# TEACHER DEVELOPMENT AND STUDENT LEARNING[1]

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*We describe and evaluate a teacher development program for in-service teachers in grades 5 to 9. The program focuses on functional relations and their multiple representations as a thread throughout the mathematics curriculum, integrating a wide range of topics commonly treated in an isolated fashion. It encourages teachers to consider students' own representations and mathematical ideas. Classroom observations and mandated assessment results show a substantial improvement in participating teachers' ways of teaching and on their students' learning.*

## INTRODUCTION

Since Shulman's (1986) analysis of pedagogical content knowledge, most mathematics teacher development programs have aimed at promoting teacher content and pedagogical knowledge by offering courses in mathematics and courses in education. Still, basic questions remain about the kinds of teacher development programs most suited for the development of such expertise, improved teaching, and student learning.

Previous studies suggest that teacher development programs contribute, for example, to teachers' performance on written tests of mathematical knowledge for teaching (Hill and Ball, 2004, see also Hill, Ball, and Schilling, 2008), that teachers' mathematical knowledge for teaching mathematics, as assessed by the analysis of teachers' logs of lessons, is related to student achievement (Hill, Rowan, and Ball, 2005), and that teacher development programs that focus on mathematical content and pedagogy contribute to student achievement (Franke, Carpenter, & Levi, 2001; Saxe, Gearhardt, & Nasir, 2001; McMeeking, Orsi, and Cobb, 2012). However, most studies fail to show results regarding both teachers' ways of teaching and their students'learning (see Bautista and Ortega-Ruiz, 2015 and the collection of papers on teacher professional development across different countries edited by Bautista, 2015). Mathematics education research needs to further explore and clarify which specific characteristics of teacher development programs lead to improvements in classroom teaching and to students' proficiency in mathematics. The present report aims at helping connect these oft-separated aspects.

Analyses of teacher professional development programs (for example, Desimone, 2009) highlight the importance of content focus, active learning, coherence, duration, and collective teacher participation. In agreement with the general conclusions of these analyses, we specifically take for granted the belief that successful instruction in the middle school classroom requires that teachers

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jointly attend to mathematical content, teaching strategies, and the mathematical reasoning of their students. Teachers must understand the mathematics content they teach in depth and beyond what is required of students and should understand how these topics are inter-related within and across grade levels. They need to use appropriate representations of mathematics and should be able to use mathematics to represent ideas and situations from science and every day life. Good teachers listen to the students and explore their ideas on the topic. They take these ideas into account not only when planning lessons but also at every point of its implementation and should be able to modify the activities on the spot, depending on how the class proceeds. This requires long-term teacher development programs that integrate mathematical content learning, understanding of how students learn, and analyses of classroom activities planning and implementation.

In this paper we describe the underlying tenets and results of a program for teachers of grades 5 to 9 based on the above features. We expected the program to have an effect on teachers' ways of teaching, as evaluated by classroom observations, and on their students' performance in state mandated standard assessments.

**The Mathematical and Pedagogical Foundations**

The program emerged from collaboration among mathematicians, mathematics educators, and physicists (see details in Teixidor-i-Bigas, Schliemann, & Carraher, 2013), with input from target school districts. Its three semester-long courses are offered by Tufts University departments of Mathematics and of Education, as part of the Poincaré Institute for Mathematics Education (https://sites.tufts.edu/poincare/). The courses cover topics from the school curriculum, such as arithmetic operations, fractions, ratios, proportion, and geometry, as they relate to algebra and functions, core mathematical ideas integrating the topics. They aim at meaningful and deeper learning, not only of algebra per se, but also of key topics in the middle school curriculum as viewed through the lens of functions.

Mathematicians and mathematics educators have often argued that functions lie at the core of algebra and algebra instruction (e.g., Dubinsky & Harel, 1992; Schwartz & Yerushalmy, 1992). They also propose that a functions approach to mathematics can enrich and promote deeper learning of mathematics throughout grades K-12, minimizing difficulties exacerbated by overly computational and syntactic approaches to arithmetic and algebra (Booth, 1988; Kaput, 1995; Carraher, Schliemann, & Brizuela, 2005; Schoenfeld, 1995).

