Uncertainty analysis for water supply reservoir yields

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SUMMARY

Understanding the variability of water supply reservoir yields is central for planning purposes. The basis of this study is an empirical global relationship between reservoir storage capacity, water supply yield and reliability based on a global database of 729 rivers. Monte Carlo simulations reveal that the coefficient of variation of estimates of water supply reservoir yields depend only on the length of streamflows record and the coefficient of variation of the streamflows used to estimate the yield. We compare the results of those Monte Carlo experiments with an analytical uncertainty method First Order Variance Approximation (FOVA). FOVA is shown to produce a general, accurate and useful expression for estimating the coefficient of variation of water supply reservoir yield estimates. We also document how the FOVA analytical model can be used to determine the minimum length of streamflow record required during the design of water supply reservoirs so as to ensure that the yield delivered from reservoir falls within a prespecified margin of error.

1. Introduction

In the design of water supply reservoirs, the Storage–Reliability–Yield (SRY) relationship is the tool that has traditionally been used to determine the reservoir storage capacity required for delivery of a water supply yield with a specified reliability or alternatively, the water supply yield that can be supplied from a reservoir with known storage capacity. Here reliability is the steady-state time based value that indicates the probability that the reservoir yields are met. Methods available for determining the SRY relationship from a streamflow record may be broadly classified into (1) sequential, and (2) nonsequential procedures. Using sequential procedures to determine the appropriate storage capacity for meeting a prespecified yield requires routing of the complete streamflow record (or synthetic traces based thereupon) through the reservoir system while accounting for the necessary outflows which may include: water supply, evaporation, seepage losses, downstream releases to maintain ecological flow regimes and other operations. The two main sequential procedures that have seen wide application are Rippl's mass curve (Rippl, 1883) or its automated Sequent Peak Algorithm (SPA) (Thomas and Burden, 1963) and behavior analysis (BA) (see McMahon and Adeloye, 2005) respectively. The main difference between these two sequential procedures is that SPA

assumes no water supply failures occur over the planning period while BA is based on a prespecified reliability thus water supply failures are allowed. Here reliability is defined as the steady-state probability that the reservoir yields are met.

Use of simulation procedures to derive the steady-state SRY relationship of reservoirs is computationally intensive because a stochastic streamflow model and a reservoir simulation model must be combined and implemented repeatedly, using thousands of Monte-Carlo experiments, corresponding to nearly all possible reservoir systems. Attempts have been made to develop generalized SRY relations based on a nonsequential model of reservoir behavior such as those summarized by McMahon et al. (2007a) and others cited therein. We do not consider such nonsequential SRY models here, because the sequential character of flow series is critical to understanding the SRY relationship.

Several studies have developed generalized SRY for reservoirs either fed by actual streamflows (Adeloye et al., 2003; Adeloye, 2009a,b; Silva and Portela, 2012; McMahon et al., 2007b; Kuria and Vogel, 2014a) or inflows generated from stochastic streamflow models (Pegram, 1980; Bayazit and Bulu, 1991; Bayazit and Önöz, 2000; Bayazit, 1982; McMahon and Mein, 1986; McMahon and Adeloye, 2005; Gould, 1964; Vogel and Stedinger, 1987; Phien, 1993). A recent detailed review of generalized SRY relationships based on a sequential analysis is provided in Kuria and Vogel (2014a). Here we use the generalized empirical SRY model developed by Kuria and Vogel (2014a) because it enables us to document the variability of water supply reservoir yields of an

extremely wide range of reservoir systems. The SRY model developed by Kuria and Vogel (2014a) is an adaptation of the SRY model introduced by McMahon et al. (2007b) both of which were based on a global database of monthly streamflow series from 729 rivers.

In the design of reservoirs using generalized SRY relations the yield Y, is often assumed constant. The SRY relationship is based on Monte-Carlo simulations which attempt to represent the impact of natural variations in streamflow on the reliability of water supply yields from a reservoir system. However, such yield estimates are in of themselves random variables, because they are estimated on the basis of short and often quite variable streamflow records. Therefore due to natural variability and the limited lengths of streamflow sequences, the estimated yields and/or the storage capacity requirements are random variables because both are estimated on the basis of a limited dataset. Thus variability in water supply yield estimates arises from two primary sources, natural variability inherent in streamflow sequences and sampling variability resulting from the limited lengths of streamflow records. Thus one can expect that estimates of storage, yield and/or reliability are all random variables which can be subject to considerable variability due to our limited knowledge of future hydrologic and climatic conditions which govern the SRY relationship.

