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Key Points:
- Panel regression methods were used to better characterize spatial and temporal variability of low flows in multiple linear regressions
- Panel regression methods provided more reliable estimates of low-flow elasticity to rainfall than traditional regression methods
- Changes in low flows and stream habitat associated with projected rainfall were estimated in a case study on Maui, Hawaii

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Panel regressions to estimate low-flow response to rainfall variability in ungaged basins

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Abstract Multicollinearity and omitted-variable bias are major limitations to developing multiple linear regression models to estimate streamflow characteristics in ungaged areas and varying rainfall conditions. Panel regression is used to overcome limitations of traditional regression methods, and obtain reliable model coefficients, in particular to understand the elasticity of streamflow to rainfall. Using annual rainfall and selected basin characteristics at 86 gaged streams in the Hawaiian Islands, regional regression models for three stream classes were developed to estimate the annual low-flow duration discharges. Three panel-regression structures (random effects, fixed effects, and pooled) were compared to traditional regression methods, in which space is substituted for time. Results indicated that panel regression generally was able to reproduce the temporal behavior of streamflow and reduce the standard errors of model coefficients compared to traditional regression, even for models in which the unobserved heterogeneity between streams is significant and the variance inflation factor for rainfall is much greater than 10. This is because both spatial and temporal variability were better characterized in panel regression. In a case study, regional rainfall elasticities estimated from panel regressions were applied to ungaged basins on Maui, using available rainfall projections to estimate plausible changes in surface-water availability and usable stream habitat for native species. The presented panel-regression framework is shown to offer benefits over existing traditional hydrologic regression methods for developing robust regional relations to investigate streamflow response in a changing climate.

1. Introduction

Simple regional hydroclimatic models, with parameters robust enough to make inferences about streamflow response to varying climate conditions, are increasingly needed to estimate streamflow characteristics in ungaged basins and to evaluate the impact of climate change on streamflow. Streamflow characteristics are typically related to physical basin characteristics in multiple linear regression models (MLRs) solved using ordinary least square (OLS) or weighted least square (WLS) methods and in some cases using the generalized least square (GLS) method [*Tasker*, 1975, 1982; *Tasker and Stedinger*, 1989]. Such regional MLRs have been developed to estimate a range of streamflow characteristics in ungaged basins during average conditions for many regions in the United States [*Ries et al.*, 2007] and other regions throughout the world [*Blöschl et al.*, 2013]. In addition to providing streamflow information at ungaged locations, the coefficients of MLRs can be interpreted as elasticities [*Vogel et al.*, 1999]—that is, the nondimensional sensitivity of streamflow to a particular explanatory variable. If climate variables are included in the MLR, the model coefficients can provide an estimate of the elasticity of streamflow to climate and provide valuable information for regional climate-change assessments. For this reason, and using the concept that space can be substituted for time, MLRs provide an alternative approach to more complex physically based hydrologic models for the evaluation of the sensitivity of hydrologic response to changes in climate [*Gyawali et al.*, 2015, *Singh et al.*, 2011].

Estimated coefficients of MLRs offer important information about the elasticity of the dependent variable to the explanatory variables; however, robust estimates of the coefficients are often complicated by two factors: (1) multicollinearity—the correlation among the explanatory variables [*Bloomfield et al.*, 2009], and (2) omitted-variable bias—the bias introduced when the model leaves out important causal factors, usually due to lack of information to quantify all potential explanatory variables (see Appendix B in *Farmer et al.*

[2015]). When multicollinearity is present among included explanatory variables, traditional regression (TR) methods such as ordinary least squares (OLS) are often thought to result in estimates of model coefficients with high variances, unrealistic (biased) coefficient values, and improper model selection [*Helsel and Hirsch*, 2002]. An evaluation by *Kroll and Song* [2013] found that when the form of the model is unknown, which is usually the case in practice, TR methods such as OLS with a stepwise selection do not usually result in models with a high level of multicollinearity and, furthermore, perform as well as other more complicated and often biased methods designed to guard against the impacts of multicollinearity such as variance inflation factor screening, principal components regression, and partial least squares regression. However, the TR methods evaluated by *Kroll and Song* [2013] were found, in another assessment of over 20 methods for MLR formulation, to consistently perform worse than methods such as the least absolute shrinkage and selection operator (LASSO) and ridge regression [*Dormann et al*., 2013].

In the development of regional hydrologic models, the occurrence of omitted-variable bias implies that important climatic and/or hydrologic information about the watersheds in the gaged network is ignored and such information could impact the interpretation of regression coefficients for predictors included in the model. *Farmer et al*. [2015] document that MLR provides an effective approach for accounting for confounding variables leading to the mitigation of omitted-variable bias. *Farmer et al*. [2015] provide several examples of how TR can reduce the effects of omitted-variable bias thus leading to an improved understanding of the spatial scaling of streamflow. The issues of developing meaningful regional hydrologic models using TR remain complex and unresolved. Until our models have perfect explanatory power, they will typically exhibit some degree of both multicollinearity and omitted-variable bias. Thus, resulting MLRs may be limited for understanding the elasticity of the dependent variable (e.g., streamflow) to future climate conditions and therefore models may not be reliable for climate-change applications. One of the primary goals of this study is to explore promising new methods for developing MLRs, not considered by either *Kroll and Song* [2013] or *Farmer et al*. [2015], which have the potential to reduce the remaining deleterious impacts of both multicollinearity and omitted-variable bias.

Panel regression (PR) is a statistical method that has the potential to improve upon TR to develop more robust models that can be applied in regional hydrologic climate-change impact assessments. PR is widely used in econometric analyses [*Croissant and Millo*, 2008], yet has seen only limited use in hydrology. To our knowledge, *Steinschneider et al*. [2013] were the first to suggest promising applications of PR to hydrologic applications by demonstrating its benefits in identifying robust generalized relations between anthropogenic landscape alterations and hydrologic response. In this study, MLR always refers to a model and TR and PR refer to the methods used to fit the MLR.

The PR approach offers numerous potential advantages over TR, and these advantages are briefly outlined here. PR provides a framework to pool multidimensional data across gaged streams and through time (termed panel data) into a single regression to enable identification of response characteristics both unique to each stream and common across streams. This single regression can pool basin characteristics that are static during a time period, such as morphological and land-cover characteristics, and temporally variable, such as climatic variables. In contrast, MLRs developed using TR relate streamflow characteristics during a fixed period to basin characteristics at gaged sites to develop average regional MLRs and are thus ill-equipped to evaluate both temporal and spatial responses. PR can still be affected by multicollinearity in the same way as TR, but the ability of PR to parse the effect of static basin characteristics from temporally variable basin characteristics helps ensure that the model coefficients in resulting MLRs are consistent with the temporal and spatial responses. This major advantage of PR provides MLRs that are expected to be more appropriate for both temporal and spatial streamflow estimation than TR. Another important advantage of the PR framework is that it enables us to identify the effect of omitted variables. An omitted variable that is important for characterizing the spatial heterogeneity of streamflow may not be readily observable or included in a MLR model developed using TR. The PR framework tests whether the unobserved effect is correlated with model explanatory variables, and thus accounts for the influence of omitted variables in MLRs. We thus expect that PR can be used to overcome both the statistical challenges of multicollinearity and omitted-variable bias in MLRs. In addition, PR enables use of data across sites and through time simultaneously in a single regression, thus increasing the sample size for model error and coefficient estimation, and leading to improvements in the efficiency of the model-coefficient estimates and model-error variance [*Steinschneider et al*., 2013]. TR often imposes constraints on data such as requiring streamflow characteristics computed from concurrent records, which sometimes leads to extending records at short-term sites or even disqualifying gaged sites that may be critical in data-poor regions. In contrast, PR is not constrained in this way and offers distinct advantages over TR for data-poor studies. In PR, because temporally varying response and explanatory variables are paired, data from gaged sites can be pooled without a common base period (assuming that the relation between response and explanatory variables is time invariant), allowing the use of a maximum number of sites with different record periods and lengths.