How can algebra and functions be introduced to young learners while avoiding premature formalization and overreliance on syntax? Our general approach has been to build on students' interpretations of carefully designed problems about contextualized situations involving physical quantities and sets of values. Variables, are introduced in natural language forms, so as to enable a shift in focus from individual values to sets of possible values and relations among sets of values.

From a pedagogical point of view, the program builds upon Piaget's theory of cognitive development and Vygotsky's ideas on the role of cultural tools and social interaction in cognitive development. As such, we emphasized the development of activities that start by taking into account students' views as they explore and discuss situations and events, reflect upon how physical quantities interrelate, and come to appropriate cultural practices, tools, and mathematical conventions. This approach is complemented by findings from our decade-long classroom studies (see http://ase.tufts.edu/education/earlyalgebra/about.asp) on the introduction of algebraic concepts and representations from grades 3 to 8 (see Brizuela & Schliemann, 2004; Carraher & Schliemann, 2007, 2015; Carraher, Schliemann, & Brizuela, 2005; Carraher, Schliemann, & Schwartz, 2008; Carraher, Schliemann, Brizuela, & Earnest, 2006; Schliemann, Carraher, & Brizuela, 2007, 2012; Schliemann, Carraher, Goodrow, Caddle, & Porter, 2013).

## PROGRAM IMPLEMENTATION AND CONTENT

Since 2011, the program has offered three online and face-to face graduate level courses to three cohorts of 60 teachers each, from nine school districts in the Northeastern USA (five in Massachusetts, three in New Hampshire, and one in Maine). Teachers met once a semester at the university campus and weekly in their schools. Instructors participated in online discussions, provided online feedback, and visited school districts once a month.

Teachers invested an average of 10 hours per week on course assignments. They read and discussed written notes on mathematical content and solved and discussed math challenges in online groups of eight to ten teachers. Working in small groups of three to five teachers in their schools, they interviewed students about particular topics to understand their ways of thinking and planned, implemented, evaluated, and improved classroom activities. They also analyzed examples of videotaped lessons on the content of the courses in terms of mathematical concepts and representations, teaching strategies, and students' ideas and achievements. They discussed how course topics related to the lessons they teach and considered how to modify prescribed curriculum activities based upon what they were learning in the program.

The written notes, challenge questions, and course activities focused on the role of functions as a unifying concept throughout the K-12 curriculum, from the standpoint of mathematics and of mathematics teaching and learning. The courses emphasized multiple representations, including verbalizations, geometric tools (in particular, number lines and Cartesian graphs), function tables, and arithmetical-algebraic notation, as representations of the same abstract mathematical objects, to deepen the understanding of arithmetic operations, fractions, ratio, proportion, and the syntax of arithmetic and algebra. Teachers dealt with equations and inequalities as comparisons between functions, solved equations using transformations of the plane, and were introduced to notions of change and invariance in terms of the behavior of functions.

The first course builds on the idea of numbers from representations of quantities to more abstract conceptions of numbers as mathematical objects. Teachers work on fractions and decimals, rational and irrational numbers, and the many ways numbers can be represented, for example, as points on a line or an oriented segment. They then consider arithmetic operations as functions of a single or of two variables. The second course deals mostly with transformations of the line and of the plane as a way to understand functions and the solution of equations as the comparison between two functions. The third course compares linear to non-linear functions and takes a closer look at non-linear functions and rates of change. Questions regarding divisibility appear in the study of numbers (course 1), in the solution of Diophantine equations (course 2) and in the factorization of polynomials for solving polynomial equations (course 3).

The program devotes less time to the mechanics of algorithms and far more to understanding why these algorithms work and how mathematical concepts are interconnected. There is an emphasis on historic and typical middle school students' conceptions about the topics, on how mathematics is used to model science and real life situations, and on how different representations can engage different types of learners and emphasize different properties of each model or tool.

