample to specialize in business ownership. These findings are also consistent with the idea that many entrepreneurs in developing countries are skill-constrained, and take up business ownership as a supplemental income source.

This paper can be extended in at least two important ways. First, the entrepreneurial production function can be altered to allow for the possibility of exclusive wage-workers.[23] Second, the framework can also be extended to include risk aversion, utility from leisure, or disutility from variance in income to study diversification motive behind the holding of multiple jobs.

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Appendix

Proposition 1

The solution to the optimization defined in 1 is

$$h^* = \left\{ \begin{array}{c} h_u \text{ if } (k_u < \lambda z \;\&\; h_u < T) \Leftrightarrow (\theta < B(z,w) \;\&\; \theta < A(w)) \\ \hat{h} \text{ if } (k_u > \lambda z \;\&\; \hat{h} < T) \Leftrightarrow (\theta > B(z,w) \;\&\; \theta < D(z,w)) \\ T \text{ if } \left\{ \begin{array}{c} (\hat{k} < \lambda z \;\&\; h_u > T) \Leftrightarrow (\theta < C(z) \;\&\; \theta > A(w)) \\ (\hat{k} > \lambda z \;\&\; \hat{h} > T) \Leftrightarrow (\theta > D(z,w) \;\&\; \theta > C(z)) \end{array} \right\} \end{array} \right\} \tag{10}$$

$$k^* = \left\{ \begin{array}{c} k_u \text{ if } (k_u < \lambda z \;\&\; h_u < T) \Leftrightarrow (\theta < B(z,w) \;\&\; \theta < A(w)) \\ \hat{k} \text{ if } (\hat{k} < \lambda z \;\&\; h_u > T) \Leftrightarrow (\theta < C(z) \;\&\; \theta > A(w)) \\ \lambda z \text{ if } \left\{ \begin{array}{c} (k_u > \lambda z \;\&\; \hat{h} < T) \Leftrightarrow (\theta > B(z,w) \;\&\; \theta < D(z,w)) \\ (\hat{k} > \lambda z \;\&\; \hat{h} > T) \Leftrightarrow (\theta > D(z,w) \;\&\; \theta > C(z)) \end{array} \right\} \end{array} \right\} \tag{11}$$

Proof

The Lagrangian for the problem is

$$\max_{h_b,k,\lambda_j} L(h_b,k) = w(T-h) + \theta k^\alpha h^\beta + r(z-k) + \lambda_1 k + \lambda_2 h + \lambda_3[\lambda z - k] + \lambda_4[T-h]$$

The Kuhn-Tucker conditions:

$$k \geq 0, \lambda_1 \geq 0, \lambda_1 k = 0$$

$$h \geq 0, \lambda_2 \geq 0, \lambda_2 h = 0$$

$$\lambda z - k \geq 0, \lambda_3 \geq 0, \lambda_3[\lambda z - k] = 0$$

$$T - h \geq 0, \lambda_4 \geq 0, \lambda_4[T-h] = 0$$

$$\frac{dL}{dh} = -w + \theta k^\alpha \beta h^{\beta-1} + \lambda_2 - \lambda_4 = 0$$

$$\frac{dL}{dk} = \theta \alpha k^{\alpha-1} h^\beta - r + \lambda_1 - \lambda_3 = 0$$

Case 1:

Suppose k and h are interior solutions. This means that $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$ as none of the constraints bind. At the optimum, marginal product is equal to marginal cost of each input.

$$\theta \alpha k^{\alpha-1} h^\beta = r$$

$$\theta k^\alpha \beta h^{\beta-1} = w$$

Let $y = k^\alpha h^\beta$.

$$\implies \theta\alpha\frac{y}{k} = r, \quad \theta\beta\frac{y}{h} = w$$

$$\implies k^* = \frac{\theta\alpha}{r}y, \quad h^* = \frac{\theta\beta}{w}y$$

At the optimum,

$$y^* = \left(\frac{\theta\alpha}{r}y^*\right)^\alpha \left(\frac{\theta\beta}{w}y^*\right)^\beta$$

$$\implies y^{*1-\alpha-\beta} = (\frac{\theta\alpha}{r})^\alpha(\frac{\theta\beta}{w})^\beta$$

$$\implies y^* = (\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}$$

Capital investment is

$$k^* = \frac{\theta\alpha}{r}(\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}} = (\frac{\theta\alpha}{r})^{\frac{1-\alpha-\beta+\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}$$

$$\implies k^* = (\frac{\theta\alpha}{r})^{\frac{1-\beta}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}} \equiv k_u$$