The focus of this study is to examine the potential for PR to overcome the limitations of TR for developing MLRs that can be used to estimate low-flow characteristics in ungaged basins under varying rainfall conditions. We consider only rainfall as a time-varying climatic explanatory variable because rainfall was the only variable with readily available annual data concurrent with the streamflow records and projected for future time periods for the study area. This study thus aims to accurately estimate rainfall elasticities in regional MLRs. To our knowledge, this is the first study to apply PR for the purpose of developing regional MLRs that both quantify the elasticity of streamflow to climate and provide estimates of low-flow characteristics in ungaged areas. A primary goal is to show that if the appropriate PR assumptions for fitting MLRs are met, the resulting MLRs provide more robust estimates of low-flows in varying rainfall conditions than using methods that do not characterize the temporal and spatial variability in the data simultaneously. If PR can limit the effects of multicollinearity between rainfall and static explanatory variables and overcome issues of omitted-variable bias common in MLRs developed for estimating low flows using TR, then resulting MLRs can be applied to varying rainfall conditions and the respective coefficients can be used for assessing regional sensitivities of low flows to changes in rainfall.

Another goal of this work is to document how PR can be useful for regionalization and exploratory analyses for regional climate-change impact assessments by applying PR in a case study on Maui, Hawaii. This application can serve as an example for other regions. Regionalization of low flows in Hawaii streams provides a particularly useful case study for testing our methods, because streamflow, climate, and basin characteristics exhibit extreme spatial and temporal variability in an area where many streams are ungaged. Tools to estimate surface-water availability under changing climate conditions in ungaged basins, particularly during low-flow conditions, are necessary to develop adaptive surface-water management strategies. Island settings can experience large interannual rainfall variations and rainfall variability affects the beneficial uses of surface water, including supplying freshwater for irrigation and domestic uses, providing freshwater for traditional and customary practices, and maintaining habitat for sensitive stream species.

We begin by presenting the study area data and stream classification method and defining the PR modeling methods and evaluation criteria. Explanatory variables for MLRs to estimate annual low-flow duration discharges in ungaged streams in Hawaii are selected using PR criteria and MLRs are fit using TR and three PR methods. The goodness of fit of the MLRs developed using the four methods are compared to discuss the benefits and limitations of the proposed PR approach and evaluate if PR can address issues of multicollinearity between rainfall and static explanatory variables and omitted-variable bias. Finally, the MLRs fit using PR are applied in a case study of the island of Maui, Hawaii, to estimate changes in low flows and usable habitat for native stream fauna resulting from a range of projected changes in rainfall for the mid to late-21st century.

2. Data and Methods

2.1. Data Analyzed

2.1.1. Selected Gaged Streams and Drainage-Basin Characteristics

Daily mean discharge data were obtained from stream-gaging stations maintained by the U.S. Geological Survey (USGS) on the islands of Kauai, Oahu, Maui, and Hawaii. A total of 69 perennial streams (14 on Kauai, 17 on Oahu, 26 on Maui, and 12 on Hawaii) with at least 10 complete water years of record between 1921 and 2007 were selected and used to develop regional MLRs to estimate annual low-flow duration discharges in ungaged regions and analyze the sensitivity of streamflow to changes in rainfall. Low-flow duration discharges are not estimated for nonperennial streams. To identify these nonperennial streams, a method is developed using additional data from 17 nonperennial streams (1 on Kauai, 7 on Oahu, 3 on Maui, and 6 on Hawaii) with at least 5 complete water years of record between 1921 and 2007. The location of selected streams is presented in Figure 1a. For this study, streams with flow at least 90 percent of the

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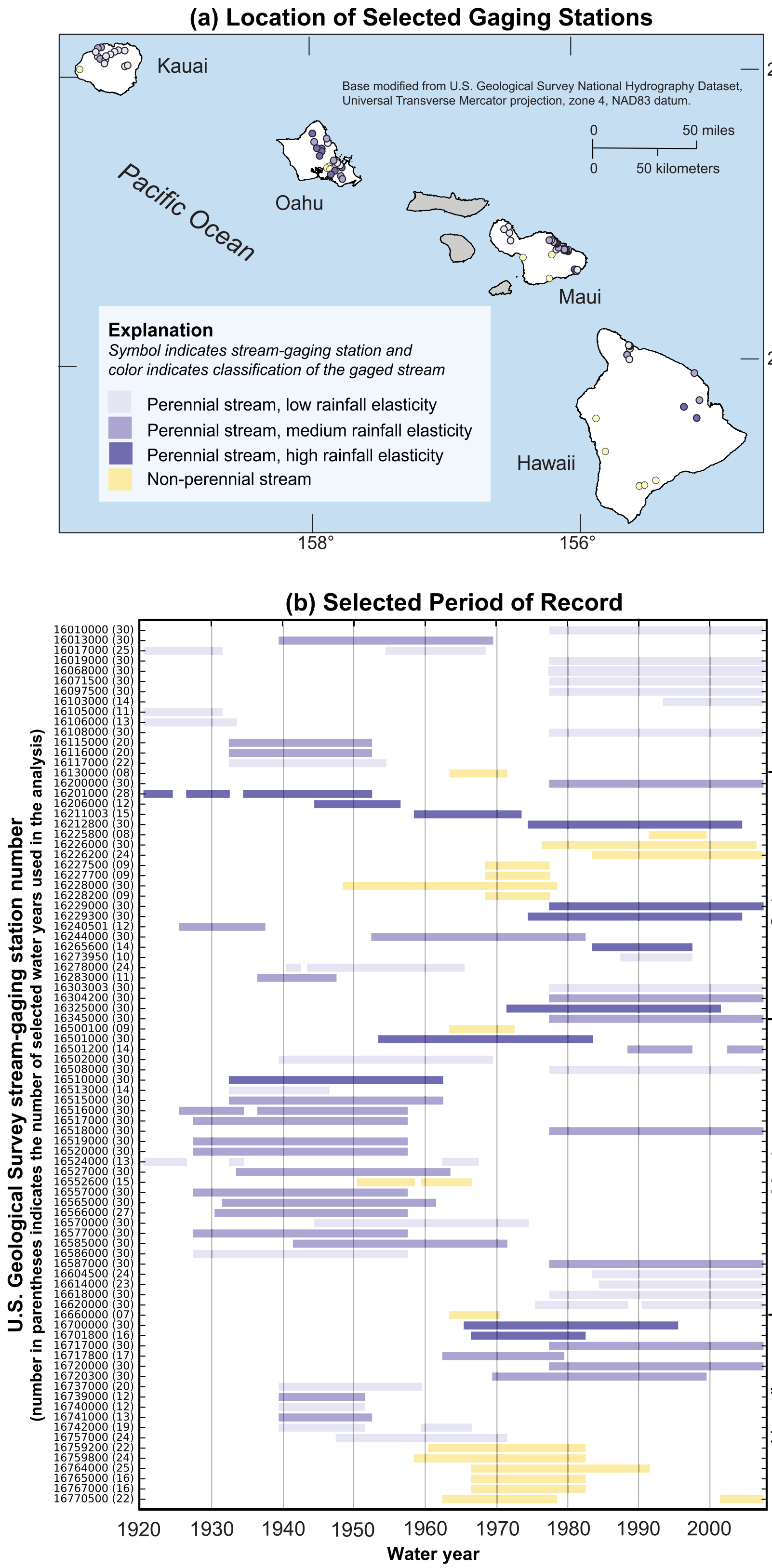