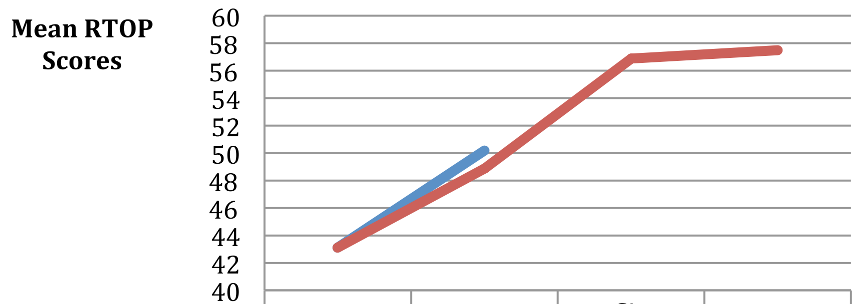
## ANALYSIS OF THE PROGRAM'S IMPACT

External evaluators from the Intercultural Center for Research in Education (INCRE) evaluated changes in classroom teaching by directly observing teachers as they each taught one lesson of their choice, at the start and at the end of the three course series. They also observed a random sub-sample of teachers six months and a year after program completion. Observations were ranked according to the Reformed Teaching Observation Protocol (RTOP) and included reliability-tested measures of (a) teachers' support for students' discussions and uses of multiple representations, (b) the in-class time teachers spent addressing students' mathematical reasoning, and (c) teachers' design and discussion of mathematics problems from various perspectives and representations.

Student learning was analyzed in terms of changes in the percentage of proficient and advanced students of grades 5 to 8 in the Massachusetts Comprehensive Assessment System (MCAS) test, a standardized, state-mandated measure of mathematical proficiency. Teachers in five Massachusetts districts started the courses in the spring of 2011. By the end of the 2013-2014 school year, 34% of those in grades 5 to 8 in the five target districts had completed the three courses. Each of these districts was matched to five districts of similar number of students, demographic variables, and MCAS results in 2010-2011. We compared changes in the results of target districts from 2010-2011 to 2013-2014 to changes in similar districts and in the whole state over the same period.

### Results

Systematic classroom observations (RTOP) by the external evaluators offer clear evidence that the first cohort of teachers in the program had improved their ways of teaching mathematics (Figure 1).




| | Start (Feb. 11) | End (Apr. 12) | Six months later (Oct. 12) | A year later (Apr. 13) |
|---|---|---|---|---|
| Cohort 1 (N=48) | 43.1 | 50.2 | | |
| Cohort 1 Subset (N=23) | 43.1 | 48.9 | 56.9 | 57.5 |

Figure 1: Mean RTOP scores for cohorts 1 and 2 at beginning and end of courses and, for a sub-sample of cohort 1, six months and a year later (source: INCRE, External Evaluation).

There were significant improvements in RTOP scores from the beginning to the end of the teacher development program. Furthermore, the additional in-class observations revealed that RTOP scores improved six months after the teachers completed the program. By then, their average score was nearly a full standard deviation higher than at the beginning of the program. These improvements remained stable when evaluated one year after completing the program.

Concerning student learning, when the program started (spring of 2011, Figure 2), the percentages of students at the Proficient and Advanced levels in the target districts were similar to those of matched districts. Three years later, target districts had narrowed the gap with regard to the state from 7.1 to 4.5 percentage points and had surpassed matched districts by 5.1 points, with changes significantly higher than changes in matched districts (W=144, z=2.68, p=.0037).