Entrepreneurial labor is

$$h^* = \frac{\theta\beta}{w}(\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}} = (\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{1-\alpha-\beta+\beta}{1-\alpha-\beta}}$$

$$\implies h^* = (\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{1-\alpha}{1-\alpha-\beta}} \equiv h_u$$

k^* and h^* are guaranteed to be positive if θ, α, β and r are positive. To be feasible, k^* must be less than λz and $h^* < T$.

$$h_u \leq T$$

$$(\frac{\theta\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{1-\alpha}{1-\alpha-\beta}} \leq T$$

$$\theta^{\frac{\alpha+1-\alpha}{1-\alpha-\beta}}(\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{\beta}{w})^{\frac{1-\alpha}{1-\alpha-\beta}} \leq T$$

$$\theta^{\frac{1}{1-\alpha-\beta}} \leq T(\frac{r}{\alpha})^{\frac{\alpha}{1-\alpha-\beta}}(\frac{w}{\beta})^{\frac{1-\alpha}{1-\alpha-\beta}}$$

$$\theta \leq T^{1-\alpha-\beta}(\frac{r}{\alpha})^\alpha(\frac{w}{\beta})^{1-\alpha} \equiv A(w)$$

Therefore,

$$h_u \leq T \Leftrightarrow \theta \leq A(w)$$

and

$$k_u \leq \lambda z$$

$$(\frac{\theta\alpha}{r})^{\frac{1-\beta}{1-\alpha-\beta}}(\frac{\theta\beta}{w})^{\frac{\beta}{1-\alpha-\beta}} \leq \lambda z$$

$$\theta^{\frac{1-\beta+\beta}{1-\alpha-\beta}}(\frac{\alpha}{r})^{\frac{1-\beta}{1-\alpha-\beta}}(\frac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}} \leq \lambda z$$

$$\theta^{\frac{1}{1-\alpha-\beta}} \leq \lambda z(\frac{r}{\alpha})^{\frac{1-\beta}{1-\alpha-\beta}}(\frac{w}{\beta})^{\frac{\beta}{1-\alpha-\beta}}$$

$$\theta \leq (\lambda z)^{1-\alpha-\beta}(\frac{r}{\alpha})^{1-\beta}(\frac{w}{\beta})^{\beta} \equiv B(z,w)$$

Therefore,

$$k_u \leq \lambda z \Leftrightarrow \theta \leq B(z,w)$$

Case 2:

Suppose at the optimum, a household chooses $k = \lambda z$ and $0 < h < T$. For this to be an equilibrium,

$$k > 0, \lambda_1 = 0$$

$$h > 0, \lambda_2 = 0$$

$$\lambda z - k = 0, \lambda_3 \geq 0$$

$$T - h > 0, \lambda_4 = 0$$

$$\frac{dL}{dh} = -w + \theta(\lambda z)^{\alpha}\beta h^{\beta-1} = 0$$

$$\implies h = \Big(\frac{\theta(\lambda z)^{\alpha}\beta}{w}\Big)^{\frac{1}{1-\beta}} \equiv \hat{h}$$

For $\hat{h} < T$

$$\Big(\frac{\theta(\lambda z)^{\alpha}\beta}{w}\Big)^{\frac{1}{1-\beta}} < T$$

$$\implies \theta < T^{1-\beta}\frac{w}{(\lambda z)^{\alpha}\beta} \equiv D(z,w)$$

Therefore,

$$\hat{h} < T \Leftrightarrow \theta < D(z,w)$$

With respect to capital;

$$\frac{dL}{dk} = \theta\alpha(\lambda z)^{\alpha-1}h^{\beta} - r - \lambda_3 = 0$$

$$\theta\alpha(\lambda z)^{\alpha-1}\Big(\frac{\theta(\lambda z)^{\alpha}\beta}{w}\Big)^{\frac{\beta}{1-\beta}} - r - \lambda_3 = 0$$

$$\theta^{1+\frac{\beta}{1-\beta}}(\lambda z)^{\alpha-1+\frac{\alpha\beta}{1-\beta}}\alpha\Big(\frac{\beta}{w}\Big)^{\frac{\beta}{1-\beta}} - r = \lambda_3$$

where $\lambda_3 \geq 0$.