Figure 1. (a) Location of U.S. Geological Survey stream-gaging stations included in this study and (b) periods of record analyzed in the regression analysis. Classification of gaged stream (see section 3.1) is indicated by the line color.

Table 1. Selected Drainage-Basin Physical Characteristics and Range of Values for Gaged Stream Classes[a]

			P_l Low Elasticity Perennial Streams			P_m Medium Elasticity Perennial Streams			P_h High Elasticity Perennial Streams			*NP* Nonperennial Streams		
Variable	Definition	Unit	min	mean	max	min	mean	max	min	mean	max	min	mean	max
Climatic characteristics [*Giambelluca et al.*, 2013]														
x_1	Average annual rainfall 1978–2007	mm	1990	3891	6318	2653	4339	6334	1843	4095	6946	568	2073	3166
Morphometric characteristics [10-m USGS DEM]														
x_2	Basin drainage area	km^2	0.22	8.25	47.94	0.65	5.83	31.19	2.83	18.76	99.57	1.56	12.13	38.92
x_3	Drainage basin perimeter	km	4.05	17.14	45.01	4.43	18.61	50.61	10.38	28.73	80.20	7.42	27.01	80.02
x_4	Basin shape factor related to basin perimeter and drainage area	-	1.36	2.02	2.82	1.47	2.38	4.03	1.60	2.16	3.22	1.54	2.28	3.63
x_5	Mean basin elevation	m	112	753	1594	365	740	1483	135	717	2171	346	1052	2315
x_6	Drainage basin relief (difference in maximum and minimum elevation)	m	216	900	2055	250	1023	2337	431	1139	3121	632	1301	2947
x_7	Length of drainage network at a 20,000 cell threshold	m	0*	4.2	23	0*	4.4	24	0*	12	55	0*	9.0	42
x_8	Length of drainage network at a 900 cell threshold	m	0.6	23	128	1.2	19	100	8	64	360	2.5	42	169
x_9	Mean slope of the drainage network at a 20,000 cell threshold	%	8	34	76	12	28	59	8	28	44	10	26	36
x_{10}	Drainage density at a 900-cell threshold	m^{-1}	1.7	2.9	4.0	1.6	3.2	4.3	2.1	3.2	4.1	1.6	3.0	4.4
x_{11}	Incision index (ratio of the mean elevation of the drainage basin perimeter to the mean elevation of the drainage network at a 20,000 cell threshold)	-	0.17	0.76	0.98	0.33	0.87	1.00	0.47	0.75	0.98	0.57	0.84	1.09
Soil characteristics [*U.S. Department of Agriculture*, 2008]														
x_{12}	Soil permeability at 610mm depth	mm/h	27.4	117	239	15.9	133	330	62.3	133	287	22.3	159	394
Land-cover characteristics [*U.S. Geological Survey*, 2010]														
x_{13}	Percentage of forested area	%	40	84	100	33	90	100	33	81	96	3	68	100
x_{14}	Percentage of shrubland area	%	0*	10	29	0*	5.5	25	1.1	6.2	16	0.02	10	60
x_{15}	Percentage of grassland area	%	0*	2.6	21	0*	1.5	7.0	0*	4.1	40	0.004	10	7
x_{16}	Percentage of barren area	%	0*	0.1	1.5	0*	1.8	48	0*	2.5	15	0*	9.0	65
x_{17}	Percentage of wetland area	%	0*	0.5	3.0	0*	0.0	1.0	0*	0.02	0.2	0*	0*	0*
Geologic characteristics [*Sherrod et al.*, 2007]														
x_{18}	Percentage of sedimentary rocks	%	0*	8.3	80	0*	1.7	21	0*	9.5	55	0*	3.8	13
x_{19}	Percentage of shield-stage volcanics	%	0*	45	100	0*	30	100	0*	62	100	0*	71	100
x_{20}	Percentage of post-shield-stage volcanics	%	0*	38	100	0*	69	100	0*	24	100	0*	25	100
x_{21}	Percentage of rejuvenated-stage volcanics	%	0*	8.5	100	0*	0.2	3.8	0*	4.2	21	0*	0*	0*
x_{22}	Percentage of Hana volcanics	%	0*	3.6	60	0*	12	84	0*	8.4	54	0*	2.4	25

[a]%, percent; mm, millimeters; m, meters; km, kilometers; h, hour; -, dimensionless; * indicates that 0 values were assigned a value of 10^{-5}during logarithmic transformation.

time (or with no flow less than 10 percent of the time) are considered perennial, whereas streams with no flow more than 10 percent of the time are considered intermittent or ephemeral and are termed here non-perennial. Among selected nonperennial streams, 10 have no flow more than 50 percent of the time. Selected streams are not known to be influenced by groundwater withdrawals and are nonregulated upstream of the gaging station [*Fontaine*, 1996], except three gaging stations for which complete daily mean diverted flow was also gaged by the USGS and reliable records of total streamflow representing nonregulated conditions could be reconstructed. Drainage basins were delineated for each selected gaging station using the USGS StreamStats application for Hawaii [*Rosa and Oki*, 2010]. Selected physical basin characteristics (Table 1) were computed for each drainage basin. Basin morphometric characteristics were determined using a USGS 10 m digital elevation model (DEM). Area-weighted soil, geology, and land-cover characteristics were computed using digital maps of soil properties [*U.S. Department of Agriculture*, 2008], surface geology [*Sherrod et al.*, 2007], and land-cover type [*U.S. Geological Survey*, 2010]. In addition, average annual rainfall during 1978–2007 was computed for each drainage basin from a monthly time series (1920–2007) of 250 m resolution raster rainfall data sets [*Frazier et al*., 2015] and values were consistent with the rainfall atlas of Hawaii [*Giambelluca et al*., 2013].