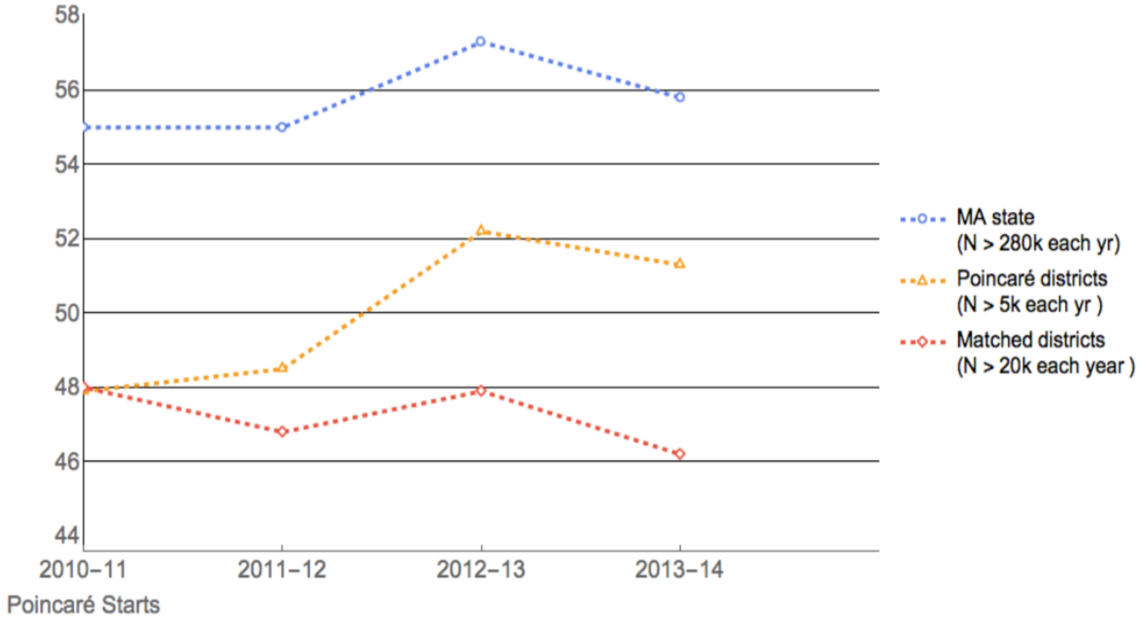



Figure 2: Percent of students at the Advanced and Proficient levels in the MCAS test.

These results actually underestimate the impact of the program on students, given that target district results included all of their students, even if their teachers had not taken part in the program. In fact, the districts varied in the percentage of teachers who had taken the three courses. This allowed investigating whether performance differences between target and matched districts varied as a function of the districts' percentages of teachers who completed the program.

We found that the greater the percentage of teachers in a district, the greater the advantage in MCAS scores over matched districts (see Figure 3). This advantage was significantly correlated with the number of teachers who had taken the courses (Spearman's r =.54, p=.007). This provides additional evidence that performance differences between target and matched districts were related to the teacher development program.

The preponderance of data points in quadrant I of Figure 3 highlights the generally superior results of target districts as compared to the matched districts. The regression line shows that the advantage was, overall, directly proportional to the percentage of teachers who had completed the program, in each grade level, in each district.

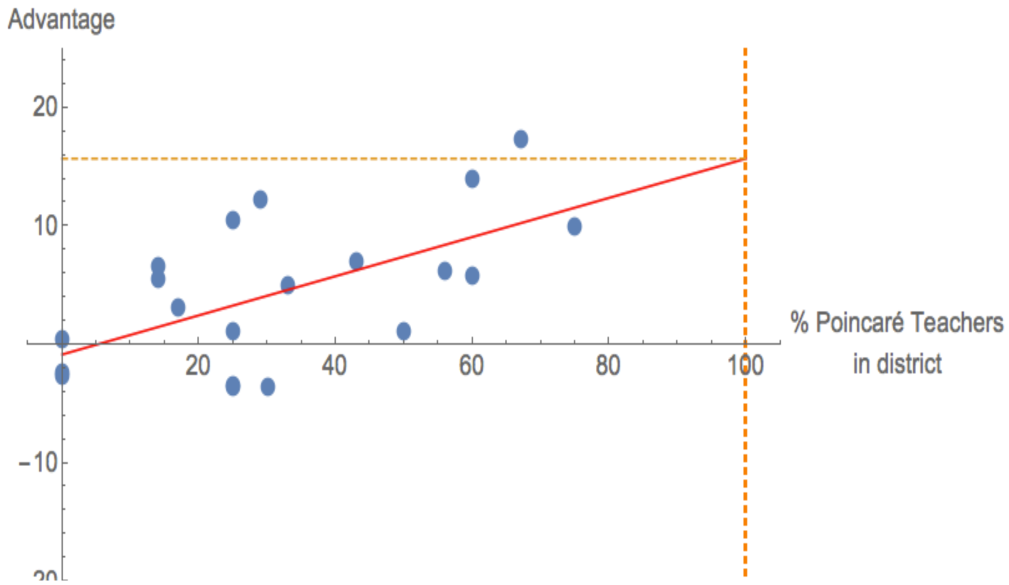



Figure 3: Target districts' advantage in 2014 over similar districts, as a function of percentage of teachers who completed the program. Each point represents one of four grade levels in five different districts (two points are hidden by other points).