Considering the importance of water supply planning in the context of surface water reservoirs, remarkably few studies have attempted to document the sampling variability of estimates of the storage, yield and/or reliability associated with surface water supply systems. A few studies have documented the variability of estimates of the reliability and the storage capacity of a surface water reservoir, yet fewer studies have considered the variability of yield estimates. Klemes (1979) examined the variability of estimates of the reliability of reservoir with fixed capacity and draft based on 25 year flow record for a single river with a coefficient of variation Cv = 0.3 corresponding to the sequence of annual streamflows. He found that the 95% confidence interval for reliability was in the range 0.8–1 while the point estimate was 0.97. Using first order uncertainty analysis, Phatarford (1977) derived approximations for the bias and variance of estimates of storage capacities for reservoirs fed by independent Gamma flows. His results indicate extreme bias and variance associated with estimates of storage capacities. For example, he found that with n = 100 years of streamflow with inflow Cv = 2 and yield ratio Y/μ = 0.9 (μ corresponds to mean annual streamflow), upward bias and root mean square error were 196% and 131% respectively. Phatarford (1977) findings led Vogel (1985) and Vogel and Stedinger (1988) to examine more generally, the sampling variability of estimates of the reservoir storage capacity when inflows follow an autoregressive lognormal (AR (1) LN) model. The only study we could locate which derived a confidence interval for estimates of water supply yield was a study by Vogel and Hellstrom (1988) which used Monte-Carlo simulations to reveal the variability of estimates of 'safe yield' for a specific system – Boston water supply system in Massachusetts, USA. Though the average 'safe yield' of the system was reported as 300 million gallons per day (mgd), they found that a 99% confidence interval of the "safe" yield was 232–370 mgd. Thus previous studies have documented the instability of an estimated storage–reliability–yield (SRY) relationship for specific systems. Kuria and Vogel (2014b) carried out a rigorous and general evaluation of the sampling properties of water supply yield for a wide range of reservoir systems subject to the type of hydrologic variations and conditions which can be expected anywhere in the world using Monte Carlo (MC) simulations. Our primary goal is to develop a generalized analytical methods that can be used to document the variability of yield estimates while avoiding the numerous computations involved in MC simulations. Results from an analytical uncertainty analysis methods will be compared with the MC simulations results from Kuria and Vogel (2014b). Further,

we apply the analytical uncertainty method to illustrate how minimum streamflow record lengths can be determined so as to ensure that reservoir yields fall within a specified range with prespecified reliability during the actual performance of the reservoirs.

2. Uncertainty analysis methods

The probability density function (pdf) provides a complete description of the probabilistic behavior of a random variable. Unfortunately, in most practical problems such a pdf cannot easily be derived especially when the random variable is a function of several other correlated random variables as is the case here. Other methods that can be used to describe the dispersion of a random variable include confidence intervals and moments. In this study we use the statistical properties of a pdf: variance and coefficient of variation, Cv, to describe the uncertainty in surface water reservoir yield estimates. We compare an analytical uncertainty method for determining the Cv and variance of water supply yields with the results based on Monte Carlo simulations presented in Kuria and Vogel (2014b).

One attractive analytical uncertainty approach is the Mellin Transform which was first introduced to the field of hydrology and hydraulics by Tung (1990). It is one of the analytical methods used to derive exact moments of any random variable if the following conditions are met: (1) the random variable is a multiplicative function of the uncertain variables and (2) the uncertain variables are independent of one another as well in time (Tung, 1990; Tung and Yen, 2005). The method is further limited for use with functional relations whose input stochastic variables have pdfs with known Mellin Transform. The method has seen limited application in water resources studies perhaps due to the lack of independence of random variables or because many models have stochastic variables with pdfs with no known Mellin Transform. Our yield equation meets the first condition for application of Mellin Transform. The random variables needed to estimate water supply reservoir yield are estimates of the mean, variance and skewness of the annual streamflows. Since reservoir inflows are serially correlated and their moments often exhibit cross correlation the second property required for use of a Mellin Transform is violated thus we do not summarize its use here. Interested readers are referred to Kuria (2015) who did an attempt to apply the Mellin Transform to this problem.

First Order Variance Approximation FOVA is an approximate analytical method that is widely used to derive estimates of the mean and variance for any given random variable which is a function of several other random variables. The method is based on first order Taylor series approximation of the random variable about the mean of the stochastic variables. Benjamin and Cornell (1970) and Tung and Yen (2005) provide a detailed description of how FOVA is carried out. For completeness we present the details here. Consider a model output Y, related to n random variables as:

$$Y = f(X_1, X_2, \ldots, X_n) \tag{1}$$

Given μ_{xi} is the mean of each random variable X_i, i = 1,2,...,n, by the first order approximation of the Taylor series expansion about the mean of each random variable, the expectations and variance of Y can be approximated by:

$$E[Y] = E[f(X)] \cong f(E[X]) = f(\mu_x) \tag{2}$$

$$Var[Y] \cong \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial Y}{\partial X_i}\bigg|_{\mu_{x_i}} \frac{\partial Y}{\partial X_j}\bigg|_{\mu_{x_j}} Cov(X_i, X_j) \tag{3}$$

For bivariate input random variables Eq. (3) reduces to

$$Var[Y] \cong \left(\frac{\partial Y}{\partial X_1} \bigg|_{\mu_{x_1}} \right)^2 Var(X_1) + \left(\frac{\partial Y}{\partial X_2} \bigg|_{\mu_{x_2}} \right)^2 Var(X_2)$$

$$+ 2 \frac{\partial Y}{\partial X_1} \bigg|_{\mu_{x_1}} \frac{\partial Y}{\partial X_2} \bigg|_{\mu_{x_2}} Cov(X_1, X_2) \qquad (4)$$

The assumptions often cited for the FOVA method to yield good results are (1) near linearity in functional relations, (2) the random variables of the equation should follow a nearly normal distribution, and (3) the variability of the random input variables should be small (Benjamin and Cornell, 1970; Cornell, 1972; Melching, 1995). According to Moore and Clarke (1981) "FOVA is rarely likely to be justifiable with models containing nine or twelve parameters". Note that FOVA requires covariance terms which are often neglected, which is probably why Moore and Clarke (1981) came to that the above referenced conclusion. Several studies for example Garen and Burges (1981), Walker (1982), Burges (1979), Scavia et al. (1981), Gardner et al. (1981), Gardner and O'Neill (1983), Smith and Charbeneau (1990), Burges and Lettenmaier (1975), Melching (1995), Gardner et al. (1981), and Gardner and O'Neill (1983) have attempted to evaluate the accuracy of variance estimates derived using FOVA with contradictory results. Despite these conceptual limitations the method is perhaps the most widely used method for uncertainty analysis and has successfully been used for uncertainty analysis studies in hydrologic design (Tang and Yen, 1972; Yen et al., 1980; Tung and Mays, 1980), water quality modeling (Burges and Lettenmaier, 1975; Scavia et al., 1981; Chadderton et al., 1982; Melching and Anmangandla, 1992), watershed modeling (Garen and Burges, 1981; Bates and Townley, 1988) and subsurface flow and contaminant transport (Sagar, 1978; Dettinger and Wilson, 1981; Devary and Doctor, 1982; Townley and Wilson, 1985) among others.

