

Poincaré Institute: Impact on Mathematics Achievement of Ethnic Groups

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Abstract

We assess how a teacher development program offered to educators in a target district may have contributed to African-American, Hispanic-Latino, and White students' performance on a state-mandated test of mathematics. More than half of the mathematics classroom teachers, special education teachers, coaches, and interventionists in the target district participated in the program. We compare students' performance gains in the target district to those of students in 10 comparison districts with similar demographics and prior test performance and to students throughout the state. In the target district, all three ethnic groups showed statistically significant gains. In the comparison districts and across the state, only White students showed significant gains.

On average, students from certain minority groups regularly underperform white students on tests of academic achievement, a discrepancy referred to as an "achievement gap". Attempts to explain such gaps draw on a wide range of socio-economic, cultural, psychological, and educational potential factors. While

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recognizing that achievement gaps reflect cultural and socio-economic differences, we take the view that quality of teaching is a core factor associated with minority students' achievement. Here, we explore the possible contribution of an intensive teacher development program to reducing the achievement gap between White and minority students, specifically, African-American and Hispanic/Latino students.

The program aimed at (a) increasing teachers' content knowledge through a functions approach to mathematics that unifies topics across the elementary, middle, and high school curriculum and (b) promoting teaching and learning through open-ended contextualized problems, eliciting students' ideas and classroom discussions, building upon students' thinking, and focusing on a deeper understanding of mathematics content, generalizations, and multiple representations for relationships among sets of numbers or physical quantities. Throughout the courses, in online and face-to-face meetings, teachers jointly discussed mathematical content, solved mathematics problems, analyzed videotaped classroom activities, interviewed their students, planned and implemented classroom lessons, and received mentors' feedback. During a school year more than half of the educators in the target district participated in the development program. We contrast gains in mathematics performance on a state standardized test by White, African-American, and Hispanic/Latino students in the target district to gains in the whole state and in demographically similar

comparison districts, thus evaluating the potential contribution of the program to narrowing the achievement gap.

### **The Achievement Gap as Inequalities in Educational Opportunities**

Despite policies and investment to narrow ethnic, racial, and socio-economic achievement gaps (Kitchen & Berk, 2016; Noguera, 2009), success stories are rare. Musu-Gillette et al. (2017) found no measurable closure of the achievement gap from 1990 to 2015 for 8<sup>th</sup> graders African-Americans and Hispanic/Latinos in the National Assessment of Education Progress (NAEP). The same is true for 4<sup>th</sup> and 8<sup>th</sup> graders tested in 2017 (see [https://www.nationsreportcard.gov/math\\_2017/#nation/gaps?grade=4](https://www.nationsreportcard.gov/math_2017/#nation/gaps?grade=4)). The Stanford Center for Education Policy Analysis (<http://cepa.stanford.edu/educational-opportunity-monitoring-project/achievement-gaps/race/>) reveals that, even though achievement gaps have narrowed somewhat in the 1970's and 1980's, by 2012 they still remain substantial, ranging from 0.5 to 0.9 standard deviations. Grades 3-8 Massachusetts Comprehensive Assessment System (MCAS) results from 2007 to 2014 show that, each year, at least 27% more Whites performed at or above proficiency levels in comparison to African-American or Hispanic/Latino students.

Historically, differences in academic achievement across different populations have been attributed to cultural deprivation and cultural disadvantages of underperforming groups (Bloom, Davis, & Hess, 1965; Brooks-

Gunn & Duncan, 1997; Coleman, 1966; Duncan et al., 1998) and seen by many, including teachers (Bol & Berry, 2005), as resulting from students' characteristics (for example, motivation, work ethic, and support received from family members). In addition, research has shown that access to teachers of low-SES students tend to be weaker in mathematics and pedagogical knowledge (Bol & Berry, 2005, Hill & Lubienski, 2007, 2008). As Hill and Lubienski (2007) observed, "the percentage of free lunch-eligible students *in a school* is significantly related to teachers' [...] score" on a test of mathematical knowledge and to their qualifications (p. 761). Currently, recommendations for closing the achievement gap include culturally responsive teaching, outreach to families, more funding, longer school days, and investing in teacher quality, and quality development (<http://www.nea.org/home/13550.htm>, Akiba et al., 2007, Hirsh, 2005; Ladson-Billings, 2000; Schoenfeld, 2002).

Researchers have proposed that a focus on conceptual learning, in-class time spent on teaching, teachers' analyses of students' difficulties and strategies, attention to students' ideas, classroom discussions, students' interactions while answering open ended questions, activities on measurement and geometry, and teacher collaboration may contribute to closing the achievement gap (e.g., Boaler & Staples, 2008; Brown, 2012; Desimone & Long, 2010; Fernandes, Crespo, & Civil, 2017). Indeed, Boaler & Staples (2008) reported positive results in narrowing the achievement gap through instruction that incorporated these

characteristics. Duncan & Murnane (2014) also report on success stories among schools in urban areas where low-income minorities who were granted access to high quality schools could perform at high levels.

Nevertheless, most public schools in the United States have not been able to show evidence of improvement in the effectiveness of instruction for minority students.

### **Mathematics Teachers' Development and the Achievement Gap**

Teacher development is an imperative to improving quality of teaching (Darling-Hammond, 1996; Hirsh, 2005; Santagata, Kersting, Givvin & Stigler, 2010; Schoenfeld, 2002) and student achievement (Ball & Cohen, 1999; Cohen & Hill, 2000; Corcoran, Shields, & Zucker, 1998; Darling-Hammond & McLaughlin, 1995; Desimone, 2009; Phelps, Kelcey, Jones & Liu, 2016).