$$\implies \theta^{\frac{1}{1-\beta}}(\lambda z)^{\frac{\alpha+\beta-1}{1-\beta}}\alpha(\frac{\beta}{w})^{\frac{\beta}{1-\beta}} - r = \lambda_3 \geq 0$$

$$\theta^{\frac{1}{1-\beta}} \geq \frac{r}{\alpha}(\frac{w}{\beta})^{\frac{\beta}{1-\beta}}(\frac{1}{\lambda z})^{\frac{\alpha+\beta-1}{1-\beta}}$$

$$\theta \geq (\frac{r}{\alpha})^{1-\beta}(\frac{w}{\beta})^{\beta}(\frac{1}{\lambda z})^{\alpha+\beta-1}$$

$$\theta \geq (\frac{r}{\alpha})^{1-\beta}(\frac{w}{\beta})^{\beta}(\lambda z)^{1-\alpha-\beta} \equiv B(z,w)$$

Households with $\theta \geq B(z,w)$ and $\theta < D(z,w)$ will invest $k = \lambda z$ and $h = \hat{h}$ in their business.

Case 3:
Suppose at the optimum, $0 < k < \lambda z$ and $h = T$. If so, the following must hold;

$$k > 0, \lambda_1 = 0$$

$$h > 0, \lambda_2 = 0$$

$$\lambda z - k > 0, \lambda_3 = 0$$

$$T - h = 0, \lambda_4 \geq 0$$

$$\frac{dL}{dk} = \theta \alpha k^{\alpha-1} T^{\beta} - r = 0$$

$$\implies \theta \alpha k^{\alpha-1} T^{\beta} = r$$

$$k^* = \Big(\frac{r}{\theta \alpha T^{\beta}}\Big)^{\frac{1}{\alpha-1}}$$

$$k^* = \hat{k} \equiv \Big(\frac{\theta \alpha T^{\beta}}{r}\Big)^{\frac{1}{1-\alpha}}$$

$\hat{k}$ must be feasible. That is, $\hat{k} \leq \lambda z$;

$$(\frac{\theta \alpha T^{\beta}}{r})^{\frac{1}{1-\alpha}} \leq \lambda z$$

$$\implies \theta < (\lambda z)^{1-\alpha} \frac{r}{\alpha T^{\beta}} \equiv C(z)$$

Therefore,

$$\hat{k} \leq \lambda z \Leftrightarrow \theta < C(z)$$

Optimality condition with respect to h is

$$\frac{dL}{dh} = -w + \theta k^{\alpha} \beta - \lambda_4 = 0$$

where λ_4 must be non-negative. If so,

$$-w + \theta (\frac{\theta \alpha T^{\beta}}{r})^{\frac{\alpha}{1-\alpha}} \beta \geq 0$$

$$\theta^{1+\frac{\alpha}{1-\alpha}} (\frac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} \beta T^{\frac{\beta+\alpha-1}{1-\alpha}} - w \geq 0$$

$$\theta^{\frac{1}{1-\alpha}} \geq \frac{w}{\beta} (\frac{r}{\alpha})^{\frac{\alpha}{1-\alpha}} \frac{1}{T^{\alpha+\beta-1}}$$

$$\theta \geq T^{1-\alpha-\beta} (\frac{r}{\alpha})^{\alpha} (\frac{w}{\beta})^{1-\alpha} \equiv A(w)$$

Case 4:
Finally, suppose $k = \lambda z$ and $h_b = T$ at the optimum. If so,

$$k > 0, \lambda_1 = 0$$

$$h > 0, \lambda_2 = 0$$

$$\lambda z - k = 0, \lambda_3 \geq 0$$

$$T-h=0, \lambda_4 \geq 0$$

$$\frac{dL}{dh} = -w + \theta(\lambda z)^{\alpha}\beta T^{\beta-1} - \lambda_4 = 0$$

where λ_4 must be non-negative

$$\implies \theta(\lambda z)^{\alpha}\beta T^{1-\beta} \geq w_a$$

$$\theta \geq \frac{w_a}{(\lambda z)^{\alpha}\beta} T^{1-\beta} \equiv D(z,w)$$

Finally, with respect to k:

$$\frac{dL}{dk} = \theta\alpha(\lambda z)^{\alpha-1}T^{\beta} - r + \lambda_3 = 0$$

where λ_3 must be non-negative.

$$\implies \theta\alpha(\lambda z)^{\alpha-1}T^{\beta} \geq r$$

$$\theta \geq \frac{r}{\alpha(\lambda z)^{\alpha-1}T^{\beta}}$$

$$\theta \geq (\lambda z)^{1-\alpha}(\frac{r}{\alpha T^{\beta}}) \equiv C(z)$$

For households with $\theta \geq D(z,w)$ and $\theta \geq C(z)$, $k^* = \lambda z$ and $h^* = T$.

