# THE IMPACT OF A TEACHER DEVELOPMENT PROGRAM ON 7$^{TH}$ GRADERS' LEARNING OF ALGEBRA

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*We examine the impact of a teacher development program based on a functions approach to algebra on 7$^{th}$ grade students' understanding of equations. We focus on how students' score gains relate to their teachers' initial level of mathematics performance. Students from participating teachers' and their control peers completed a mathematics assessment at the start and at the end of the school year the teachers were taking the second and third of three courses. Students of participating teachers made greater gains than controls regardless of their teachers' initial level of mathematical understanding.*

## BACKGROUND

Algebra, a central topic in the mathematics curriculum (Moses & Cobb, 2001; National Council of Teachers of Mathematics, 2000; Common Core State Standards Initiative, 2010), has become a gatekeeper for higher education and a roadblock to access to science careers (Kaput, 1998; Moses & Cobb, 2001). Although requirements across states in the U.S. may differ, all states require that all students take mathematics and science to graduate from high school, at least from the middle school years. However, most students lose interest in mathematics when, in middle school, algebra is first introduced.

Research has repeatedly documented middle and high school students' difficulties with algebra, which are often attributed to the inherent abstractness of algebra and to levels of cognitive development (see reviews by Carraher and Schliemann, 2007 and by Kieran, 2007). Students often view the equals sign as a unidirectional operator, focus on computing specific answers, find difficult to use mathematical symbols to express relationships between quantities, do not use letters as generalized numbers or as variables, and do not operate on unknowns. Generating equations from word problems and using those equations to solve the problem constitutes a major challenge for 11 to 15 year-olds. Even when 6$^{th}$ and 7$^{th}$ grade students can generate the equation to represent a word problem, they often use methods other than algebra syntactic rules for manipulation of symbols to solve the equation.

In contrast, recent studies of early algebra show that, given relevant experiences, elementary school children succeed in understanding basic algebraic principles and representations (Cai & Knuth, 2011; Carraher & Schliemann, 2007; Kaput, Carraher, & Blanton, 2007). Such findings strongly support Booth's (1988) suggestion that students' difficulties with algebra in middle and high school are due to the traditional

2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 5, pp. 161-168. Vancouver, Canada: PME.

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computational approach to algebra in the mathematics curriculum, rather than to developmental limitations.

Mathematics education researchers (Kaput, 1998; Schoenfeld, 1995; Schwartz & Yerushalmy, 1992) have argued that a functions approach to algebra has the potential to better prepare students for a deep understanding of algebra. Within a functions approach, equations are considered as comparisons between two functions. Instead of starting by learning to compute the unknown values in an equation, students are introduced early on to variables and to the analysis of relations between sets of numbers. In doing so, they are introduced to multiple representations of functions, such as verbal statements, number lines, data tables, Cartesian graphs, and algebraic notation. Students move between multiple representations of functions and consider the process of solving equations as the comparison between two functions. Within this approach, we argue that students will become better prepared to solve word problems by representing problem statements as functions and as equations and to understand how the transformations in an equation towards its solution corresponds to transformations of the graphs of the two functions in the Cartesian space.

The implementation of a functions approach for teaching algebra represents a departure from the traditional path of teaching algebra by solely focusing on the manipulation of equations. As such, it requires the preparation of teachers for doing so, as well as close evaluation of the impact of this preparation on students' learning. This study evaluates the impact of a teacher development program focused on a functions approach to algebra on $7^{th}$ graders' ability to understand algebra equations and to represent and solve verbal problems using equations.

The Poincaré Institute (http://sites.tufts.edu/poincare/), a program supported by the National Science Foundation (grant #0962863), offers three semester-long online graduate level courses to teachers in three New England states (see Teixidor-i-Bigas, Schliemann, & Carraher, 2013 for details). The courses, offered to $5^{th}$ to $9^{th}$ grade teachers, covered algebra and functions, their multiple representations, and modelling and applications. Course 1 dealt with functions and relations and the representation of functions on the real line and on the plane. Course 2 focused on fractions and divisibility as they relate to functions, transformations of the line, and transformations of the plane, and on the use of transformations to analyse graphs of functions and for solving equations as the comparison of two functions represented in the plane. Course 3 included the representation of problems as equations, work on linear, quadratic and higher order equations, the relation between factoring and roots of equations, and slope and rate of change. Course 1 alternated weeks of study of mathematical content with weeks on examining classroom lessons related to the topics. Courses 2 and 3 were each structured across four three-week units plus two weeks dedicated to a final project. The first two weeks of each unit were dedicated to mathematics and pedagogical content; in the third week, groups of three to five teachers designed a learning activity (course 2) or interviewed individual students to explore their thinking (course 3). For the final project of courses 1 and 2, each teacher designed, implemented, and analysed a lesson.