Effective and sustained programs are needed to support teachers. This can no longer be “the kind of scattershot, single-session, workshop-style programs that pass for professional development in so many school districts” (Singham, 2003, p. 590). Nevertheless, studies demonstrating that teacher development programs have led to increase in student performance are rare (see Gersten et al., 2014). In one of these rare cases, Briars & Resnick (2000) and Schoenfeld (2002) report on the three-year implementation of a reform curriculum in Pittsburg, MA schools, where teachers were offered development programs built around NCTM standards. The implementation focused on coherent and connected mathematical

content and on high expectations for all students in reasoning, representation, communication, problem solving, and making connections. Brian & Resnick's results show a narrowing of the gap among fourth graders' African-American and White students in the district. McMeeking, Orsi & Cobb's (2012) also shows that gains in proficiency for all students can occur when teacher development aims at deepening teachers' mathematical content knowledge and pedagogy.

In this paper we describe how a mathematics teacher development program offered to a whole district may have improved learning among three ethnic groups and reflect upon its possible contribution towards reducing the achievement gap between White students and their African-American and Hispanic/Latino peers.

A previous analysis of the possible impact of the program in 10 school districts showed improvements in teachers' practice and in students' standard tests' performance (Schliemann, Carraher, & Teixidor, 2016; submitted). Moreover, positive results in narrowing the achievement gap were observed in one of the districts with a large percentage of minority students (Hotomski & Schliemann, 2016).

This time, we examine changes in mathematics achievement on standardized tests for students in grades 3-8 in a single Massachusetts multi-ethnic district, over the academic year more than half of their teachers or other educators were engaged in the first two of the three courses in the program. Even though standardized tests are a limited measure of learning and understanding,

their results are the only available measures of student achievement allowing comparisons across districts and progress over the years for large number of students. We do believe that teaching that addresses conceptual learning, instead of just computation and procedures, would prepare students to deal with a variety of problems, including those in standardized tests.

### **The Program's Foundations**

The program offers an integrated approach to mathematics and pedagogy with applications to science and everyday situations. Based on early algebra studies (Carraher & Schliemann, 2016), the mathematical development of topics uses functional relations and their representations as a thread unifying arithmetic operations, fractions, ratios, proportions, algebra, and geometry. Building upon Piaget's theory of cognitive development and Vygotsky's ideas on the role of cultural tools and social interaction, the program promotes teaching that takes into account students' views as they discuss and reflect upon relations between quantities and as they appropriate new mathematical tools, procedures, and representations.

An external evaluation of teachers in three previous cohorts showed that, during classroom discussions, students more often engaged in discussions, put forth their own ideas, respected each other's ideas, and showed interest, motivation, and on-task behavior. Moreover, one year after completion of the program, these characteristics were associated with a "substantive" effect size of



0.69 (see <https://sites.tufts.edu/poincare>; Teixidor, Schliemann, & Carraher, 2013; and Schliemann, Carraher, & Teixidor, 2016 and submitted).

The semester-long courses are hosted mostly online, with a smaller face-to-face component. Teachers in this study invested approximately 10 hours per week in course assignments. Activities included reading and discussing notes about mathematics content and students' reasoning, solving and discussing mathematical problems in online groups of eight to ten teachers, and working in smaller groups of 2-4 teachers (usually from the same school) in pedagogical activities. Four times per course, teachers analyzed videotaped classroom lessons, interviewed their students about particular topics, and planned, implemented, and evaluated classroom activities. At the end of each course, teachers individually implemented and analyzed their students' evolving learning during a lesson they had designed with their peers. Through feedback given by mentors, teachers increasingly (1) posed open-ended problem questions, (2) elicited students' ideas, and (3) built subsequent teaching on these ideas.

The content of the courses, although generally aligned with the topics discussed in middle and beginning of high school mathematics courses, was not matched to the particular materials used by the schools or state assessments. The approach to content focused mainly on understanding the logic and relations behind mathematical procedures, rather than memorizing procedural steps.

The first course introduces functions and its multiple representations through verbal, tabular, graphical and algebraic representations. It uses the lens of functions to examine numbers including fractions and arithmetic operations. Variables were introduced to emphasize relations among physical quantities, as opposed to relations among pure numbers.

The second course focuses on equations and inequalities as comparisons between two functions. Equations are represented algebraically and through the graphs of the functions that give rise to the equations. Solutions to equations correspond to one of the coordinates of the points of intersection of the graphs. Transformations are introduced as a special type of functions mapping a set, usually the line or the plane onto itself and are connected to the operations of addition and multiplication. Steps in solving equations are interpreted as the application of the same transformation on two functions, one on each side of the equal sign of the equation or as the application of the same transformation to each graph of the functions in the equations. Divisibility for integers ties up with divisibility for polynomials and solution of polynomial equations.

Pedagogical materials and activities require teachers to examine student reasoning about course topics in research reports, classroom videos, student interviews, and lessons they had designed and implemented. The activities and mentoring feedback address mathematical content and teachers' responses to

students' ideas and understanding. They promote classroom discussion of open-ended questions, starting from students' proposals.

A distinctive feature of the program is that, in each course unit, teachers discuss classroom videos from early algebra research and from teachers in previous cohorts, featuring third to seventh grade students making sound generalizations, using variable notation and graphs of linear functions to solve problems, and producing and analyzing graphs of non-linear functions. Teachers use these videos as a source for planning their own classroom activities and for how to respond to students' own ideas. Teachers are encouraged to introduce new conventional representations, moving from students' verbal statements or intuitive drawings to number line diagrams, data tables, graphs, algebraic expressions, and equations as models of relations among physical quantities. In this process, the contribution of each student is taken into account and discussed by their peers and by the teacher.

### **Method**

From August 2015 to December 2016, the program enrolled its fourth cohort, consisting of 53 elementary, middle, and high school teachers of mathematics and 10 special education teachers, coaches, or interventionists from a target district in the Greater Boston Area, with a large proportion of African-American and Hispanic/Latino students. Teachers volunteered to enroll in the program following encouragement by the district's curriculum coordinators.

Financial support for tuition, stipends, and computers were provided by the National Science Foundation.