2.1.2. Annual and Time-Averaged Low-Flows and Rainfall Time Series

Data are analyzed in water years to ensure that coherent rainfall-streamflow relations are preserved [*Fu et al*., 2011]. A water year starts on 1 October and ends on 30 September of the following year and is named according to the year during which it ends. For each perennial stream k, annual low-flow duration discharges ($Q_{50_{k,t}}$, $Q_{70_{k,t}}$, and $Q_{90_{k,t}}$) were computed for each water year t during 1921–2007 with complete daily flow record for that year. $Q_{x_{k,t}}$ represents the flow that is equaled or exceeded x percent of the time during the water year t considered and is expressed as a daily mean discharge. The mean annual low-flow duration discharge, termed $\overline{Q_{x_k}}$ is also computed and is simply the mean value of $Q_{x_{k,t}}$ during the period of record considered. $\overline{Q_{x_k}}$ represents $Q_{x_{k,t}}$ for an average or typical year during the averaging period. Such annual flow-duration curve characteristics are now in common usage ever since the introduction of the mean and median annual flow-duration curve by *Vogel and Fennessey* [1994]. Time series of annual anomalies relative to $\overline{Q_{x_k}}$, termed $\delta_{x_{k,t}}$ and expressed as a percentage, were also computed to analyze the annual temporal variability in $Q_{x_{k,t}}$. Concurrent annual rainfall ($RF_{k,t}$) values were determined for each drainage basin of stream k from raster rainfall data sets [*Frazier et al*., 2015]. The periods of record used for each selected streamflow-gaging station are presented in Figure 1b. For sites with long-term data, only the most recent 30 water years of data are used to develop MLRs because significant changes in basin characteristics other that rainfall may have contributed to long-term trends in low flows [*Bassiouni and Oki*, 2013] and long-term data for basin characteristics such as land cover are not available to include in the MLRs. Constraining the period of record to a maximum of 30 years limits potential effects of these long-term unobserved factors. A number of nonparametric tests were made using data at each stream individually to test the stability of the annual relation between low-flow duration discharge and rainfall. Results indicated that there were no trends in this relation during the periods of record considered. We assume that within a selected 30 year period, basin characteristics are stationary enough to not have a significant effect on the annual relation between low-flow duration discharge and rainfall. We recognize that average low flows determined using data from the period of record prior to the documented shift in streamflow in the early 1940s [*Bassiouni and Oki*, 2013] may not be representative of low flows for current conditions. The effect of using data for the early 20th century in this analysis is uncertain. We recognize that errors may occur in the developed models if current land cover and other unobserved basin characteristics are not representative of the historical period of record. However, for this study, the benefit of maximizing the number of sites and increasing the spatial coverage of the gaged streams in the analysis was thought to outweigh the cost of introducing potential errors by including the historical data.

2.2. Classification of Streams

MLRs assume that each of the explanatory variables has the same effect on streamflow response for all streams in a region or class. Identifying distinct streamflow regimes and stratifying dominant hydrologic processes in hydroclimatically variable study areas such as Hawaii is critical for developing interpretable predictive models and providing accurate streamflow estimates in ungaged areas [*Sanborn and Bledsoe*, 2006]. For the purpose of this study, streams are grouped into classes of similar streamflow sensitivity to changes in rainfall and separate MLRs are developed for each stream class. Rainfall elasticity (ε_{x_k}), for a stream k (where the subscript x refers to the flow-duration percentile x considered) is a useful indicator of effects of rainfall variability on streamflow characteristics [*Schaake*, 1990] and is defined here as the proportional change in annual low-flow duration discharge ($Q_{x_{k,t}}$) divided by the proportional change in annual rainfall ($RF_{k,t}$).

$$\varepsilon_{x_k} = \frac{dQ_{x_{k,t}}/Q_{x_{k,t}}}{dRF_{k,t}/RF_{k,t}} \quad (1)$$

The model coefficient of the explanatory variable annual rainfall in a MLR in log-space (similar to the MLRs developed in this study), in which annual low-flow duration discharge is the dependent variable, represents the regional rainfall elasticity and reflects the generalized sensitivity of low flows to rainfall for the region considered [*Vogel et al*., 1999; *Sankarasubramanian et al*., 2001].

2.2.1. Classification of Gaged Streams From Observed Local Rainfall Elasticity

Local values for ε_{50_k}, ε_{70_k}, and ε_{90_k} were estimated for each gaged perennial stream k. Local rainfall elasticity values for each stream were derived from observed annual data using the power-law or log-linear relation

$$Q_{x_{k,t}} = \alpha RF_{k,t}{}^{\varepsilon_{x_k}} \tag{2}$$

where α is a constant. The bivariate power-law formulation and estimation of rainfall elasticity is of equivalent robustness to the bivariate nonparametric estimator introduced by *Sankarasubramanian et al*. [2001] and is preferred in this study because periods of record in this study are not concurrent for each stream and rainfall elasticity is not defined around a mean value using the power-law formulation.

K-means clustering was used to partition the 69 perennial streams into three classes in which each stream belongs to a class with the nearest mean [ε_{50}, ε_{70}, ε_{90}]. The objective K-means approach was preferred to grouping streams within subjectively selected ranges of rainfall elasticity because streams are thus clustered around objective mean values, which are representative of the study area, the cluster centroids. The coefficient of the explanatory variable rainfall in the MLRs that will be developed to estimate annual low-flow duration discharges (methods explained in next section) for each perennial stream class is expected to be consistent with the centroid of the stream class to which it belongs. The median rainfall elasticity value for each stream class and flow-duration discharge (ε_{x_m}) is used as a baseline reference that is independent of the developed models. Rainfall elasticity derived from the MLRs will be compared to ε_{x_m} as a criterion to evaluate the effect of multicollinearity and omitted-variable bias in the MLRs because local rainfall elasticity values are well constrained by equation (2) and the variation of rainfall elasticity values within a stream class is small. We expect that rainfall elasticity derived from MLRs most affected by multicollinearity and omitted-variable bias will diverge from ε_{x_m} and not be consistent with local rainfall elasticities within a stream class.

2.2.2. Classification of Ungaged Streams

Linear discriminant analysis was used to assign ungaged streams to one of the three perennial stream classes or the nonperennial stream class using basin characteristics. All 21 basin characteristics and average annual rainfall (1978–2007) were considered for the discriminant analysis. All data were natural-log transformed to ensure reasonable compliance with the assumption of normality. Log-transformed data were further standardized so that each variable had a mean of 0 and a standard deviation of 1. A backward stepwise procedure was used to identify discriminators among the selected basin characteristics providing the best classification scores for cross-validation sets. Cross-validation sets were randomly resampled 50 times from all 86 gaged streams, with 75% of the gaged streams used to fit the classification function and 25% of the gaged streams used to compute the classification score for each of the 50 samples. The classification score is the percentage of streams classified correctly with the linear decision function from the discriminant analysis. The discriminant analysis was computed using the Scikit-Learn tools in the Python computing language [*Pedregosa et al*., 2011].