Though several studies have been carried out with the objective of attempting to verify the assumptions of the FOVA, it appears that no general conclusions can be drawn from the comparisons of FOVA and MC simulations. When Gamma distribution is the assumed model of streamflows, using the central limit theorem (CLT), estimates of the mean and standard deviation of the inflows may be assumed to follow a normal distribution. Here the CLT applies approximately because estimates of the mean and variance of annual streamflows are made up of the sum of many individual streamflows. We show that when normal distributions are assumed in the MC simulations, the results of output uncertainty analysis are in agreement with the FOVA. Burges (1979) found accurate agreement between FOVA and MC when the Cv of the input variables was found to be about 0.2. However he also used a non-normal distribution for the flood frequency: extreme value type I. Thus it remains unclear if it is the effect of Cv or the non-normal distribution.

3. Previous comparisons of FOVA with MC simulations

Previously MC simulation has been used to check the accuracy of variance estimates from the more approximate FOVA. Studies by Burges and Lettenmaier (1975), Garen and Burges (1981), Walker (1982), Burges (1979) found that the two methods were in agreement. In the Burges and Lettenmaier (1975) study, normal distributions for the random variables was assumed in the Streeter-Phelps equation considering a Cv range of the input random variable of 0.05–0.32. However, in the Garen and Burges (1981) study, a highly non-linear rainfall–runoff model was used assuming uniform distributions for the random variables with Cv range of 0.1–0.6. In the Garen and Burges study agreements were found when the Cv of the input random variables were less than 0.3. However studies by Scavia et al. (1981), Gardner et al.

(1981), Gardner and O'Neill (1983), Smith and Charbeneau (1990) found contrary results. Scavia et al. (1981) considered a highly non-linear lake eutrophication model assuming triangular distributions with Cv range of 0.16–0.42. Gardner et al. (1981) also considered a non-linear model. Though in these studies the three cited assumptions of FOVA were not met, it is also noted that in most cases, independence of the input random variables was assumed which would have led to the observed disagreement. Scavia et al. (1981) actually confirmed that near linearity did not play a significant role in the disagreement between the FOVA and MC results. These studies illustrate that the three assumptions previously cited as necessary for a good fit of FOVA analysis are apparently not adequate or sufficient.

Ignoring higher order terms has been highlighted as one of the reasons why FOVA does not yield accurate results. However, including such higher order terms is only possible for bias estimates (see Phatarford, 1977 for a good example of how to use FOVA to derive a first estimate of bias). For variance estimates such terms requires the knowledge of higher order moments which are mostly not available. Bates and Townley (1988) suggest that inclusion of second order terms in Eq. (2) may increase the accuracy of mean estimates especially when the degree of non-linearity is significant while Scavia et al. (1981) shows that first order linearization is probably not the main reason that disagreement between variance estimates from FOVA and Monte Carlo simulations. For functional relations with strictly independent random variables Tyagi and Haan (2001) derived the correction factors FOVA estimates of mean and variance of a multiplicative and additive power function. Generally most studies conclude that the lack of agreement increased as the variability of input random variable was increased. We hypothesize that if the covariance between the input random variables is included, the FOVA may give results that are almost in agreement with MC even when these assumptions are violated. However, for most models, it is not easy to evaluate this covariance thus this is one of the main limiting factors of the application of FOVA. In general, we conclude that MC experiments are essential for evaluating the degree of accuracy obtained from FOVA, and such an approach is taken here. Once, or if, FOVA is verified, however, it can provide advantages over MC analyses because it provides more general results which are much less computational intensive than MC experiments.

4. Generalized SRY relationship based on actual streamflows

The generalized SRY that was used to derive estimates of water supply yield, Y, in this study was developed by Kuria and Vogel (2014a) from a global database of 729 unregulated rivers all of which had continuous monthly streamflow data for at least 25 years (see their Fig. 1 for the spatial coverage of the dataset). The median mean annual runoff (MAR) = 354 mm, coefficient of variation (Cv) = 0.314 and skewness (γ) = 0.561 while the maximum MAR = 5370 mm, Cv = 2.97 and γ = 0.119 and minimum MAR = 0.393 mm, Cv = 0.0619 and γ = −2.22. See Tables 1 and 2 of McMahon et al. (2007b) for detailed statistical descriptions of these rivers. This is the same dataset used by McMahon et al. (2007b) to develop a generalized SRY for estimating reservoir storage capacity S. Kuria and Vogel (2014b) found that when the SRY model developed by McMahon et al. (2007b) for estimating S, is used (solved) to estimate yield, Y, the resulting estimates often led to yield ratios (yield/mean annual of the flows) $Y/\mu > 1$ which are infeasible in practice. The following approach was used by McMahon et al. (2007b) to obtain the values of the variables needed to develop their generalized SRY model. For each monthly flow series, behavior analysis (McMahon and Adeloye, 2005, pg 80–81) was used to calculate storage capacities for hypothetical
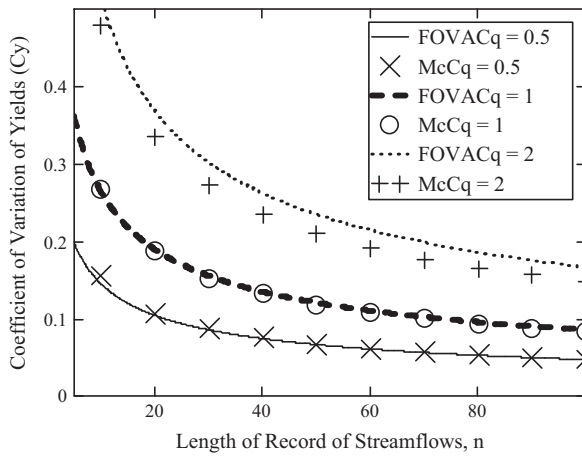
Fig. 1. Comparison of coefficient of variation of yield using FOVA and MC simulation.