We examined mathematics achievement of students in grades 3-8 over the academic year 2015-2016, when their teachers were taking the first and second courses in the program. Achievement was measured by the Partnership for Assessment of Readiness for College and Careers (PARCC, see [http://profiles.doe.mass.edu/state\\_report/parcc.aspx](http://profiles.doe.mass.edu/state_report/parcc.aspx)). Students were taught mathematics by 83 teachers in the district. They also received direct or indirect input from the other 21 professionals assigned to their classrooms. Of these 104 educators, 63 (61%) enrolled in the program.

We compare changes in student PARCC's mathematics results in the target district, from June 2015 (before teachers entered the program) to June of 2016 (when teachers had finished the second course), to changes in 10 similar comparison school districts. These were selected according to (a) similar percentages of students at the highest levels of achievement (levels 4 and 5) in 2014-2015 and (b) characteristics of districts and student population.

Because we focused on mathematics achievement during the year teachers were taking the two courses, we only analyzed PARCC's results in mathematics, instead of using the Student Growth Percentile (SGP), a state measurement of the overall progress of students in all content areas over a number of years. Thus, our list of comparison districts differs from the lists of similar districts included in

reports produced by the state on students' progress over the years

<http://profiles.doe.mass.edu/Default.aspx>.

In year 2014-2015, only 37% of the students in the target district and 35.6% of those in the 10 comparison districts were classified as proficient in mathematics (levels 4 and 5); this was well below the state percentage (51.5%). The target district and each comparison district had between 6,988 and 7,125 students, with median family income ranging from \$50,762 to \$54,795, and 14.9 to 14.11% of the families classified as living below the poverty line. Table 1 shows that target and comparison districts were also closely matched in terms of percentage of English language learners, economically disadvantaged students, students with disabilities, and students with high needs. However, it was not possible to closely match the target district to comparison districts on two measures: the target district had relatively more minority students (69.2% vs 47.4%) and students whose first language was not English (58.6% vs. 28.7%).

Table 1: Student Characteristics (2015-2016)

Districts	Minority Students	First Language not English	English Language Learner	Economically Disadvantaged	With Disabilities	High Needs
Target	69.2	58.6	16.0	42.1	15.1	61.9
Comparison	47.4	28.7	13.1	39.7	17.9	54.5
State	37.3	19.0	9.0	27.4	17.2	43.5

Teacher qualifications were similar in the target and comparison districts. Also, average salaries in the target district at \$76,262 were \$3,373 higher.

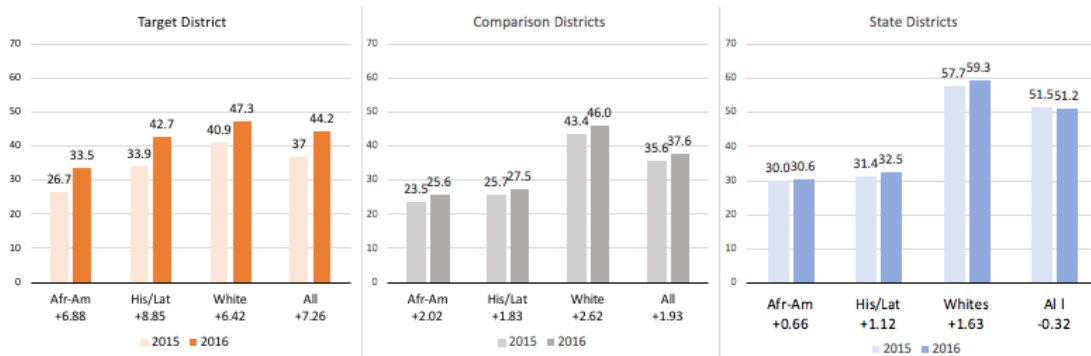
## **Results**

We first examine the proficiency for African-American, Hispanic/Latino, and White students in grades 3-8 from 2014-2015 (labeled henceforth as 2015) to 2015-2016 (labeled as 2016) school year. We complement this with an analysis on mean performance levels. Finally, we investigate shifts in performance across the five PARCC levels. Detailed data by grades and PARCC levels are available in the Appendix ([Tables A and B](#) and [Figures A and B](#)).

PARCC results are reported as one of five levels : L1 – Did Not Yet Meet Expectations, L2 – Partially Met Expectations, L3 – Approached Expectations, L4 – Met Expectations, and L5 – Exceeded Expectations. An achievement gap may be expressed as the difference between the percent of white students and minority students at levels L4 and L5. It may alternatively be expressed as the difference in mean performance level, to be defined below.

### **Changes in Proficiency Levels**

Figure 1 shows percentages of students at proficiency levels (L4+L5) in 2015 and 2016, in the state, target, and comparison districts. The change from 2015 to 2016 is indicated under each histogram bar.



Note: The negative change of -0.32 for All Students in the state is due to a drop in performance among Asian students, a sub-group not included in the analysis.

Figure 1. Performance Above the Proficiency Line L4+L5 in 2015 and 2016.

Across the state there were marginal increases in proficiency for each ethnic group (0.66% for African-Americans, 1.12% for Hispanic-Latinos, and 1.63% for Whites). The increases for comparison districts were also small (2.02% for African-American, 1.83% for Hispanic/Latinos and 2.62% for Whites). In contrast, in the target district there were more substantial increases in performance in all three ethnic subgroups, with slightly greater increases for the two minority groups (6.88% for African-Americans, 8.85% for Hispanic/Latinos) than for Whites (6.42%). Thus, the gap in proficiency between African-Americans and Whites decreased slightly, from 14.2 percentage points to 13.8, and the gap between Hispanic/Latinos and Whites decreased from 7.0 points to 4.6 points.

### Changes in Mean Performance

Table 2 shows mean performance for each group in 2015 and 2016, as well as changes in mean performance from one year to the next.

A district's mean performance is determined by treating each student's PARCC level as a score from 1 to 5: a group's mean performance is simply the average of all students' level scores. For example, in 2016, among the 492 African-American students in the target district, 51 students performed at level L1, 117 at L2, 159 at L3, 152 at L4, and 13 at L5. The calculation  $(1*51 + 2*117 + 3*159 + 4*152 + 5*13) / (51 + 117 + 159 + 152 + 13) = 1435/492$  results in an average performance of 2.917 for the sub-group.