## DISCUSSION

Even though the focus of the courses was on functions and algebraic reasoning, the impact of the program was found among teachers using a variety of textbooks across a wide range of grade levels and among students taking a test on all content areas of the grades 5 to 8 curriculum. This suggests that improving teachers understanding of mathematics using a functions approach to curriculum topics, promoting their understanding of students' ways of thinking and learning, and providing them with opportunities for jointly planning lessons and participating in discussions about mathematics and about classroom teaching leads to better teaching and student performance. These gains are likely to emerge from the integration of, among others, the following five aspects.

1. *The length of the program*: Unlike most professional development initiatives, this program lasts for three 15 week semesters with an expected workload of 10 hours per week. This allows for teachers broader and deeper view of a large number of topics, for understanding and appropriation of new concepts and representations, and for changing their ways of teaching. As teachers themselves expressed to external evaluators, time to mature was important in their development.

2. *The interconnection and depth of analysis of mathematical topics afforded by the algebra and functions approach*: This allowed for a deeper understanding of arithmetic operations, fractions, ratio, proportion, and geometry and appropriation of multiple mathematical representations.

3. *The integration of mathematics and pedagogical content*: Pedagogical materials and activities were presented in tandem with mathematics content and took into account students' ways of approaching specific topics and representing everyday and science situations they had access to. In their discussions and assignment reports teachers revealed their surprises about their students initial approaches and how questions they raised had helped their students.

4. *The focus on concepts and understanding rather than algorithms and computations*: There was no attempt to teach how to write formal mathematics, as is the case for many mathematics courses

developed for undergraduates. But teachers were made aware of the necessity to clarify their ideas and justify their assertions in a way that works in any situation rather than for specific numbers and of the need to be precise and clear in their assumptions. Preliminary data from ongoing analyses show that, in the post-assessment, teachers seem far more likely to give general arguments and use variables to justify a statement than they were in the pre-assessment.

5. *Teachers interactions with their colleagues*: Throughout the courses, teachers held online and face-to-face discussions with their peers as they solved mathematical problems and as they planned, implemented, discussed, and evaluated lessons they taught. In their written evaluations of the courses at the end of each semester, teachers highlighted this as one of the most important features of the program.

6. *The intense interaction with instructors*: Course assignments required a preliminary draft and at least one improved version, written after each teacher received feedback from instructors and colleagues. The amount of feedback provided by the group mentors was fairly high compared with more standard courses but, still, differed from group to group due to different instructors' styles. We are currently examining whether teachers in groups that received more feedback made more progress. This information is relevant and likely to be applicable to other on-line teaching, given its repercussion on the cost of this form of instruction in terms of number of required instructors.

We are aware of the need for further analyses to clarify how the different aspects of the program contributed to teacher and student performances. Our data on student assessment were taken from general reports by grade and by district. Issues of confidentiality prevented us from obtaining individual student reports and to analyze how their results related to their teacher's performance in course activities. Fine grained analysis of teachers online work, reports, program assessments, and classroom videos are currently underway and will provide further information on the matter.

## References

Bautista, A. (Ed.) Teacher Professional Development: International Perspectives and Approaches. Psychology, Society and Education, Special Issue, Vol. 7(3).

Bautista, A., & Ortega-Ruíz, R. (2015). Teacher professional development: International perspectives and approaches. Psychology, Society and Education, 7(3), 240-251.

Booth, L. R. (1988). Children's difficulties in beginning algebra. In A. F. Coxford & A. P. Shulte (Eds.), The ideas of algebra, K-12: 1988 Yearbook (pp. 20-32). Reston, VA: National Council of Teachers of Mathematics.

Brizuela, B.M., & Schliemann, A. D. (2004). Fourth graders solving linear equations. For the Learning of Mathematics, 24(2), 33–40.