The goal was to help teachers deepen their understanding of mathematics and of students' mathematical thinking and to enhance students' mathematical learning in their classrooms. Weekly meetings of teachers contributed to the development of a strong professional learning community to examine and improve their practices.

While, with a few exceptions, previous research more often has described the impact of teacher development programs on changes on teachers' ways of teaching, not many studies have focused on the possible contribution of these programs to the students of participating teachers. This study (a) examines the impact of the Poincaré Institute program on how the students of participating teachers understand and create equations to solve word problems and (b) analyses how students' score gains relate to teachers' initial level of mathematics understanding.

## METHOD

We focus on 7th grade teachers and their 12- to 13-year old students because, at this level, the content of the program closely matched their mathematics curriculum.

Of the 56 cohort teachers in the program, only 13 who taught 7th grade were included in this analysis. A total of 319 students of these teachers (cohort students) and 267 from non-cohort teachers were given the mathematics assessment at the start (September 2011) and at the end (June 2012) of the school year during which cohort teachers were taking the second and third courses of the program, respectively. Teachers from both cohorts also took an online mathematics assessment on algebra, functions, and graphs at the start of the program (January 2011).

We focus on four problems (see Figures 1 to 4) from the 15-problem written assessment. The first problem, the "Liam and Tobet problem", is a multi-step algebraic word problem that requires representing two functions, setting them equal, thus generating an equation, solving the equation, and interpreting the equation solution. The second, the "Amusement Park problem", is a multiple-choice problem about the relationship between the dependent and independent variables in an equation. The third, the "Cases problem", is another multiple-choice problem about which equation satisfies three sets of 'x' and 'y' values. The fourth and final problem, the "Finding X" problem, asks students to find the value for 'x' for a given equation.

Each of the five subparts of the Liam and Tobet problem and each of the three other problems was scored as "0" when answers were missing or incorrect and "1" when it was correct; hence, the minimum score a student could receive for the four problems was 0 and the maximum was 8.

Teachers' initial level of mathematics understanding was determined through the 24 problems in their written assessment. Some of the problems had multiple parts, leading to scores that could vary from 0 to a maximum of 47. These scores were then recoded (using the 33.3 and 66.6 percentiles) as three levels, where low corresponded to scores from 0 to 35, medium to 36 to 40, and high scores to 41 to 47.

Liam and Tobet are going to walk in a fund-raising event to raise money for their school.

- Liam's mother promised to donate to the school $4 per mile that Liam walks, plus an additional $30.
- Tobet's father promised to donate to the school $6 per mile that Tobet walks, plus an additional $20.

a) If Liam walks 15 miles during the event, what is the total amount of money his mother will donate? Show or explain how you got your answer.

b) Write an equation that represents *y*, the total amount of money Liam's mother will donate if Liam walks *x* miles during the event.

c) Write an equation that represents *y*, the total amount of money Tobet's father will donate if Tobet walks *x* miles during the event.

After the event, Liam and Tobet compared their results. Liam had walked the same number of miles as Tobet. Liam's mother had donated the same amount of money as Tobet's father.

d) Using your two equations from parts (b) and (c), determine the number of miles Liam and Tobet each walked during the event. Show or explain how you got your answer.

e) Using your answer from part (d), determine the total amount of money Liam's mother and Tobet's father **each** donated. Show or explain how you got your answer.

Andrea went to an amusement park.

- The cost of admission was $5.
- The cost for each ride was $0.75.

The equation below shows *c*, Andrea's total cost to go to the amusement park and go on *r* rides.

$$c = 5 + 0.75r$$

Based on the equation, which of the following statements is true?