Table 2. Mean Performance and Changes from 2015 to 2016

Ethnic Group	Target District				Comparison Districts				State			
	2015	2016	Change	p	2015	2016	Change	p	2015	2016	Change	p
Afr-Am	2.759	2.917	0.158	<.02	2.675	2.690	0.014	n.s.	2.821	2.813	-0.008	n.s.
N	469	492			6,484	6,742			16,551	27,712		
%	17.3	17.7			19.5	19.5			10.0	9.6		
His/Lat	2.930	3.123	0.193	<.0001	2.719	2.713	-0.006	n.s.	2.855	2.843	-0.012	n.s.
N	1,116	1,215			8,130	8,867			28,084	61,389		
%	41.2	43.9			24.4	25.7			17.9	2.1		
White	3.092	3.249	0.157	<.001	3.174	3.212	0.038	<.001	3.482	3.509	0.027	<.0001
N	891	844			15,946	16,098			103,966	166,526		
%	32.9	30.5			47.8	46.6			63.0	57.8		
Other	---	---			---	---			---	---		
N	231	156			2773	3997			134463	32772		
%	8.5	5.8			8.32	11.6			10.0	11.4		
All Stud	3.009	3.165	0.156	<.0001	2.973	2.989	0.016	<.006	3.338	3.313	-0.025	<.0001
N	2,707	2,769			33,333	34,557			165,023	288,399		
%	100	100			100	100			100	100		

Note: The number of students in the State is smaller for 2015 than for 2016 because fewer districts took PARCC's assessment in 2014-2015. Probability levels were determined by Mann-Whitney's U test.

Consistent with the analysis of proficiency levels, the three ethnic groups in the target district showed significant positive changes in mean performance from 2015 to 2016. In contrast, in the state and comparison districts only White



students showed positive changes. These, however, were much smaller than in the target district.

As shown in the table (and highlighted by [Figure A](#)), in the target district the change in mean performance for African-Americans matched the change for Whites and the change for Hispanic/Latinos was greater than that for Whites. Changes in the achievement gap, however, were small: the gap between African-American and Whites in the target district remained essentially the same and the gap between Hispanic/Latinos and Whites decreased by 0.04 points. In the comparison districts, the gaps increased by 0.02 for African-Americans and by 0.04 points for Hispanic/Latinos. In the state, they increased by 0.03 points and by 0.04 points, respectively.

An Analysis of Variance with time, (2015 vs. 2016), district (target vs. comparison), and ethnic group (White, African-American, and Hispanic/Latino) as main factors, and PARCC proficiency level as the dependent variable, revealed that the differences between means for each of the main factors was significant. The interaction between district (target vs. comparison) and time (2015 vs. 2016) was also significant ( $F_{(1, 1)} = 28.96$ ,  $p < 0.0001$ ), as illustrated by the steeper slopes for the target district graphs in Figure 2. The interaction between districts (target vs. comparison) and ethnic groups was also significant ( $F_{(1, 2)} = 46.98$ ,  $p < 0.0001$ ).

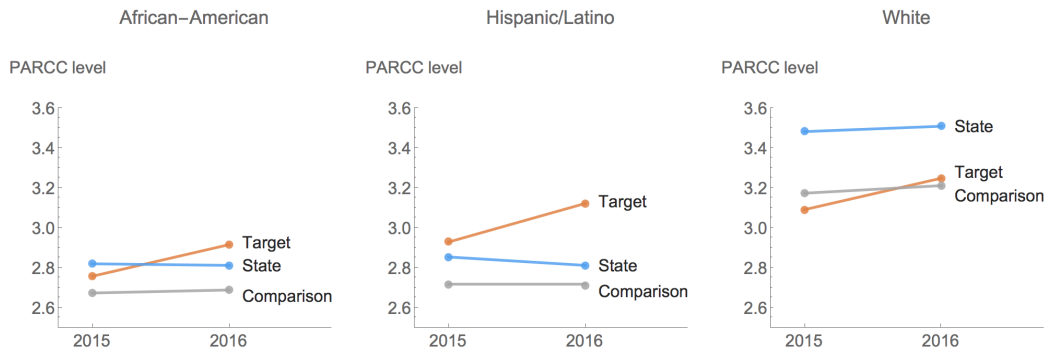


Figure 2. Mean Performance in 2015 and 2016 for Target, Comparison, and State Districts

With the exception of African-American in grades 3 and 7 and White students in grades 3, all grade levels in the target district showed larger increases (or smaller decreases) than the comparison groups (see [Table A](#)).

The effect sizes in the target district (z-scores calculated by dividing the change in mean performance by the standard deviation obtained by pooling the 2015 and 2016 data) were 0.154 for African-Americans, 0.188 for Hispanic/Latinos, and 0.154 for Whites. Effect sizes were small but the before-after difference was, as described above, significant. These measures are conservative, given that only 60% of the educators from the target district had enrolled in the program.

### Changes Across PARCC Levels

Figure 3 shows the percentage of students in each ethnic group, at each PARCC level, from 2015 to 2016. The figure highlights the fact that, generally,

drops in the percentages of students performing at levels L1 and L2 indicate improvement in performance, as do increases in percentages at levels L4 and L5. The latter were more pronounced in the target district (see [Figure B](#)).