2.3. Definition of MLRs and Modeling Methods

Several PR structures can be used depending on the characteristics of the heterogeneity in the dependent variable between streams and assumptions on model residuals. Random-effects panel regression (RE-PR) is the focus in this study because it is a PR structure that can be used for estimation in ungaged basins. Other PR structures, including fixed-effects (FE-PR), and pooled (P-PR) panel regression, are presented and evaluated in this study for comparative purposes. PR structures are briefly defined below and conditions necessary for different PR structures are discussed and compared in detail in sections 2.3.1 and 2.3.2. An introduction to the use of PR in hydrologic applications is provided by *Steinschneider et al*. [2013] and similar procedures are followed in this study. Computation of PR was done using the package *plm*, which was developed in the statistical computing language R [*Croissant and Millo*, 2008] and supports the unbalanced panels in this study composed of data from streams with record lengths varying between 10 and 30 years.

Random-effects panel regression (RE-PR) accounts for both between and within basin heterogeneities. The general form of the RE-PR applied in this study is:

$$y_{k,t} = \sum_{i=1}^{N} \beta_i \; x^i_{k,t} + \beta_o + \mu_k + r_{k,t} \tag{3}$$

where $y_{k,t}$ is the logarithm of $Q_{x_{k,t}}$, β_i is the ith regression coefficient, $x^i_{k,t}$ is the logarithm of the ith explanatory variable value for stream k during water year t, β_o is the global regression intercept, constant across time and between streams, μ_k is the individual effect for stream k, a random independent variable, constant in time for stream k, that characterizes the time-averaged heterogeneity in $y_{k,t}$ between streams, and $r_{k,t}$ is the zero-mean independent noise term for stream k during water year t, assumed Gaussian in this study.

RE-PR provides coefficient estimates that are consistent with the spatial and temporal effects of the explanatory variables if the appropriate assumptions on model terms are met, in particular if the individual effect is uncorrelated with the included explanatory variables. We discuss these assumptions in detail and describe the methods to test them in section 2.3.1. To estimate MLR coefficients with RE-PR, values of μ_k and $r_{k,t}$, which represent the between stream and within stream error terms, respectively, are combined into a single error term ($v_{k,t}$). Due to the cross correlation among the values of the dependent variables at each watershed and across watersheds, RE-PR is solved using feasible GLS based on the estimation of the variance of μ_k and $r_{k,t}$. The use of GLS methods to estimate coefficients in RE-PR ensures that the sum of the variances of μ_k and $r_{k,t}$ is equal to the variance of $v_{k,t}$ [*Swamy and Arora*, 1972].

For MLRs in this study, we characterize annual temporal variations in low-flow duration discharges using annual rainfall alone, the only explanatory variable which is a time series. The spatial variability in average low-flow duration discharges across streams during their selected period of record is characterized by selected static basin characteristics and annual rainfall during this period. The terms $x^i_{k,t}$ are constant within a stream for static basin characteristics used as explanatory variables in the MLR. Since rainfall is the only time-variant explanatory variable included in the MLRs, ours analysis can only demonstrate how RE-PR addresses effects of omitted-variable bias and the issues of multicollinearity between rainfall and the static explanatory variables included in the model. By including the individual effect for each stream, the effects of rainfall on low flows over time can be separated from time-averaged variations in low flows across streams that could be explained by other omitted variables. This study is not concerned with evaluating multicollinearity among static explanatory variables because the primary goal is to accurately estimate rainfall elasticity in the MLRs. The coefficient on rainfall (elasticity) may partially compensate for the effect of other time-varying variables that are not included in the models.

Fixed-effects panel regression (FE-PR) assumes omitted factors controlling the behavior of the streamflow characteristics $y_{k,t}$ in the PR structure are correlated with other explanatory variables included in the model. To account for such fixed effects, the individual effects μ_k in equation (3) cannot be considered random variables, the assumption for RE-PR, and are estimated for each gaged stream. μ_k is the unknown intercept for each stream and estimation of such fixed effects is equivalent to the use of dummy variables in OLS regression, where each value of μ_k is estimated as the regression coefficient for a binary [0, 1] variable corresponding to the watershed of interest. Time-invariant explanatory variable are perfectly collinear with the individual effects (or dummy variables) and are not included in FE-PR. Equation (3) for FE-RE models is rewritten as

$$y_{k,t}=\sum_{i=1}^{N}\beta_i\ x^i_{k,t}+\mu_k+r_{k,t} \tag{4}$$

Pooled panel regression (P-PR) assumes no heterogeneity between watersheds in the PR structure and thus the individual effects (μ_k) in equation (3) are equal to zero and all model coefficients are homogenous. Equation (3) for P-PR models is rewritten as

$$y_{k,t}=\sum_{i=1}^{N}\beta_i\ x^i_{k,t}+\beta_o+r_{k,t} \tag{5}$$

P-PR is equivalent to the development of a single OLS regional hydrologic regression for estimating time series of annual values of the dependent variable for all streams. This method has previously been employed for example, to estimate runoff characteristics over time [*Samaniego and Bárdossy*, 2005].

Space-for-time traditional regression (ST-TR), the most common regional hydrologic modeling approach, estimates streamflow characteristics at ungaged locations for average conditions and is written as

$$\overline{y_k}=\beta_o+\sum_{i=1}^{N}\beta_i\overline{x}^i_k+r_k \tag{6}$$

where $\overline{y_k}$ is the logarithm of $\overline{Q_{x_k}}$, β_o is the global regression intercept, β_i is the ith regression coefficient, $\overline{x}^i_k$ is the logarithm of the time-averaged ith explanatory variable value for stream k, and r_k is the zero mean independent noise term for stream k. To represent the approach most commonly used in regional hydrologic regression studies, ST-TR models are solved using OLS. Such an approach is now in widespread usage across the entire U.S. as evidenced by the StreamStats application developed by the U.S. Geological Survey [*Ries et al.*, 2008].

2.3.1. Definition of Necessary Assumptions for PR

The development of meaningful and sensible MLRs depends critically upon the behavior of model residuals as emphasized and documented in *Helsel and Hirsch* [2002]. A critical assumption needed to perform statistical inference on an MLR is that the model residuals are homoscedastic. We contend that one of the primary advantages of PR methods is their ability to overcome issues relating to heteroscedasticity, which can arise for a number of reasons. Two common tests of homoscedasticity are considered. *The Breusch-Pagan Lagrange multiplier (BPLM)* tests the statistical significance of the heterogeneity in $y_{k,t}$ between basins [*Breusch and Pagan*, 1980], that is, whether the model residuals have constant variance across watersheds or whether model residuals are heterogeneous. If the BPLM test determines that heterogeneity in $y_{k,t}$ between basins is statistically significant, a RE-PR or FE-PR structure is necessary, wherein the random variable μ_k accounts for this heterogeneity, and P-PR is not appropriate. *The Honda Lagrange multiplier (HLM)* tests the statistical significance of explanatory variable heterogeneity [*Honda*, 1985]. We employ the HLM to determine whether this heterogeneity can be accounted for in a RE-PR.