A) As the value of *r* increases, the value of *c* increases.
B) As the value of *r* decreases, the value of *c* stays the same.
C) As the value of *c* decreases, the value of *r* increases.
D) As the value of *c* increases, the value of *r* stays the same.

Figure 1: Liam and Tobet Problem    Figure 2: The Amusement Park Problem

| | x | y |
|---|---|---|
| Case A | 0 | -3 |
| Case B | 1 | -1 |
| Case C | 2 | 1 |

Which equation is true for the values of *x* and *y* shown in Cases A, B, & C above?

A) $3x + 2 = y$
B) $3x - 2 = y$
C) $2x + 3 = y$
D) $2x - 3 = y$
E) $x - 3 = y$

Find the value of *x* if $12x - 10 = 6x + 32$

Show your work.

Answer:_____

Figure 3: The Cases Problem    Figure 4: The Finding X Problem

## RESULTS

The mean number of correct answers for all students in the two groups, for the eight items, increased from 2.88 (SD = 2.285) in September 2011 to 4.53 (SD = 2.541) in June 2012. The initial mathematics scores for all teachers, in January 2011, ranged from 20 to 44, with a mean of 36.92 (SD = 6.726).

The program had a positive impact on the 7th grade students' average mathematics scores for all 15 problems (31 sub-items) in the assessment: the interaction effect of time (September 2011 vs. June 2012) by cohort (Cohort vs. Non-Cohort teachers' students) on all students' mathematics scores, after controlling for any differences between groups at time 1 was statistically significant ($F(1, 583) = 13.63$, $p < .001$,

$\eta^2_p = .023$). The analysis that follows will only deal with the four problems that directly relate to the main content of the courses (see above).

Similar to the general results, students of cohort teachers performed better than the 7th grade students of non-cohort teachers after controlling for group differences at time 1 (Figure 5). The interaction effect of time and cohort on students' mathematics scores was, again, statistically significant ($F(1, 583) = 22.93$, $p < .001$, $\eta^2_p = .038$).
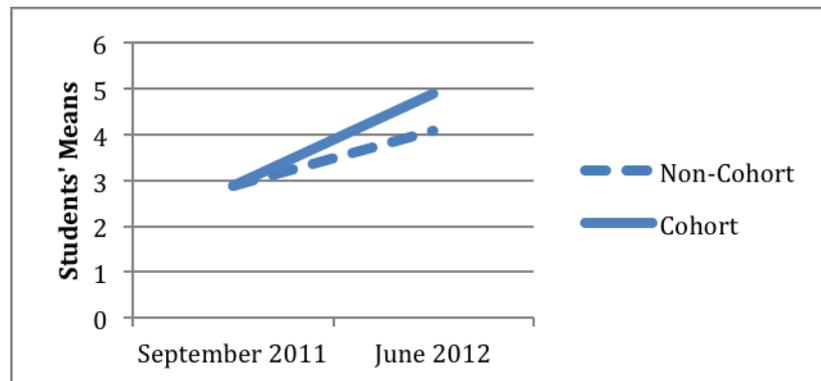


Figure 5: The interaction between Time and Cohort membership on students' scores for the four selected problems.

Students' average score gains in the eight selected items, by cohort and non-cohort teachers' initial mathematics level at the start of the program, were obtained by taking the average difference between students' scores in September 2011 and June 2012. Figure 6 shows that 7th graders' average score gains at the end of the school year were higher for students of cohort teachers, regardless of the teachers' initial levels of mathematics understanding. Here, students of teachers classified at the low level performed nearly as well as those of teachers at the high level. In contrast, the score gains for the non-cohort group closely related to the teachers' initial levels of mathematics, with students of non-cohort teachers in the low initial level showing less gains than students of teachers with high initial mathematics level. The interaction effect of cohort membership and teacher initial mathematics levels on student score gains was statistically significant ($F(2, 585) = 6.53$, $p = .002$). Thus, regardless of cohort teachers' initial mathematics level at the start of the program, their students showed somewhat similar and higher score gains. For the non-cohort group, only students of teachers with high initial mathematical levels showed gains that approached those of students of cohort teachers.