□

Other proofs

Proposition 3

Define gross income from entrepreneurship as $q^E \equiv \theta k^{\alpha}$. The model predicts that the expected gross income conditional on $M=0$, characteristics (z,x) and the model parameters ψ is

$$\begin{aligned} E(q^E|M=0) &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha}} T^{\frac{\beta}{1-\alpha}} E(\theta^{\frac{1}{1-\alpha}}) \tfrac{\Phi(\frac{\sigma}{1-\alpha}-a)-\Phi(\frac{\sigma}{1-\alpha}-c)}{\Phi(c)-\Phi(a)} P_{q_1^E} \\ &\quad + (\lambda z)^{\alpha} T^{\beta} E(\theta) \tfrac{\Phi(\sigma-\max(c,d))}{\Phi(-\max(c,d))} P_{q_2^E} \end{aligned}$$

where

$$\begin{aligned} E(\theta^{\frac{1}{1-\alpha}}) &= \exp(\tfrac{\bar{\theta}}{1-\alpha} + \tfrac{\sigma^2}{2(1-\alpha)^2}) \\ E(\theta) &= \exp(\bar{\theta} + \tfrac{\sigma^2}{2}) \\ P_{q_1^E} &= \tfrac{1_{A<B}\{\Phi(c)-\Phi(a)\}}{1-P(M=1)} \\ P_{q_2^E} &= \tfrac{1_{A>B}\{1-\Phi(c)\}+1_{A<B}\{1-\Phi(d)\}}{1-P(M=1)} \end{aligned}$$

Proof

$$\begin{aligned} E(q^E|M=0) &= E(q^E|M=0, h^*=T, k^*=\hat{k})P(h^*=T, k^*=\hat{k}|M=0) \\ &\quad + E(q^E|M=0, h^*=T, k^*=\lambda z)P(h^*=T, k^*=\lambda z|M=0) \end{aligned}$$

From Proposition 1, $h^* = T, k^* = \hat{k}$ is equivalent to $A(w) < \theta < C(z)$, and $h^* = T, k^* = \lambda z$ is equivalent to $\theta > \max(C(z), D(z,w))$. Therefore,

$$\begin{aligned} E(q^E|M=0,h^*=T,k^*=\hat{k}) &= E(\theta(\tfrac{\theta\alpha T^\beta}{r})^{\frac{\alpha}{1-\alpha}}T^\beta|h^*=T,k^*=\hat{k},M=0) \\ &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha}}T^{\frac{\beta}{1-\alpha}}E(\theta^{\frac{1}{1-\alpha}}|h^*=T,k^*=\hat{k},M=0) \\ &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha}}T^{\frac{\beta}{1-\alpha}}E(\theta^{\frac{1}{1-\alpha}})\frac{\Phi(\frac{\sigma}{1-\alpha}-a)-\Phi(\frac{\sigma}{1-\alpha}-c)}{\Phi(c)-\Phi(a)} \end{aligned}$$

$$\begin{aligned} E(q^E|M=0,h^*=T,k^*=\lambda z) &= E(\theta(\lambda z)^\alpha T^\beta|h^*=T,k^*=\lambda z,M=0) \\ &= (\lambda z)^\alpha T^\beta E(\theta|h^*=T,k^*=\lambda z,M=0) \\ &= (\lambda z)^\alpha T^\beta E(\theta)\frac{\Phi(\sigma-\max(c,d))}{\Phi(-\max(c,d))} \end{aligned}$$

Finally,

$$\begin{aligned} P(h^*=T,k^*=\hat{k}|M=0) &= \frac{P(A(w)<\theta<C(z))}{P(M=0)} \\ &= \frac{1_{A<B}\{\Phi(c)-\Phi(a)\}}{1-P(M=1)} \\ P(h^*=T,k^*=\lambda z|M=0) &= \frac{P(\theta>\max(C,D))}{P(M=0)} \\ &= \frac{1_{A>B}\{1-\Phi(c)\}+1_{A<B}\{1-\Phi(d)\}}{1-P(M=1)} \end{aligned}$$