A critical benefit of pooling both time-series data as well as watershed-specific data is the ability to control for effects specific to each watershed, which are likely to be unobservable, but may be correlated with variables included in the model. *The Hausman test* was developed to test for this condition to determine if model residuals are truly exogenous and not correlated with variables already included in the model [*Hausman and Taylor*, 1981]. RE-PR assumes the residuals ($r_{k,t}$) in (3) are correlated within each stream and that the individual effects are uncorrelated across streams. With the assumption that unobserved effects are not correlated with either the model explanatory variables or the $r_{k,t}$, static basin characteristics included in the MLR can be used to estimate the time-averaged response in $y_{k,t}$ across streams, thus allowing the estimation of low-flow duration discharges in ungaged areas. We perform the Hausman test to determine whether there is evidence that the individual effects are not exogenous, and thus correlated with other explanatory variables. In such instances, effects of static explanatory variables estimated using RE-PR may be biased, low-flow duration discharges cannot be accurately estimated in ungaged areas, and only FE-PR can be used.

2.3.2. Comparison of PR Modeling Methods Considered

P-PR is only appropriate for cases in which the spatial heterogeneity in the dependent variable between streams can be characterized by the pooled model. If BPLM and/or HLM indicate that the model residuals are heteroscedastic, then the heterogeneity in the dependent variable between streams cannot be accounted for by the model explanatory variables, and the OLS estimator of the model coefficients would not be consistent (meaning that they do not converge to their true value).

FE-PR and RE-PR use individual effects to account for the unobserved spatial heterogeneity in the dependent variable, thus removing potential bias in the estimation of MLR coefficients. In FE-PR, the individual effect is estimated for each gaged stream and model coefficients are estimated for only time-varying explanatory variables, thus FE-PR cannot be used for low-flow estimation in ungaged streams unless a separate method is developed for estimation of the individual effects in ungaged streams. At a gaged site as was considered by *Steinschneider et al*. [2013], RE-PR is even more general than FE-PR because the individual effects are embedded into the covariance matrix of the model residuals and estimated as a separate variance component in addition to the model error variance. Using RE-PR, model coefficients for both stationary and time-varying explanatory variables can be accurately estimated, so that such methods may be quite promising for developing regional hydrologic models that can estimate the effect of climatic variability. When conditions for RE-PR cannot be met (Hausman test indicates that individual effects are not exogenous), FE-PR is used to provide consistent estimates of rainfall elasticity for each region and flow-duration percentile to model the relative annual variability of annual low-flow duration discharges, as represented by $\hat{\delta}_{x_{k,t}}$, in ungaged areas without including static explanatory variables. FE-PR is, however, less efficient than RE-PR, because FE-PR requires an estimate of the fixed effect for each stream, thus RE-PR results in fewer model coefficients.

Comparison of results of MLR using ST-TR and the time-averaged results of MLRs fit using RE-PR provides an indication of the performance and robustness of rainfall elasticity estimates derived from MLRs over long-term average climate conditions. Comparison of MLRs fit using P-PR and RE-PR provides an indication of the temporal performance and the accuracy of rainfall elasticity estimated by accounting for between-basin heterogeneity. If P-PR and RE-PR provide equivalent results, this provides indication that individual effects approach zero given the selected explanatory variables in the MLR, and thus a pooled model could be adequate. Comparison of MLRs fit using FE-PR and RE-PR provides an indication of the effect of omitted variables and if the available data are appropriate for both temporal and spatial estimations using RE-PR. If FE-PR and RE-PR provide equivalent results, this may be an indication that explanatory variables were appropriately selected and MLR coefficients resulting from RE-PR are unbiased.

2.4. Developing MLRs to Estimate Low-Flow Characteristics and Rainfall Elasticities

MLRs are developed to estimate $Q_{50_{k,t}}$, $Q_{70_{k,t}}$, and $Q_{90_{k,t}}$ and obtain estimates of ε_{50}, ε_{70}, and ε_{90} for each perennial stream class (a total of nine MLRs). RE-PR criteria are used to select explanatory variables for each MLR. Each MLR is fit using RE-PR, FE-PR, P-PR, and ST-TR and the goodness of fit of each resulting model is evaluated to discuss the efficiency of each method for regional climate-change assessments. The same selected explanatory variables and data were used for each model structure. For FE-PR, the effects of static explanatory variables are incorporated in the individual effects and coefficients for static explanatory variables were not estimated. Although these different modeling methods may provide better results using different explanatory variables and constraining data with additional criteria, it is important to contrast comparable MLRs in this study.

2.4.1. Evaluating MLRs

Estimated values of y_{kt} using MLRs fit using RE-PR, FE-PR, and P-PR were converted back to real space ($\hat{Q}_{x_{k,t}}$) and the mean of the annual low-flow duration discharge estimates was computed and termed $\overline{\hat{Q}_{x_k}}$. Estimated annual anomalies relative to the $\overline{\hat{Q}_{x_k}}$ were also computed and termed $\hat{\delta}_{x k,t}$. For ST-TR estimated values of $\overline{y_k}$ were converted back to real space, $\overline{\hat{Q}_{x_k}}$. The Nash-Sutcliffe Efficiency (NSE) is used as a measure of both the variance and the bias of model annual low-flow duration discharge estimates for RE-PR and P-PR. The spatial goodness-of-fit (NSE_s) is computed from $\overline{\hat{Q}_{x_k}}$ and $\overline{Q_{x_k}}$ and is evaluated for RE-PR, P-PR, and ST-TR. The spatial goodness-of-fit is not evaluated for FE-PR because FE-PR cannot be applied spatially to estimate low-flow duration discharge in ungaged basins. The annual temporal goodness-of-fit (NSE_a), computed from $\hat{\delta}_{x k,t}$ and $\delta_{x_{k,t}}$ of all streams in a class combined, assesses the fit of model coefficients for the temporally varying explanatory variable (rainfall) only and is evaluated for RE-PR, FE-PR, and P-PR. The local temporal goodness-of-fit (NSE_a^k), computed from $\hat{\delta}_{x_{k,t}}$ and $\delta_{x_{k,t}}$ for each stream individually, assesses the temporal model performance for each gaged stream and is evaluated for RE-PR, FE-PR, and P-PR. ST-TR cannot be used to estimate annual anomalies and can only estimate the time-averaged values. Goodness-of-fit metrics were computed a second time with a leave-one-out approach to evaluate model robustness and aid in the selection of explanatory variables in a final MLR. For the leave-one-out approach, MLRs were fit leaving out all data from one gaged stream at a time, and the resulting MLR was used to estimate annual and average low flows for the stream that was left out. Validation of regional rainfall elasticity values is necessary to evaluate whether the developed MLR can be reliably used for varying climate conditions. NSE_a^k for the 1978–2007 reference period and the 1921–1977 validation period at seven sites with long-term records were computed using annual and 30 year moving average values of $\hat{\delta}_{x_{k,t}}$ and $\delta_{x_{k,t}}$ to evaluate the annual and long-term temporal model performance.