Table 1
Coefficient of variation of estimated reservoir yield, Cy resulting from Monte-Carlo (MC) and first-order (FOVA) analysis for reservoirs fed by Gamma annual inflows with coefficient of variation, Cq.

n	MC ($C_q = 0.5$)	FOVA ($C_q = 0.5$)	MC ($C_q = 1$)	FOVA ($C_q = 1$)	MC ($C_q = 2$)	FOVA ($C_q = 2$)
10	0.155	0.145	0.268	0.264	0.482	0.514
20	0.107	0.104	0.188	0.189	0.335	0.368
30	0.087	0.085	0.152	0.155	0.273	0.302
40	0.075	0.074	0.132	0.135	0.235	0.262
50	0.067	0.066	0.118	0.121	0.21	0.235
60	0.061	0.061	0.108	0.11	0.191	0.214
70	0.057	0.057	0.1	0.102	0.177	0.199
80	0.053	0.053	0.093	0.096	0.165	0.186
90	0.05	0.05	0.088	0.09	0.156	0.175
100	0.047	0.047	0.083	0.085	0.148	0.166

Table 2
The Lower Intervals (LI) and Upper Interval (UI) of reservoir true yield estimated from Eq. (1) for $R = 0.8$ and 0.95 and $S/\mu = 0.5$, 1 and 2.5.

	True yield, Y (m³/yr)	LI (m³/yr)	UI (m³/yr)
$R = 0.8, S/\mu = 0.5$	277	246	318
1	319	283	366
2.5	384	341	441
$R = 0.95, S/\mu = 0.5$	226	200	259
1	260	230	298
2.5	313	278	359

reservoirs required to meet yields in the range of 30–80% of mean historical flow for three steady state reliabilities of 90%, 95% and 98%. A constant yield was assumed during each simulation. These experiments resulted an overall dataset with 12, 413 combinations of S, R and, Y across all 729 rivers.

McMahon et al. (2007b) used Weighted Least Squares (WLS) regression to develop two sets of generalized SRY models based on storage estimates derived from SPA and BA. WLS regression was employed to account for the fact that each streamflow record had a different record length leading to estimates of S, R, and Y with varying accuracy. The weights were proportional to the record lengths giving greater weight to rivers with more streamflow information. Their resulting multivariate regression equations relate the storage capacity, S, as a function of numerous independent variables including: the mean μ, standard deviation σ, and skew γ of the annual inflows and those parameters describing the system yield Y and reliability R. Their equation for storage capacity based on BA summarized in Table 4 of their paper is:

$$S = 1.932\mu^{-3.254}\sigma^{1.599}\gamma^{-0.074}Y^{2.67}Z_R^{1.445} \quad (R^2 = 0.899) \tag{5}$$

where S is storage capacity, μ, σ and γ are the mean, standard deviation and skew coefficient of the annual inflows, Y is the yield which all have units in millions of m³. Z_R is the standardized normal variate with R equal to the system reliability (for example, a system with reliability $R = 0.95$ corresponds to a value of $Z_R = 1.645$). Kuria and Vogel (2014a) evaluated Eq. (5) more fully by comparing a more robust form of regression known as Iterative Reweighted Least Squares (IRLS) and concluded that Eq. (5) compared favorably to their slightly improved SRY relationship. However, they found that when Eq. (5) is solved for Y, that resulting estimates of the yield ratio were often greater than unity, which is physically not possible. Thus Kuria and Vogel (2014a) developed a generalized SRY relationship using yield as the dependent variable. They employed IRLS regression to minimize the impact of outliers, thus producing a more robust regression. Their generalized SRY relationship is:

$$Y = 0.631\mu^{1.135}\sigma^{-0.342}\gamma^{0.017}S^{0.203}Z_R^{-0.306} \quad (R^2 = 0.992) \tag{6}$$

where all the variables have the same meaning and units as described in Eq. (5).

When the yield ratio is computed using Eq. (6), Kuria and Vogel (2014a) found that all values were less than unity as expected. Because the models in Eqs. (5) and (6) are based on an actual global streamflow database of 729 rivers, they are able to reproduce both the empirical distribution of streamflows and the resulting empirical SRY relationships globally. Importantly, the database used to develop Eqs. (5) and (6) consists of monthly flow records from rivers from different parts of the world: from USA, to Europe, to South Africa among others, with varying hydrologic characteristics: see McMahon et al. (2007a,b,c) for a detailed description of these characteristics. Thus both models may be considered for application worldwide.