$\square$

Proposition 4

Define gross income from entrepreneurship as $q^E \equiv \theta k^\alpha$. The model predicts that the expected gross income conditional on $M=1$, characteristics (z,x) and the model parameters ψ is

$$\begin{aligned} E(q^E|M=1) &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}E(\theta^{\frac{1}{1-\alpha-\beta}})\frac{\Phi(\min(a,b)-\frac{\sigma}{1-\alpha-\beta})}{\Phi(\min(a,b))}P_{q_1^M} \\ &\quad +(\lambda z)^{\frac{\alpha}{1-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\beta}}E(\theta^{\frac{1}{1-\beta}})\frac{\Phi(\frac{\sigma}{1-\beta}-b)-\Phi(\frac{\sigma}{1-\beta}-d)}{\Phi(d)-\Phi(b)}P_{q_2^M} \end{aligned}$$

where

$$\begin{aligned} E(\theta^{\frac{1}{1-\alpha-\beta}}) &= \exp(\frac{\bar{\theta}}{1-\alpha-\beta}+\frac{\sigma^2}{2(1-\alpha-\beta)^2}) \\ E(\theta^{\frac{1}{1-\beta}}) &= \exp(\frac{\bar{\theta}}{1-\beta}+\frac{\sigma^2}{2(1-\beta)^2}) \\ P_{q_1^M} &= \frac{1_{A>B}\Phi(b)+1_{A<B}\Phi(a)}{P(M=1)} \\ P_{q_2^M} &= \frac{1_{B<D}[\Phi(d)-\Phi(b)]}{P(M=1)} \end{aligned}$$

Proof

$$\begin{aligned} E(q^E|M=1) &= E(q^E|M=1,h^*=h_u,k^*=k_u)P(h^*=h_u,k^*=k_u|M=1) \\ &\quad +E(q^E|M=1,h^*=\hat{h},k^*=\lambda z)P(h^*=\hat{h},k^*=\lambda z|M=1) \end{aligned}$$

From Proposition 1, $h^*=h_u, k^*=k_u$ is equivalent to $\theta < \min(A(w), B(z,w))$, and $h^*=\hat{h}, k^*=\lambda z$ is equivalent to $B(z,w) < \theta < D(z,w)$. Therefore,

$$\begin{aligned} E(q^E|M=1,h^*=h_u,k^*=k_u) &= E(\theta^{\frac{1}{1-\alpha-\beta}}(\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}|h^*=h_u,k^*=k_u,M=1) \\ &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}E(\theta^{\frac{1}{1-\alpha-\beta}}|h^*=h_u,k^*=k_u,M=1) \\ &= (\tfrac{\alpha}{r})^{\frac{\alpha}{1-\alpha-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\alpha-\beta}}E(\theta^{\frac{1}{1-\alpha-\beta}})\frac{\Phi(\min(a,b)-\frac{\sigma}{1-\alpha-\beta})}{\Phi(\min(a,b))} \end{aligned}$$

$$\begin{aligned} E(q^E|M=1,h^*=\hat{h},k^*=\lambda z) &= E(\theta(\lambda z)^\alpha\hat{h}^\beta|h^*=\hat{h},k^*=\lambda z,M=1) \\ &= (\lambda z)^{\frac{\alpha}{1-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\beta}}E(\theta^{\frac{1}{1-\beta}}|h^*=\hat{h},k^*=\lambda z,M=1) \\ &= (\lambda z)^{\frac{\alpha}{1-\beta}}(\tfrac{\beta}{w})^{\frac{\beta}{1-\beta}}E(\theta^{\frac{1}{1-\beta}})\frac{\Phi(\frac{\sigma}{1-\beta}-b)-\Phi(\frac{\sigma}{1-\beta}-d)}{\Phi(d)-\Phi(b)} \end{aligned}$$

Finally,

$$\begin{aligned}
P(h^* = h_u, k^* = k_u | M = 1) &= \tfrac{P(\theta < \min(A(w), B(z,w)))}{P(M=1)} \\
&= \tfrac{1_{A>B}\Phi(b) + 1_{A<B}\Phi(a)}{P(M=1)} \\
P(h^* = \hat{h}, k^* = \lambda z | M = 1) &= \tfrac{P(B(z,w) < \theta < D(z,w))}{P(M=1)} \\
&= \tfrac{1_{B<D}[\Phi(d) - \Phi(b)]}{P(M=1)}
\end{aligned}$$

$\square$