2.4.2. Selecting Explanatory Variables

MLRs for all combinations of up to seven explanatory variables (selected from a total of 21 static explanatory variables) were considered and evaluated with the leave-one-out approach. A final MLR for each stream class and flow-duration percentile was selected based on best judgment and the following criteria:

1. All model coefficients were physically meaningful and statistically significant with 5% significance level.
2. Selected explanatory variables for flow percentiles 50, 70, and 90 within a stream class were either the same or physically similar. This ensures, for example, that for a particular stream the MLRs for Q_{90} do not estimate a value greater than from a MLR for Q_{70}.
3. All conditions in section 2.3.1 were met with a 5% significance level for the final MLR and for at least 90% of the MLRs in the leave-one-out approach.
4. The root mean square error associated with the leave-one-out approach was the smallest.

3. Results

3.1. Local Rainfall Elasticity and Stream Classification

For each low-flow duration percentile, the ranges of average low-flow duration discharge ($\overline{Q_{x_k}}$), local rainfall elasticity (ε_{x_k}), and the R_k^2 of the power-law relation between annual low-flow duration discharge ($Q_{x_{k,t}}$) and

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annual rainfall ($RF_{k,t}$) for gages within each of the three stream classes are summarized in Figure 2. The numbers of streams clustered using k-means into perennial stream classes representing low (P_l), medium (P_m), and high (P_h) rainfall elasticity are 28, 29, and 12, respectively, and the stream class for each stream is reported in Figure 1b.

For a particular low-flow duration discharge, estimates of local rainfall elasticity for each stream were generally lower than 1 (termed inelastic) for streams with the greatest average low-flow duration discharge values. For individual P_l and P_m streams, local rainfall elasticity decreased with decreasing flow-duration discharge (ε_{50_k} was greater than ε_{70_k} and ε_{90_k}). A majority of streams in the P_h class have characteristics of ephemeral streams. Discharge is more controlled by rainfall than groundwater discharge in such streams and rainfall elasticity generally increased with decreasing flow-duration discharge. Figure 2c indicates that a log-linear model to represent the response of low flows to rainfall is reasonable. For a particular stream class, the strength of the relation between annual low-flow duration discharges and annual rainfall decreased with decreasing flow-duration discharge, and for a particular flow-duration discharge the strength of the relation generally was greatest for P_h streams and lowest for P_l streams. Local rainfall elasticity was weakly and negatively correlated with average low-flow duration discharge. The Pearson's correlation coefficients between $\overline{Q_{50_k}}$ and ε_{50_k}, $\overline{Q_{70_k}}$ and ε_{70_k}, and $\overline{Q_{90_k}}$ and ε_{90_k} were -0.26, -0.37, and -0.38,

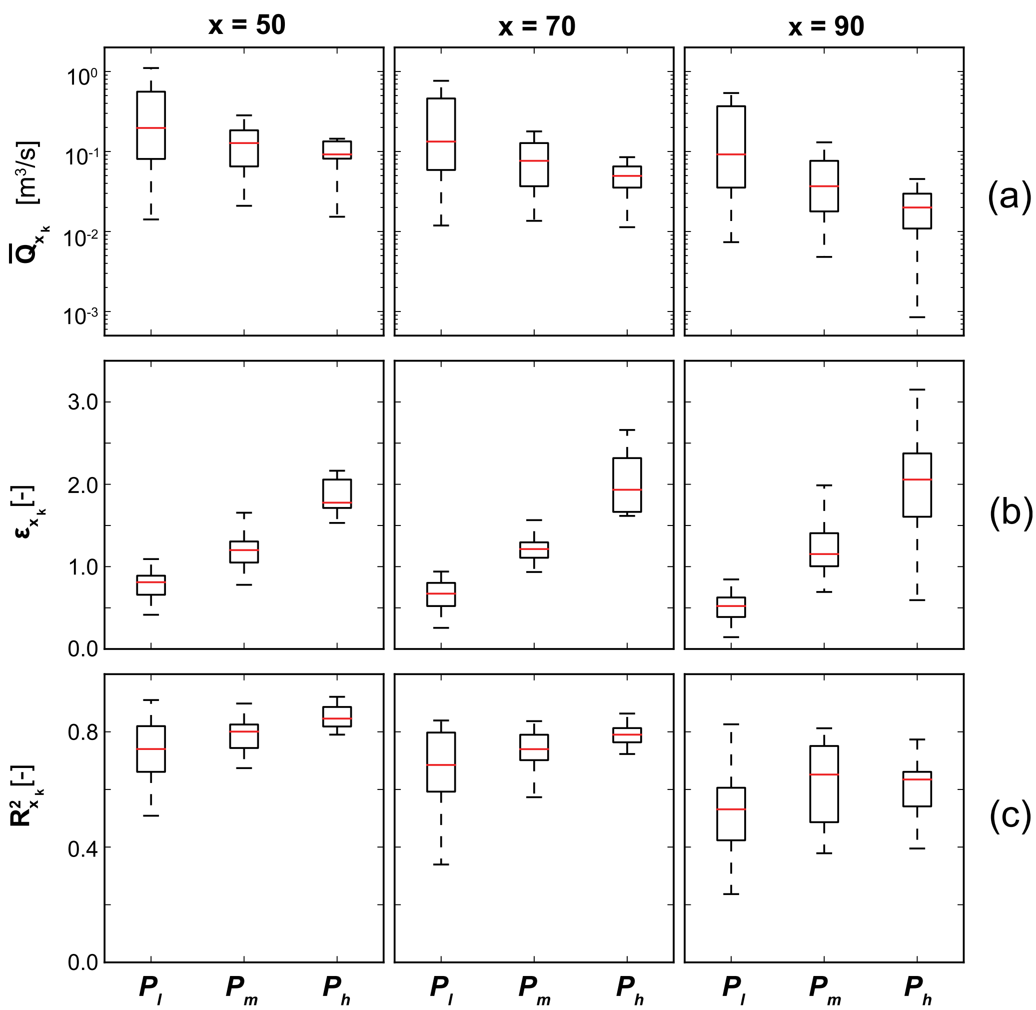

Figure 2. Characteristics of perennial stream classes with low (P_l), medium (P_m), and high (P_h) rainfall elasticity. (a) Ranges of average annual low-flow duration discharges $\overline{Q_{x_k}}$ at gaged sites, (b) associated ranges of rainfall elasticity (ε_{x_k}), and (c) strength of the power-law relation between annual low-flow duration discharges and annual rainfall ($R_{x_k}{}^2$). The subscript x refers to the flow-duration percentile considered and takes the values of $x = 50$, 70, and 90. Box plots indicate interquartile ranges, whiskers indicate the maximum and minimum values, and the median value is marked by a red line.

respectively, and the relations were statistically significant at the 5% significance level. Basin characteristics correlated with local rainfall elasticity using a significance level of 5% are generally associated with groundwater and surface-water interactions and included percentage of area covered in wetland, basin-incision index, length of stream network, and drainage density, and correlation coefficients were small (ranged from -0.35 to 0.3). Local rainfall elasticity was not significantly correlated with average rainfall.

All basin characteristics except mean elevation, relief, length of stream network, average slope, and percentage of forested area were selected to determine the stream class in ungaged areas using linear discriminant analysis. The discriminant analysis classified 76% of the streams correctly and on average 50% of the streams in the cross-validation sets.