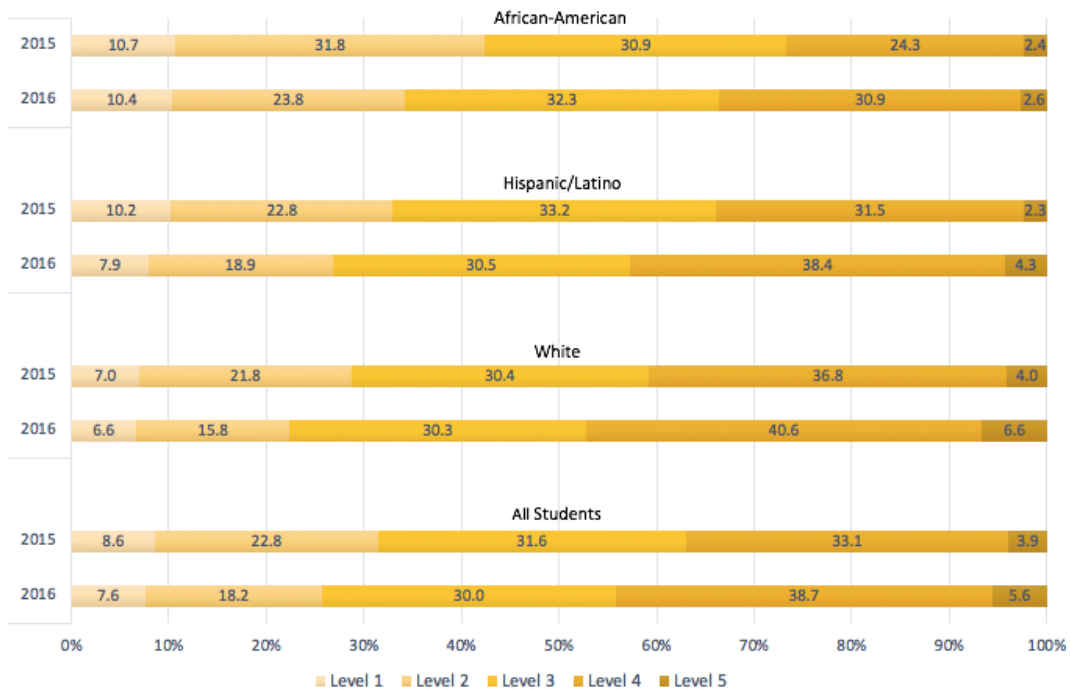


Figure 3: Percentage of Target District Students at each PARCC’s Performance Levels in 2015 and 2016.

### Summary and Discussion

As a whole, the gains in all three target-district ethnic groups outpaced those of the comparison districts and of the state. Within the target district, statistically significant gains emerged in the African-American, Hispanic-Latino, and White groups. Within the Comparison districts and across the state, only the White groups showed significant gains.

The gains for African-Americans (0.158) in the target district matched gains by White students (0.157). By this standard, the achievement gap between Whites and African-Americans was essentially unchanged. The gains by Hispanic/Latino students (0.193) were larger than those for White students. Thus, the achievement gap of Hispanic-Latinos in the target district slightly diminished. By contrast, in the comparison districts the achievement gap between whites and minorities slightly increased. In the state, the gaps slightly decreased for African-Americans and increased for Hispanic/Latinos.

If we look at the achievement gap as the difference in mean performance by minority students in comparison to the performance of White students in the state, the gap between Hispanic/Latino students in the target district and all students in the state was reduced by more than half, while the gap between African-American in the district and all students in the state was cut by more than a third. From this perspective, the gaps for both minority groups had narrowed from 2015 to 2016.

As a whole, our data suggest that the teacher development program played a positive role in students' learning and points to a possible contribution of our approach to narrowing the achievement gap. Determining the factor(s) responsible for significant gains for all groups in the target district is, however, fraught with difficulty, given the numerous features of the program. We will nonetheless

attempt to speculate on this matter, focusing on what appear be the most prominent characteristics of the teacher development program.

The program aimed at increasing teachers' mathematical and pedagogical knowledge. These two aspects are not unrelated. As teachers felt more secure in their knowledge and more aware of a variety of approaches to mathematics problems, they were able to better take into account students' reasoning, even when it deviated from standard approaches. As students perceived that their ideas were valued, they were able to participate in discussions and consider new ideas and representations. This may have led to deeper learning, especially for students from disadvantaged backgrounds that could feel neglected in more traditional classrooms. Increasing teacher's mathematical knowledge and their capacity for giving all students a voice and building on their individual ideas created a more equitable classroom, which in turn, we speculate, were key contributors to achievement by all groups in the target district. These assumptions are consistent with results from previous studies (Boaler & Staples, 2008 and Duncan & Murnane, 2014). We further hypothesize that, as students' mathematical ideas were valued and used as a basis for new learning, their attitudes and beliefs about their own capacity as mathematical learners may have increased. Teachers' confidence in what their students could achieve may also have increased as they analyzed research videos of diverse classroom showing third and fourth grade students making sound generalizations and using variable notation and graphs of

linear functions to solve problems. As Rosenthal & Jacobson (1968) found, “when teachers expected that certain children would show greater intellectual development, those children did show greater intellectual development (p. 20)”.

Institutional support may have been another factor behind the program’s contribution. When invited to participate in the program, the administrators and mathematics coordinators in the district had just implemented a new curriculum to address Common Core requirements and adopted new textbooks. They encouraged all teachers in grades 3 to 10 to join the program. According to publicly available information from the state department of education and from districts, the comparison districts also implemented changes to address Common Core requirements and promote their students’ success. However, we found no information suggesting that they undertook development programs as long and intensive as the one offered to the target district. The much smaller gains of comparison district students support the conclusion that gains in the target district are, at least in part, due to the program’s mathematical and pedagogical foundations.

While our results suggest that high quality teacher development in mathematics may lead to more equitable teaching and narrowing of the achievement gaps, we must acknowledge certain limitations of our analysis:

1. For confidentiality reasons, we did not have access to individual student results and were not able to separate results of students whose teachers had enrolled in the program from those of the remaining students.
2. We did not keep track of other teacher development programs and initiatives in comparison districts during the period we evaluated.
3. The measurement of student achievement by written standardized tests may not measure deep understanding.
4. The effect size of gains in the target district groups, even though statistically significant, were modest.
5. It is not clear why the Hispanic/Latino students gains were larger than those of African-Americans. Possible factors to explore in the future are the greater concentration of Hispanic-Latino students which may make them feel at home in their school, the larger number of Hispanic/Latino teachers than African-American teachers, and the initial higher performance level of Hispanic/Latinos.
6. We did not directly evaluate changes in classroom teaching which would help explain specific changes in performance.