Eq. (6) can be generalized as:

$$Y = KS^{\beta_1}Z_R^{\beta_2}\mu^{\beta_3}\sigma^{\beta_4}\gamma^{\beta_5} \tag{7}$$

where K and β_i, $i = 1,\ldots,5$ are the constants of the model. Thus where the streamflow data is available for say n years, estimates of Y that can be met from a storage capacity S with a specified reliability R can be evaluated using Eq. (8)

$$\widehat{Y} = KS^{\beta_1}Z_R^{\beta_2}\hat{\mu}_Q^{\beta_3}\hat{\sigma}_Q^{\beta_4}G_Q^{\beta_5} \tag{8}$$

where $\widehat{Y}$ is the estimate of Y, $\hat{\mu}_Q$, $\hat{\sigma}_Q$ and G_Q are the estimates of mean, standard deviation and skewness of the annual inflows into the reservoir respectively. For Gamma flows $G_Q = 2C\hat{v}$ where $C\hat{v} = \frac{\hat{\sigma}_Q}{\hat{\mu}_Q}$. Substituting into Eq. (8) and collecting like terms results in:

$$\widehat{Y} = 2^{\beta_5}KS^{\beta_1}Z_R^{\beta_2}\hat{\mu}_Q^{\beta_3-\beta_5}\hat{\sigma}_Q^{2\left(\frac{\beta_4+\beta_5}{2}\right)}. \tag{9}$$

Letting;

$$a = 2^{\beta_5}KS^{\beta_1}Z_R^{\beta_2}$$

$$b = \beta_3 - \beta_5 = 1.118$$

$$c = \frac{\beta_4 + \beta_5}{2} = -0.163$$

And substituting for these constants in Eq. (9) results in

$$\widehat{Y} = a\hat{\mu}_Q^{1.118}(\hat{\sigma}_Q^2)^{-0.163} \tag{10}$$

Eq. (10) is used to analytically derive the variance of yield estimates from a given reservoir of storage capacity S and reliability R fed by Gamma inflows. Since Eq. (10) is a power law equation, its exponents can be interpreted as nondimensional sensitivity coefficients known as elasticities. Thus a unit increase in mean annual

flow increases the yield from the reservoir by 1.12% while a unit increase in variance of the flows reduces the yields from the reservoir by 0.16%. A unit change in mean annual flow has a larger impact on the yields than a unit change in variance of the streamflows. These results can be used to evaluate the impacts of climate change of surface water reservoir yields. Here Eq. (10) is used to derive the variance of estimated reservoir yields for reservoirs fed by Gamma streamflows using the FOVA analytical uncertainty analysis method.

5. Moments of estimated moments of reservoir yield

The yield model given in Eq. (10) is a function of sample mean and sample variance of the inflows into the surface water reservoir. Therefore for us to derive the variance of reservoir yield estimates, the central moments of sample a mean and sample variance are needed. We consider the cases of both normal and Gamma inflows in this section.

For a given random variable X of length n say streamflow record X_n, we use the well known estimators of sample mean and sample variance given by:

$$\text{Sample mean, } \hat{\mu}_{X_n} = \frac{\sum_n X_i}{n} \tag{11a}$$

$$\text{Sample variance, } \hat{\sigma}_{X_n}^2 = \frac{\sum_n (X_i - \hat{\mu}_{X_n})^2}{n-1} \tag{11b}$$

Eqs. (11a) and (11b) are consistent and unbiased estimators of the sample mean and sample variance. In general,

$$E\lfloor \hat{\mu}_{X_n} \rfloor = \mu_X \tag{12a}$$

$$Var(\hat{\mu}_{X_n}) = \frac{\sigma_X^2}{n} \tag{12b}$$

$$E(\hat{\sigma}_{X_n}^2) = \sigma_X^2 \tag{12c}$$

Cho and Cho (2008) derived the general solution for the $Var(\hat{\sigma}_{X_n}^2)$ as:

$$Var(\hat{\sigma}_{X_n}^2) = \frac{1}{n}\left(\mu_4 - \frac{n-3}{n-1}\mu_2^2\right) \tag{12d}$$

where μ_2 and μ_4 are the second and fourth moments of the random variable evaluated about the central moments. For a normal distribution $\mu_2 = \sigma^2$ and $\mu_4 = 3\mu_2^2 = 3\sigma^4$ (Stuart and Ord, 1987). Replacing for these moments in Eq. (12d) and solving, when streamflows follow a normal distribution Eq. (12d) reduces to the well known result that:

$$Var(\hat{\sigma}_{X_n}^2) = \frac{2\sigma_X^4}{n-1} \tag{13}$$

For a Gamma distribution, $G_Q = 2C\hat{v}$, $\mu_4 = \left(3 + \frac{6}{\alpha}\right)\sigma^4$ and $\mu_2 = \sigma^2$ where $\alpha = \frac{\mu^2}{\sigma^2}$. Replacing for these moments in Eq. (12d) and solving, when streamflows follow a Gamma distribution Eq. (12d) reduces to:

$$Var(\hat{\sigma}_{X_n}^2) = \frac{2\sigma_X^4}{n-1}\left(\frac{n\mu_X^2 + 3n\sigma_X^2 - 3\sigma_X^2}{n\mu_X^2}\right) \tag{14}$$

$$= \frac{2\sigma_X^4}{n-1}\left(1 + 3Cv_X^2\left(\frac{n-1}{n}\right)\right) \tag{15}$$

The coefficient of variation of these moments can be evaluated as:

$$Cv_{\hat{\mu}_{X_n}} = \frac{\sqrt{Var(\hat{\mu}_{X_n})}}{E(\hat{\mu}_{X_n})} = \frac{\sigma_X}{\mu_X\sqrt{n}} \tag{16}$$

$$Cv_{\hat{\sigma}_{X_n}^2} = \frac{\sqrt{Var(\hat{\sigma}_{X_n}^2)}}{E(\hat{\sigma}_{X_n}^2)} = \sqrt{\frac{2}{n-1}\left(1 + 3Cv_X^2\left(\frac{n-1}{n}\right)\right)} \tag{17}$$

The above results were used to derive the variance of yield estimates for reservoirs fed by Gamma inflows using FOVA uncertainty analysis method.