Figure 7 shows the percentage of cohort and non-cohort students who correctly answered each of the eight items in September 2011 and in June 2012. The items can be clustered in three groups in terms of the percentage of students' correct answers in September 2011. Group 1, with more than 50% of students answering correctly, include part a of the Liam and Tobet Problem (LTPa), the Amusement Park problem (APP), and the Cases Problem (CP). Group 2, with an average of 35% of correct answers, include parts b and c of the Liam and Tobet Problem (LTPb and LTPc).

Group 3 includes parts d (LTPd) and e (LTPe) of the Liam and Tobet problem and the Finding X Problem (FXP), with 11 to 16% of students answering correctly.
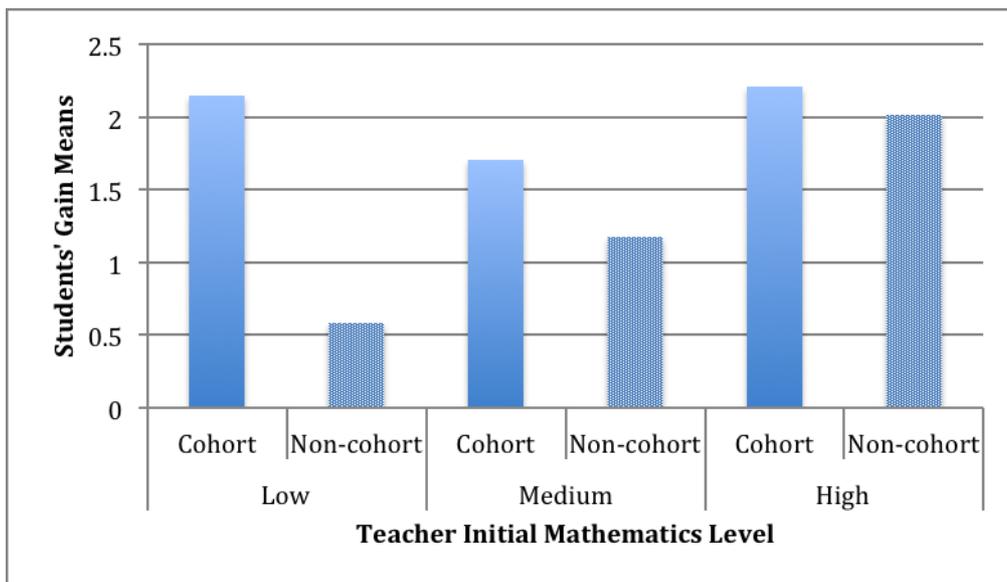


Figure 6: Student gains by Cohort membership and Teacher Initial Mathematics Levels (score gains ranged from -8 to 8).
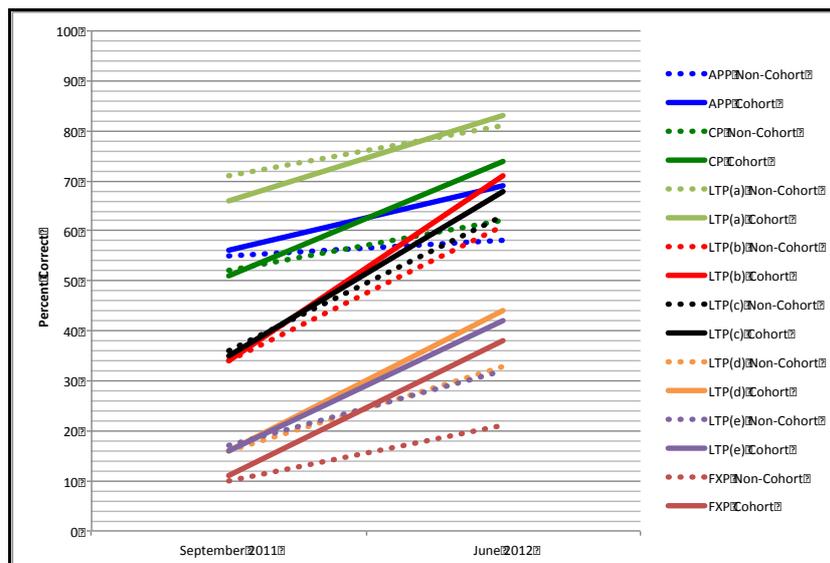


Figure 7: Percentage of students who correctly solved each item by Time and Cohort Membership.