The program involved 60% of the educators in the district. Had all of them enrolled, one would expect the benefits of the program to be even greater.

We found shifts from the lowest performance levels towards higher performance levels for all groups in the target district. The program's activities

did not include specific recommendations regarding minority students' achievement, the achievement gap, or racial awareness aspects. These aspects, if incorporated in future implementations, might lead to higher achievement gains among minority students, a hypothesis that needs to be checked experimentally.

While complementing findings from previous studies in the direction of reducing the achievement gap, our results point to the importance of long and intensive teacher development programs, that integrates mathematical and pedagogical knowledge, in promoting mathematical achievement among minority students.

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## APPENDICES

Table A. Average Performance and Change by Grade-Level in 2015 and 2016)

Average Performance	Target District			Comparison Districts			State			
	2015	(N)	2016 (N)	Change	2015 (N)	2016 (N)	Change	2015 (N)	2016 (N)	Change
<b>African American</b>										
Grade 3	3.038	( 79 )	3.044 ( 90 )	0.006	2.746 ( 1,110 )	2.857 ( 1,256 )	0.112	2.907 ( 2,324 )	2.989 ( 4,889 )	0.082
Grade 4	2.904	( 73 )	3.106 ( 85 )	0.202	2.700 ( 1,122 )	2.713 ( 1,143 )	0.013	2.762 ( 2,883 )	2.818 ( 4,720 )	0.056
Grade 5	2.692	( 91 )	3.012 ( 81 )	0.320	2.662 ( 1,107 )	2.736 ( 1,149 )	0.074	2.914 ( 2,729 )	2.897 ( 4,604 )	-0.017
Grade 6	2.863	( 80 )	3.011 ( 88 )	0.149	2.646 ( 1,046 )	2.652 ( 1,096 )	0.006	2.876 ( 2,816 )	2.770 ( 4,595 )	-0.106
Grade 7	2.608	( 74 )	2.600 ( 75 )	-0.008	2.642 ( 1,047 )	2.603 ( 1,058 )	-0.039	2.732 ( 3,075 )	2.768 ( 4,544 )	0.036
Grade 8	2.431	( 72 )	2.644 ( 73 )	0.213	2.651 ( 1,052 )	2.537 ( 1,040 )	-0.115	2.758 ( 2,724 )	2.611 ( 4,360 )	-0.147
<b>Hispanic-Latino</b>										
Grade 3	3.116	( 232 )	3.331 ( 254 )	0.214	2.884 ( 1,620 )	3.029 ( 1,734 )	0.145	2.901 ( 4,973 )	3.047 ( 11,503 )	0.145
Grade 4	3.126	( 191 )	3.372 ( 231 )	0.247	2.848 ( 1,476 )	2.868 ( 1,676 )	0.020	2.827 ( 5,217 )	2.920 ( 10,958 )	0.093
Grade 5	2.723	( 195 )	3.070 ( 200 )	0.347	2.642 ( 1,353 )	2.650 ( 1,449 )	0.009	2.938 ( 4,623 )	2.867 ( 10,199 )	-0.072
Grade 6	3.149	( 161 )	3.122 ( 197 )	-0.027	2.702 ( 1,258 )	2.615 ( 1,422 )	-0.087	2.906 ( 4,513 )	2.756 ( 10,034 )	-0.150
Grade 7	2.764	( 161 )	2.915 ( 165 )	0.151	2.600 ( 1,223 )	2.605 ( 1,275 )	0.005	2.791 ( 4,723 )	2.775 ( 9,577 )	-0.016
Grade 8	2.653	( 176 )	2.738 ( 168 )	0.085	2.563 ( 1,200 )	2.377 ( 1,311 )	-0.187	2.755 ( 4,035 )	2.635 ( 9,118 )	-0.120
<b>White</b>										
Grade 3	3.201	( 149 )	3.350 ( 157 )	0.149	3.281 ( 2,879 )	3.455 ( 2,960 )	0.174	3.554 ( 18,127 )	3.682 ( 27,718 )	0.128
Grade 4	3.315	( 130 )	3.322 ( 149 )	0.007	3.250 ( 2,676 )	3.245 ( 2,857 )	-0.005	3.392 ( 16,794 )	3.513 ( 27,543 )	0.121
Grade 5	3.021	( 144 )	3.256 ( 121 )	0.235	3.130 ( 2,676 )	3.263 ( 2,617 )	0.133	3.560 ( 18,017 )	3.511 ( 27,917 )	-0.049
Grade 6	3.236	( 140 )	3.363 ( 135 )	0.127	3.180 ( 2,633 )	3.111 ( 2,614 )	-0.069	3.531 ( 17,851 )	3.471 ( 28,369 )	-0.060
Grade 7	3.018	( 167 )	3.099 ( 121 )	0.081	3.124 ( 2,603 )	3.158 ( 2,589 )	0.034	3.411 ( 17,309 )	3.469 ( 29,170 )	0.058
Grade 8	2.826	( 161 )	3.093 ( 161 )	0.267	3.064 ( 2,479 )	2.994 ( 2,461 )	-0.069	3.433 ( 15,868 )	3.405 ( 25,809 )	-0.027
<b>All Students</b>										
Grade 3	3.168	( 505 )	3.320 ( 535 )	0.151	3.076 ( 6,110 )	3.226 ( 6,507 )	0.150	3.408 ( 28,079 )	3.475 ( 49,981 )	0.067
Grade 4	3.170	( 440 )	3.318 ( 510 )	0.147	3.042 ( 5,809 )	3.046 ( 6,181 )	0.003	3.251 ( 27,860 )	3.320 ( 48,763 )	0.069
Grade 5	2.863	( 459 )	3.150 ( 440 )	0.287	2.932 ( 5,609 )	3.003 ( 5,736 )	0.071	3.425 ( 28,445 )	3.330 ( 48,304 )	-0.095
Grade 6	3.211	( 426 )	3.211 ( 454 )	0.000	2.970 ( 5,369 )	2.909 ( 5,591 )	-0.061	3.393 ( 27,966 )	3.272 ( 48,621 )	-0.122
Grade 7	2.890	( 435 )	3.003 ( 398 )	0.113	2.919 ( 5,280 )	2.923 ( 5,350 )	0.005	3.261 ( 27,748 )	3.283 ( 48,843 )	0.022
Grade 8	2.740	( 442 )	2.910 ( 432 )	0.170	2.879 ( 5,156 )	2.765 ( 5,192 )	-0.114	3.281 ( 24,925 )	3.184 ( 43,887 )	-0.097