6. Derivation of variance of water supply yield Y using First Order Variance Approximation (FOVA)

For a bivariate function given by Eq. (10)

$$Var[\hat{Y}] \cong \left(\frac{\partial \hat{Y}}{\partial \hat{\mu}_Q}\right)^2 Var(\hat{\mu}_Q) + \left(\frac{\partial Y}{\partial \hat{\sigma}_Q^2}\right)^2 Var(\hat{\sigma}_Q^2)$$
$$+ 2\frac{\partial \hat{Y}}{\partial \hat{\mu}_Q}\frac{\partial \hat{Y}}{\partial \hat{\sigma}_Q^2}\bigg|_{\mu_{x_j}} Cov(\hat{\mu}_Q, \hat{\sigma}_Q^2) \tag{18}$$

Thus for reservoirs fed by Gamma distributed inflows, the variance of yields estimated from Eq. (10) can be derived as:

$$Var(\hat{Y}) = \left(ab\hat{\mu}_Q^{b-1}\left(\hat{\sigma}_Q^{2c}\right)\right)^2 Var(\hat{\mu}_Q)$$
$$+ \left(ac\hat{\mu}_Q^b \hat{\sigma}_Q^{2(c-1)}\right)^2 Var\left(\hat{\sigma}_Q^2\right)$$
$$+ 2\left(ab\hat{\mu}_Q^{b-1}\hat{\sigma}_Q^{2c}\right)\left(ac\hat{\mu}_Q \hat{\sigma}_Q^{2(c-1)}\right)Cov(\hat{\mu}, \hat{\sigma}_Q^2) \tag{19}$$

Now the covariance between sample mean and sample variance is derived by Stuart and Ord (1987) as; $Cov(\hat{x}, s^2) = \frac{\mu_3}{n}$. And for Gamma distribution $\mu_3 = \frac{2\sigma^4}{\mu}$. Therefore covariance between sample mean and sample variance when flows follow Gamma distribution is $Cov(\hat{x}, s^2) = \frac{2\sigma^4}{n\mu}$. Substituting for variance of sample mean, variance and their covariance,

$$Var(\hat{Y}) = \left(ab\hat{\mu}_Q^b \hat{\sigma}_Q^{2c}\right)^2 \frac{Cq^2}{n}$$
$$+ \left(ac\hat{\mu}_Q^b \hat{\sigma}_Q^{2(c-1)}\right)^2 \frac{2\sigma^4}{n-1}\left(1 + 3Cq^2\left[\frac{n-1}{n}\right]\right)$$
$$+ 4bc\left(a\hat{\mu}_Q^b \hat{\sigma}_Q^{2c}\right)^2 \frac{Cq^2}{n} \tag{20}$$

which can be simplified as:

$$Var(\hat{Y}) = b^2(\hat{Y})^2 \frac{Cq^2}{n} + c^2(\hat{Y})^2 \frac{2}{n-1}\left(1 + 3Cq^2\left[\frac{n-1}{n}\right]\right)$$
$$+ 4bc(\hat{Y})^2 \frac{Cq^2}{n} \tag{21}$$

$$Var(\hat{Y}) = \hat{Y}^2\left(b^2 \frac{Cq^2}{n} + c^2 \frac{2}{n-1}\left(1 + 3Cq^2\left[\frac{n-1}{n}\right]\right) + 4bc\frac{Cq^2}{n}\right) \tag{22}$$

Now;

$$C\hat{y} = \frac{\sqrt{Var(\hat{Y})}}{E(\hat{Y})} \tag{23}$$

Using FOVA, $E(\hat{Y}) \approx Y$. Substituting for $E(\hat{Y})$ in Eq. (19), the final form of the equation for Cy reduces to:

$$Cy = \sqrt{\frac{b^2}{n}Cq^2 + \frac{2c^2}{n-1}\left[1 + 3Cq^2\frac{n-1}{n}\right] + \frac{4bc}{n}Cq^2} \tag{24}$$
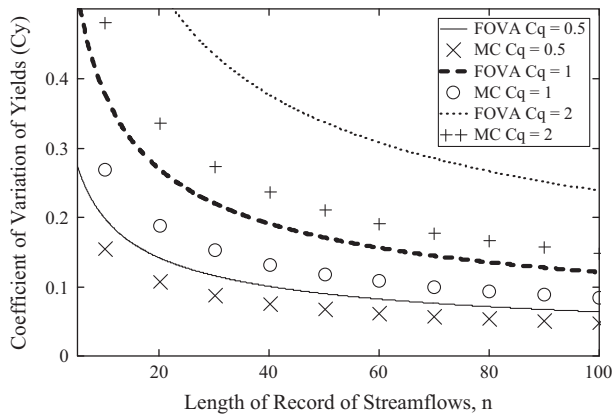
Fig. 2. Comparison of coefficient of variation of yield using FOVA and MC simulation assuming the covariance between input random variable, Cov = 0.