The graphs for each item show that, in June 2012, cohort students did better on each of the items, in comparison to non-cohort students. A Repeated Measure Mixed Design ANOVA showed that the interaction effect of time by cohort membership on the percentage of students giving correct answers was statistically significant ($F(1, 14) = 6.61$, $p = .022$, $\eta^2_p = .321$).

We also investigated, for each of the eight items, the changes in the percentage of students who correctly solved each item, for each cohort and for each group of teacher initial mathematics level.

For teachers with low initial mathematics level, the difference between cohort and non-cohort students' changes were higher for cohort students in all items, varying from 13% to 28%, Students of non-cohort teachers showed modest improvements in seven of the items and a decline in the CP problem (picking an equation that satisfy three sets of x and y values). For teachers with medium initial mathematics level, the change was 5% to 20% higher for students of cohort teachers in five of the eight items (APP, LTPb, LTPc, LTPd, and LTPe), with students of non-cohort teachers showing a decline in the APP problem (finding the relationship between the dependent and independent variables), equal increase in one problem (LTPa), and 2% to 6% higher improvement for the other two problems. For teachers with high initial performance, the change was 4% to 19% higher for students of cohort teachers on five of the eight items (CP, LTPa, LTPd, LTPe, and FXP) and from 3% to 9% higher for non-cohort students in the other problems.

In summary, students of cohort teachers with low initial mathematics level showed higher improvement in their scores on all eight items in comparison to their control counterparts. Moreover, regardless of the initial level of mathematics of their teachers, students of cohort teachers consistently outperformed non-cohort students in items LTPd and LTPe, on finding and interpreting the solution to the Liam and Tobet problem.

## DISCUSSION

Our results suggest that teachers' participation in a program focused on a functions approach to algebra and on students' reasoning contributed to 7th grade students' learning of how to represent statements in a word problem as an equation, of how to solve and interpret the solution to equations, and of how the elements in an equation relate to each other.

Students of teachers who had initially demonstrated relatively low levels of mathematical understanding benefited the most, in all the problems analysed here. The program seems to have helped all teachers better contribute to their students' learning about how to create, solve, and interpret algebra equations to solve word problems. These are important achievements, given the well-documented difficulties with algebra among middle and high school students.

Our data contribute to further advance our understanding of the impact of teacher development programs and of teachers' levels of mathematical understanding on their students' learning.

### References

Booth, L. R. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Shulte (Eds.), *The ideas of algebra, K-12: 1988 yearbook* (pp. 20-32). Reston, VA: NCTM.

Cai, J., & Knuth, E. (Eds.). (2011). *Early algebraization: A global dialogue from multiple perspectives*. New York, NY: Springer.

Carraher, D. W., & Schliemann, A. D. (2007) Early algebra and algebraic reasoning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol 2, pp. 669-705). Charlotte, NC: Information Age Publishing.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Kaput, J. (1998). Transforming algebra from an engine of inequity to an engine of mathematical power by 'algebrafying' the K-12 Curriculum. In NCTM & NRC (Eds.), *The nature and role of algebra in the K-14 curriculum* (pp. 25-26). Washington, DC: National Academies Press.

Kaput, J., Carraher, D., & Blanton, M. (Eds.). (2007). *Algebra in the early grades*. Mahwah, NJ: Erlbaum.

Kieran, C. (2007). Learning and teaching Algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol 2, pp. 707-762). Charlotte, NC: Information Age Publishing.

Moses, R., & Cobb, C.E. (2001). *Radical equations: Math literacy and civil rights*. Boston, MA: Beacon Press.

National Council for Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Schoenfeld, A. (1995). Report of working group 1. In C. B. Lacampagne (Ed.), *The algebra initiative colloquium* (Vol. 2, pp. 11-18). Washington, DC: U.S. DoE, OERI.

Schwartz, J., & Yerushalmy, M. (1992). Getting students to function on and with algebra. In E. Dubinsky & G. Harel (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (pp. 261-289). Washington, DC: Mathematical Association of America.

Teixidor-i-Bigas, M., Schliemann, A. D., & Carraher, D. W. (2013). Integrating disciplinary perspectives: The Poincaré Institute for Mathematics Education. *The Mathematics Enthusiast, 10*(3), 519-562.