Carraher, D.W. & Schliemann, A. D. (2007) Early Algebra and Algebraic Reasoning. In F. Lester (ed.) Second Handbook of Research on Mathematics Teaching and Learning: A project of the National Council of Teachers of Mathematics. Vol II. Charlotte, NC: Information Age Publishing, pp. 669-705.

Carraher, D.W. & Schliemann, A.D. (2015). Powerful Ideas in Elementary Mathematics Education. In L. English & D. Kirshner (Ed.). Handbook of International Research in Mathematics Education (3rd edition). New York: Taylor & Francis.

Carraher, D. W., Schliemann, A. D., & Brizuela, B. (2005). Treating operations as functions. In D. Carraher & R. Nemirovsky (Eds.), Monographs of the Journal for Research in Mathematics Education, XIII.

Carraher, D. W., Schliemann, A. D., & Schwartz, J. L. (2008). Early algebra is not the same as algebra early. In J. Kaput, D. Carraher & M. Blanton (Eds.), Algebra in the early grades. Mahwah, NJ: Erlbaum.

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. Journal for Research in Mathematics Education, 37(2), 87-115.

Desimone, L.M. (2009). Improving Impact Studies of Teachers' Professional Development: Toward Better Conceptualizations. Educational Researcher, Vol. 38, No. 3, pp. 181-199.

Dubinsky, E., & Harel, G. (Eds.) (1992). The concept of function: Aspects of epistemology and pedagogy. Washington, DC: Mathematical Association of America.

Franke, M. L., Carpenter, T. P., & Levi, L. (2001). Capturing teachers' generative change: A follow- up study of professional development in mathematics. American Educational Research Journal, 38, 653–689.

Hill, H. C. & Ball, D. L. (2004). Learning Mathematics for Teaching: Results from California's Mathematics Professional Development Institutes. Journal for Research in Mathematics Education, Vol. 35(5), pp. 330- 351.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge. Journal for Research in Mathematics Education, 39(4), 372–400.

Hill, H. C., Rowan, B. & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. American Educational Research Journal, 42(2), pp. 371–406

Kaput, J. (1995, April). Transforming algebra from an engine of inequity to an engine of mathematical power by "algebrafying" the K-12 curriculum. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Boston, MA.

McMeeking, L. B., Orsi, R., and Cobb, B. (2012). Effects of a Teacher Professional Development Program on the Mathematics Achievement of Middle School Students. Journal for Research in Mathematics Education, 43(2), 159–181

Saxe, G. B., Gearhart, M., & Nasir, N. S. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. Journal of Mathematics Teacher Education, 4, 55–79.

Schliemann, A. D., Carraher, D. W., & Brizuela, B. M. (2007). Bringing out the algebraic character of arithmetic: From children's ideas to classroom practice. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schliemann A. D., Carraher D. W., Brizuela B. M. (2012). Algebra in elementary school. In L. Coulange & J.-P. Drouhard (Eds.) Enseignement de l'algèbre élémentaire: Bilan et perspectives. Special Issue of Recherches en Didactique des Mathématiques, pp. 109-124.

Schliemann, A.D., Carraher, D.W., Goodrow, A., Caddle, M., & Porter, M. (2013). Equations in elementary school. In A. M. Lindmeier & A. Heinze (Eds.). Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4, pp. 161-168. Kiel, Germany: PME.

Schoenfeld, A. (1995). Report of Working Group 1. In C. B. Lacampagne (Ed.), The Algebra Initiative Colloquium: Vol. 2. Working Group Papers (pp. 11-18). Washington, DC: U.S. Department of Education, OERI.

Schwartz, J. & Yerushalmy, M. (1992). Getting students to function on and with algebra. In E. Dubinsky & G. Harel (Eds.), The concept of function: Aspects of epistemology and pedagogy (pp. 261-289). Washington, DC: Mathematical Association of America.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4–14.

Teixidor-i-Bigas, M., Schliemann, A. D., & Carraher, D. W. (2013). Integrating disciplinary perspectives: The Poincaré Institute for Mathematics Education. The Mathematics Enthusiast, 10(3), 519-562.