The results of coefficient of variation of yield estimates *Cy*, derived from FOVA are shown in Fig. 1 (labeled for example FOVA *Cq* = 0.5) evaluated at different coefficient of variation of streamflows (*Cq*) when the covariance between input random variables and Fig. 2 when this covariance is assumed to be zero. Also included in Figs. 1 and 2 are the results of Monte Carlo simulations of streamflows from the same distribution (labeled for example McCq = 0.5). There is good agreement between both methods when the covariance between the input random variables is considered. These results show that when the correlation between mean and variance is a taken into account, the MC and FOVA results are in good agreement. Therefore inclusion of covariance terms can be quite significant for obtaining good approximation of variance estimates using FOVA. We also note from Fig. 1, as did Kuria and Vogel (2014b) that a generalized understanding of the variability in estimates of water supply yield can be obtained from Eq. (20) which only depends on the coefficient of variation of the inflows and the record length. This is quite important because the result is independent of particular values of the storage capacity *S* and/or reliability *R*. A tabulation of the results of these comparisons are also presented in Table 1 which demonstrates the effects of the variability of streamflows (quantified by *Cq*) and the length of the streamflow record *n* for storage ratio, (s/mean) = 1.5 and reliability, *R* = 0.9 on the coefficient of variation of estimated reservoir yields *Cy*.

Fig. 1 and Table 1 illustrate that the agreement between MC and FOVA analyses is greatly quite good, but that agreement degrades slightly as the coefficient of variation of the streamflows *Cq*, increases and as the sample size *n*, decreases.

7. The minimum length of streamflow records for design of reservoirs

Our analysis of the variability of water supply reservoir yields results in several conclusions. (1) The constant water supply yield values used during the design of water supply reservoirs may not actually be achieved during the actual performance of the reservoir, especially when only short streamflow records are available for rivers with high variability (arid and semi-arid regions). (2) Water supply reservoir yield estimates are a random variable and can be approximated by the Gamma (GAM) probability distribution (see Kuria (2014) for further details). (3) When *Cy* is used to characterize the variability of the yield estimates, the storage ratio (S/μ) and reliability of the yield estimates do not appear to influence the variability of yield estimates. The length of record *n* and the coefficient of variation of inflows, *Cq*, are the only two factors

that appear to influence the coefficient of variation of the yield estimates, and (4) the likely intervals of yield estimates can be determined using the analytical *Cy* model derived in this paper for known reservoir storage capacities, reliability, and length of record of streamflows. These results can be used to provide guidance to reservoir design, planning and management concerning the necessary minimum length of streamflow data and other sources of information needed to obtain reservoir yields of a specified reliability. With the awareness that actual yields from water supply reservoirs are a random variable then it would be more accurate for reservoir designs to consider the range of yields expected from the reservoirs for a given reliability instead of a single value. For such designs to be carried out, then the minimum length of streamflow data to be considered during reservoir design would need to be specified. This is analogous to specifying the minimum sample size of data required for estimating parameters within a specified range which is often done in environmental and water quality studies as is described below.

Studies for determining such minimum sample sizes have been carried out in ground water monitoring studies (Nelson and Ward, 1981) and environmental pollution monitoring (Gilbert, 1987) among others. According to these studies, the minimum number of samples required to estimate the mean of a random variable depends on the confidence interval associated with a sample mean, the variance and the margin of error of the mean. Gilbert (1987) derives formulas for determining the minimum sample sizes for estimating the mean for both correlated and independent data when the data follows a normal distribution. We use this concept in this study to determine the minimum length of streamflow records required for water supply reservoirs yields to fall within a specified range for a given reliability. Because here the true yield is known *apriori*, likely intervals are considered here instead of confidence intervals. Because the probability distribution of reservoir yield estimates can be approximated by a Gamma (GAM) distribution with skewness coefficient in the range $-3 < \gamma_Y < 3$, we use the Wilson and Hilferty (1931) quantile function given in Eq. (25) to determine the yield estimate for a given quantile, *p*.

$$Y_p = \mu_Y + K_p \cdot \sigma_Y \tag{25}$$

$$K_p = \left(\frac{2}{\gamma_Y} \left(1 + \frac{Z_p \gamma_Y}{6} - \frac{2 \gamma_Y}{36} \right)^3 - \frac{2}{\gamma_Y} \right) \tag{26}$$

where Y_p is the yield estimate for a given quantile *p*, $\mu_Y = Y$ (the true yield), γ_Y and σ_Y are the skewness and standard deviation of yield estimates *Y* and Z_p is the standardized normal variate for a given percentile *p*. The standard deviation of *Y* estimated from $\sigma_Y = Cy \cdot Y$ where *Cy* is obtained from Eq. (26) and for GAM distribution, $\gamma_Y = 2Cy$. Therefore, the relative error of the yield estimates obtained from error divided by the true yield is obtained from:

$$d_p = K_p \cdot C_Y \tag{27}$$

Thus the relative error of reservoir yields depends on the coefficient of variation of the streamflows *Cq* and the length of the streamflow record, *n*. Therefore Eq. (27) can be used to determine the minimum streamflow record required for reservoir yields to fall within a specified range. Fig. 3 shows the results of minimum length of streamflow record required to ensure that water supply yields remain within *d*% of their true values 95% of the time (*p* = 0.025). Fig. 2 illustrates the lower likely interval of the yield estimates corresponding to various values of the margin of error *d*%. The results in Fig. 3 are surprising because very long streamflow records are required to ensure that yields from water supply reservoirs have small margin of errors and especially for rivers with high variability. For example when a reservoir system is designed considering 40 years of streamflow data on a river with
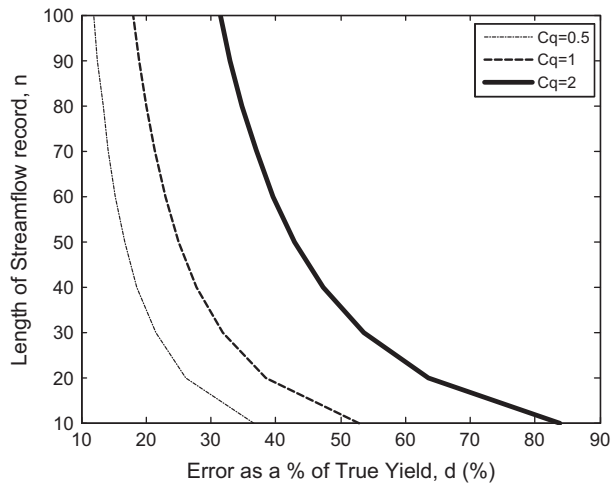
Fig. 3. Minimum length of streamflow records required for specified error of yield estimates *d*, (expressed as a % of true yield) corresponding to 95% likely range of reservoir yields (*p* = 0.025 for two sided likely range).