Table B. Percentage of Students by PARCC Performance Levels and Change in  
2015 and 2016)

PARCC Levels	Target District			Comparison Districts			State		
	2015 (N)	2016 (N)	Change	2015 (N)	2016 (N)	Change	2015 (N)	2016 (N)	Change
<b>African American</b>									
L1	10.66% ( 50 )	10.37% ( 51 )	<b>-0.295%</b>	13.74% ( 891 )	15.32% ( 1,033 )	<b>1.580%</b>	11.95% ( 1,978 )	13.86% ( 3,840 )	<b>1.906%</b>
L2	31.77% ( 149 )	23.78% ( 117 )	<b>-7.989%</b>	29.80% ( 1,932 )	27.99% ( 1,887 )	<b>-1.808%</b>	26.74% ( 4,425 )	24.75% ( 6,859 )	<b>-1.985%</b>
L3	30.92% ( 145 )	32.32% ( 159 )	<b>1.400%</b>	32.93% ( 2,135 )	31.13% ( 2,099 )	<b>-1.794%</b>	31.35% ( 5,189 )	30.77% ( 8,527 )	<b>-0.582%</b>
L4	24.31% ( 114 )	30.89% ( 152 )	<b>6.587%</b>	22.25% ( 1,443 )	23.52% ( 1,586 )	<b>1.269%</b>	27.21% ( 4,503 )	27.50% ( 7,620 )	<b>0.290%</b>
L5	2.35% ( 11 )	2.64% ( 13 )	<b>0.297%</b>	1.28% ( 83 )	2.03% ( 137 )	<b>0.752%</b>	2.76% ( 456 )	3.13% ( 866 )	<b>0.370%</b>
<b>Hispanic-Latino</b>									
L1	10.22% ( 114 )	7.90% ( 96 )	<b>-2.314%</b>	13.09% ( 1,064 )	16.14% ( 1,431 )	<b>3.051%</b>	11.49% ( 3,227 )	13.65% ( 8,381 )	<b>2.162%</b>
L2	22.76% ( 254 )	18.85% ( 229 )	<b>-3.912%</b>	29.27% ( 2,380 )	26.75% ( 2,372 )	<b>-2.523%</b>	26.01% ( 7,305 )	24.39% ( 14,972 )	<b>-1.623%</b>
L3	33.15% ( 370 )	30.53% ( 371 )	<b>-2.619%</b>	31.94% ( 2,597 )	29.58% ( 2,623 )	<b>-2.362%</b>	31.10% ( 8,733 )	29.43% ( 18,068 )	<b>-1.664%</b>
L4	31.54% ( 352 )	38.44% ( 467 )	<b>6.895%</b>	24.05% ( 1,955 )	24.72% ( 2,192 )	<b>0.674%</b>	28.33% ( 7,956 )	29.04% ( 17,828 )	<b>0.712%</b>
L5	2.33% ( 26 )	4.28% ( 52 )	<b>1.950%</b>	1.65% ( 134 )	2.81% ( 249 )	<b>1.160%</b>	3.07% ( 863 )	3.49% ( 2,140 )	<b>0.413%</b>
<b>White</b>									
L1	6.96% ( 62 )	6.64% ( 56 )	<b>-0.323%</b>	6.40% ( 1,020 )	7.29% ( 1,174 )	<b>0.896%</b>	3.58% ( 3,717 )	4.34% ( 7,219 )	<b>0.760%</b>
L2	21.77% ( 194 )	15.76% ( 133 )	<b>-6.015%</b>	18.82% ( 3,001 )	17.09% ( 2,751 )	<b>-1.731%</b>	12.31% ( 12,803 )	11.59% ( 19,306 )	<b>-0.721%</b>
L3	30.42% ( 271 )	30.33% ( 256 )	<b>-0.084%</b>	31.41% ( 5,009 )	29.62% ( 4,769 )	<b>-1.787%</b>	26.42% ( 27,473 )	24.76% ( 41,229 )	<b>-1.667%</b>
L4	36.81% ( 328 )	40.64% ( 343 )	<b>3.827%</b>	37.70% ( 6,012 )	39.07% ( 6,289 )	<b>1.365%</b>	47.66% ( 49,549 )	47.46% ( 79,033 )	<b>-0.199%</b>
L5	4.04% ( 36 )	6.64% ( 56 )	<b>2.595%</b>	5.67% ( 904 )	6.93% ( 1,115 )	<b>1.257%</b>	10.03% ( 10,424 )	11.85% ( 19,739 )	<b>1.827%</b>
<b>All Students</b>									
L1	8.61% ( 233 )	7.55% ( 209 )	<b>-1.059%</b>	9.44% ( 3,148 )	11.09% ( 3,834 )	<b>1.651%</b>	5.70% ( 9,400 )	7.27% ( 20,979 )	<b>1.578%</b>
L2	22.79% ( 617 )	18.20% ( 504 )	<b>-4.591%</b>	23.48% ( 7,825 )	21.62% ( 7,470 )	<b>-1.859%</b>	15.82% ( 26,112 )	15.59% ( 44,971 )	<b>-0.230%</b>
L3	31.62% ( 856 )	30.01% ( 831 )	<b>-1.611%</b>	31.45% ( 10,483 )	29.73% ( ##### )	<b>-1.722%</b>	26.94% ( 44,456 )	25.91% ( 74,738 )	<b>-1.024%</b>
L4	33.06% ( 895 )	38.68% ( 1,071 )	<b>5.616%</b>	31.57% ( 10,522 )	32.40% ( ##### )	<b>0.829%</b>	42.05% ( 69,392 )	40.95% ( 118,100 )	<b>-1.100%</b>
L5	3.92% ( 106 )	5.56% ( 154 )	<b>1.646%</b>	4.07% ( 1,355 )	5.17% ( 1,785 )	<b>1.100%</b>	9.49% ( 15,663 )	10.27% ( 29,611 )	<b>0.776%</b>