Cq = 2, the actual yields obtained from the reservoir can be as low as over 50% of the design yield value. Therefore, such a reservoir may rarely meet the objective of reducing the variability in the delivery of water supply from the river. On the same river if the yields from the reservoirs are to be about 30% lower than the design yield over 100 years of streamflow data would be required. Such long records of streamflow are rarely available and especially in developing countries.

8. Case study

Since this study is based on a global database, the result of this analysis can easily be used to determine the likely range of yields that can be expected on a given reservoir site on a river located in any part of the world. Since the generalized SRY model used here is developed from regression analysis, the findings in this study can only be used for rivers which have similar statistical characteristics as well as the reservoir operating systems (i.e. storage and reliability), considered in the development of the regression equations for accurate results to be obtained. Determination of likely intervals of a given random variable requires knowledge of the frequency distribution of the variable. A detailed analysis of the approximate distribution of reservoir yield is presented in Kuria (2014) which shows that a three parameter Gamma distribution, also known as the Pearson type III distribution (P3), provides the best overall goodness of fit to estimates of water supply yield. However, an LN2 or GAM model would also suffice for approximating the distribution of water supply yield, regardless of the inflow model or record length considered since all of the values of probability plot correlation coefficients were extremely high for all cases considered. Considering LN2 as the distribution of the yield estimates, the *p*th quantile yield estimate is given by:

$$Y_p = \exp(\mu_y \pm \sigma_y Z_p) \qquad (28)$$

where Z_p is the standardized normal variate for a given quantile *p*, $\mu_y = \log\left(\frac{Y}{\sqrt{1+Cy^2}}\right)$ and $\sigma_y = \sqrt{\ln(1 + Cy^2)}$. Here *Y* is estimated using Eq. (6) and *Cy* is obtained from Fig. 1 or Eq. (20) once the *Cq* of the inflows into the reservoir are known.

We illustrate our method using a river in the USA located at U.S. Geological Survey gauging station 03439500. Using annual streamflow data for 1925–1955, the annual statistics of the streamflow data are calculated as mean, $\hat{\mu} = 300.26$, standard deviation,

$\hat{\sigma}$ = 72.42, coefficient of variation, $C\hat{q} = 0.24$, and skewness, $\hat{\gamma} = 0.795$. These statistics are within the range of the global dataset considered in this study as shown in Table 2 of McMahon et al. (2007b). The units of mean μ and standard deviation, σ are million cubic meters per year. Assuming hypothetical reservoirs with storage ratios S/μ of 0.5,1 and 2.5 and *R* = 0.85 and 0.95, the likely ranges of yields that can be expected in the river are estimated as shown in Table 2.

9. Conclusion

This paper develops a first order, approximate, analytical uncertainty model that can be used to document the uncertainty in estimated water supply reservoir yields which arises from the natural variability inherent in streamflows as well as the limited length of records available. Our analysis is general because it is based on previous work (Kuria and Vogel, 2014a,b) which introduced a generalized global model describing the relationship between the storage capacity of a reservoir *S*, and its yield and reliability *R*. The global SRY model is an empirical model which leads to accurate ($R^2 = 0.99$) estimates of reservoir yield based on estimates of the mean variance, and skewness of the reservoir inflows.

A First Order Variance Approximation (FOVA) was used to develop the analytical uncertainty model for reservoir yield estimates and Monte Carlo (MC) simulations were used to check the accuracy of the variance estimates from the resulting analytical model. The results indicate good agreement between the derived uncertainty model and the MC simulations. Previously FOVA has been assumed to result into accurate results when there was near linearity in functional relations, the input variables followed a near normal distribution, and the variability of the random input variables were small. In this paper, a non-linear function was used with non-normal (Gamma) and highly variable input random variables. However since the covariance between the input random variables was included in our analysis, we conclude that perhaps the covariance of the random input variables plays a central role for obtaining accurate results from FOVA. In many cases, the covariance of the input random variables is not available which may have led to th in conclusion that FOVA does not yield accurate results in numerous previous studies.

Interestingly, we found that the coefficient of variation of estimated water supply yields were independent of the storage ratio (S/μ) and reliability of the yield. This observation allowed us to develop very general relationships which summarize the variability of yield estimates for a very wide class of reservoir systems over a wide range of reliabilities and yields. For this simple SRY model, the length of record and the coefficient of variation of the reservoir inflows, appear to be the only two factors that influence the coefficient of variation of the yield estimates. The variability of yield estimates increases as the variability of flows increases and decreases as the length of record increases. With the awareness that actual yields from water supply reservoirs are a random variable, we used the our analytical model which gives the coefficient of variation of yield, to determine the minimum length of streamflow record required for designing reservoirs that will deliver yields within a prespecified margin of error. Results indicate that relatively long streamflow records are required to ensure that the margin of error of yields delivered from reservoirs are small, particularly for watersheds with highly variable streamflows.

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