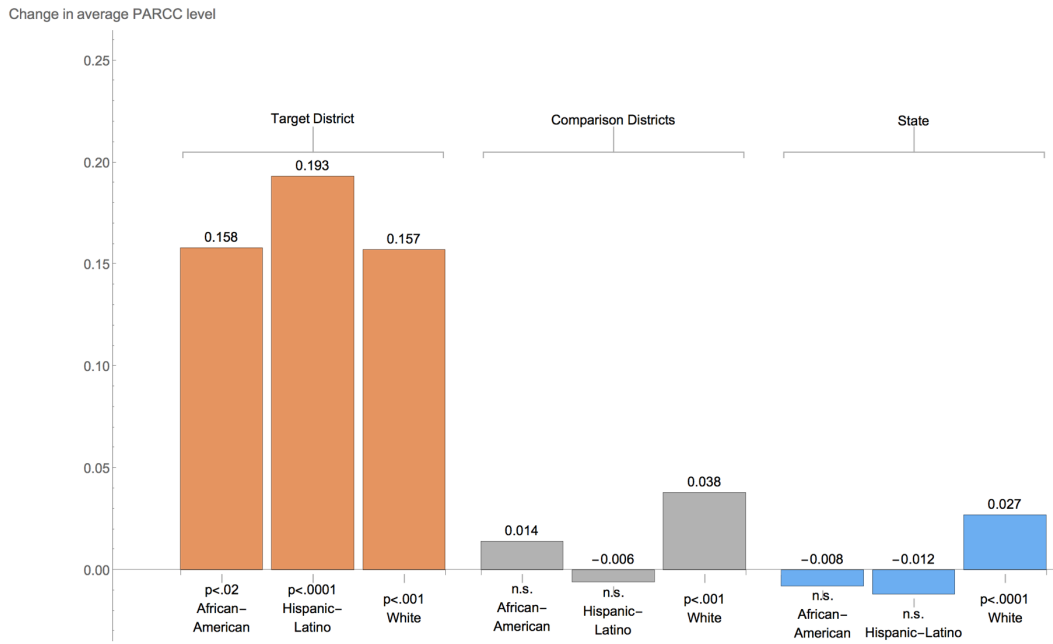


Figure A. Changes in Mean Performance Over One Year by Ethnicity in the Target, Comparison, and State Districts



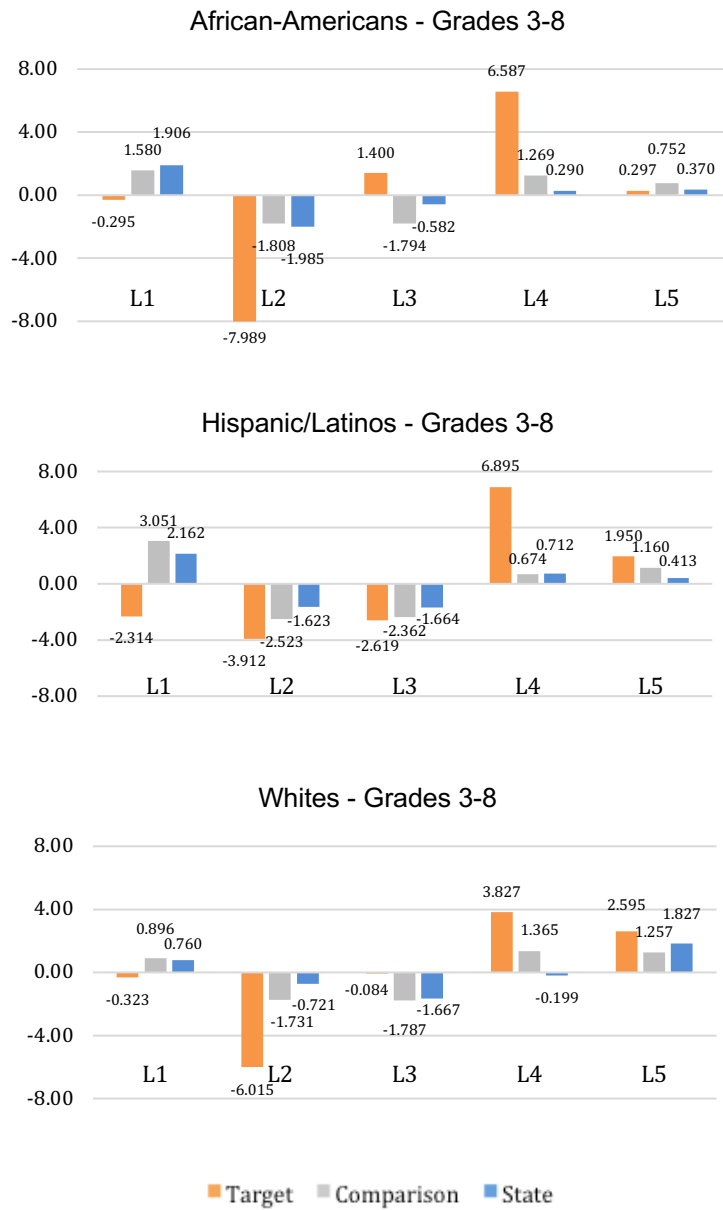


Figure B. Percent Change for Each PARCC Performance Level from 2015 to 2016