3.2. Selected MLRs

MLRs with static basin characteristics, which met all assumptions necessary for RE-PR (section 2.3.2), were identified and one MLR for each stream class and low-flow duration percentile was selected for further analysis using criteria in section 2.4.2. An important aspect of the value of the PR was demonstrated during the MLR selection. For all MLRs that met the necessary assumptions for RE-PR, the value of the coefficient on annual rainfall was stable and was not affected by the number or type of static explanatory variables (results not shown). The coefficients on rainfall therefore, in these cases, partitioned the spatial and temporal variability correctly and were not biased by the explanatory variables included or excluded in the MLRs. For the P_m stream class, few MLRs met all necessary assumptions and overall the performance of MLRs was poorest suggesting that the P_m stream class may need to be further stratified.

The selected MLRs are summarized in Table 2. The median baseline model (M-BL) in Table 2 estimates relative annual variability in low flows ($\hat{\delta}_{x_{k,t}}$) using the median rainfall elasticity values for each stream class (ε_{x_m}) and the relative change in annual rainfall and the regional and local temporal model performances are reported. Selected static basin characteristics included morphometric characteristics (x_2, drainage area; x_7 and x_8, stream length; x_{10}, drainage density; x_9, average slope; x_6, basin relief) and surface-geology characteristics (x_{18}, percentage of area mapped as sedimentary rock; x_{19}, percentage of area mapped as shield-stage volcanics; x_{20}, percentage of area mapped as post-shield-stage volcanics; x_{21}, percentage of area mapped as rejuvenated-stage volcanics). Individual selected static basin characteristics were not correlated with time-averaged rainfall in a physically meaningful way and using a 5% significance level. The variance inflation factor (VIF) for average annual rainfall (Table 2) was, however, generally above the threshold of 5. The VIF factors were greatest (up to a value of 40) for the P_l stream class and indicate that rainfall elasticity estimated from MLRs using TR for the P_l stream class may be most affected by multicollinearity. For the P_m and P_h stream classes, VIF values ranged from 6 to 13 and MLRs may be less affected by multicollinearity. The effect of multicollinearity on regression coefficients of static explanatory variables was not evaluated. Figure 3 illustrates the overall (3a), spatial (3b), and local temporal (3b) goodness of fit for $Q_{70_{k,t}}$ using RE-PR for the three stream classes. Patterns of values above and below the 1:1 line in Figure 3a. correspond to annual values for each stream, which are clustered around the time-average estimates represented in Figure 3b. Annual values for each stream generally follow a unit slope indicating that the elasticity describing the annual variability is accurate and this observation is consistent with the model residuals for each stream visualized in Figure 3c.

3.3. Analysis of Individual Effects

For all of the nine final MLRs developed using the PR-RE method, the p values for the BPLM, the HLM, and MLR coefficients were all considerably smaller than 0.001. Results of the BPLM indicate that a PR structure is necessary because heterogeneity in the dependent variable between basins is significant. Results of the HLM indicate that given the selected explanatory variables, RE-PR can account for the heterogeneity in the data. p Values for the Hausman test were greater than 0.05 (ranging from 0.06 to 0.23) (Table 2), indicating that consistent regression coefficients (β_i) can be obtained from RE-PR because individual effects (μ_k) are exogenous.

For each MLR developed using FE-PR, a summary of individual effects values is presented in Figure 4. The ranges of individual effects were greatest for the P_l stream class reflecting the greatest heterogeneity in low flows in P_l streams, which is consistent with Figure 2. Figure 4 illustrates the importance of incorporation of individual effects to account for cross-site heterogeneity, because it highlights the extreme variability in

Table 2. Selected Multiple Linear Regression (MLR) Models[a]

Stream Class	Flow	Explanatory Variables (i)	VIF_{RF}	Hausman p Value	Model Structure	β_0	$\beta_1(\varepsilon_x)$	β_2	β_3	β_4	β_5	E_1	NSE_s	NSE_a
					Selected Regression Model								Goodness of Fit	
P_l	Q_{50}	$^{(1)}RF_t$	1.5	0.06	RE-PR	−10.116	0.809	0.865	0.077			0.033	0.78 (0.73)	0.49 (0.49)
		$^{(2)}x_8$			P-PR	−12.483	1.095	0.867	0.075			0.067	0.81 (0.75)	0.41 (0.41)
		$^{(3)}x_{18}$			ST-TR	−13.39	1.344	0.817	0.088			0.425	0.79 (0.71)	
					FE-PR		0.806					0.033		0.49 (0.49)
					M-BL		0.810							0.49 (0.49)
	Q_{70}	$^{(1)}RF_t$	40	0.08	RE-PR	0.000	0.685	0.843	1.422	0.112		0.033	0.73 (0.67)	0.40 (0.40)
		$^{(2)}x_2$			P-PR	0.000	0.270	0.952	0.890	0.077		0.058	0.77 (0.71)	0.25 (0.26)
		$^{(3)}x_{10}$			ST-TR	0.000	0.298	0.906	0.869	0.088		0.353	0.75 (0.66)	
		$^{(4)}x_{18}$			FE-PR		0.690					0.033		0.40 (0.40)
					M-BL		0.666							0.40 (0.40)
	Q_{90}	$^{(1)}RF_t$	40	0.17	RE-PR	0.000	0.541	0.839	1.266	0.130		0.039	0.68 (0.61)	0.23 (0.23)
		$^{(2)}x_2$			P-PR	0.000	0.144	0.955	0.761	0.094		0.065	0.72 (0.66)	0.11 (0.11)
		$^{(3)}x_{10}$			ST-TR	0.000	0.215	0.889	0.805	0.110		0.404	0.69 (0.61)	
		$^{(4)}x_{18}$			FE-PR		0.545					0.039		0.23 (0.23)
					M-BL		0.512							0.23 (0.23)
P_m	Q_{50}	$^{(1)}RF_t$	6	0.23	RE-PR	0.000	1.202	1.053	−1.435	6.228	6.116	0.037	0.66 (0.47)	0.56 (0.56)
		$^{(2)}x_6$			P-PR	0.000	1.109	1.073	−1.406	6.016	5.920	0.050	0.65 (0.36)	0.56 (0.55)
		$^{(3)}x_9$			ST-TR	0.000	0.990	1.025	−1.340	5.222	5.125	0.341	0.64 (0.36)	
		$^{(4)}x_{19}$			FE-PR		1.207					0.037		0.56 (0.56)
		$^{(5)}x_{20}$			M-BL		1.200							0.56 (0.56)
	Q_{70}	$^{(1)}RF_t$	6	0.17	RE-PR	0.000	1.193	1.137	−1.517	6.538	6.405	0.044	0.69 (0.52)	0.49 (0.48)
		$^{(2)}x_6$			P-PR	0.000	1.084	1.164	−1.451	6.343	6.233	0.054	0.67 (0.42)	0.48 (0.48)
		$^{(3)}x_9$			ST-TR	0.000	0.853	1.110	−1.399	5.163	5.053	0.378	0.66 (0.41)	
		$^{(4)}x_{19}$			FE-PR		1.199					0.043		0.49 (0.48)
		$^{(5)}x_{20}$			M-BL	0.000	1.204							0.49 (0.49)
	Q_{90}	$^{(1)}RF_t$	6	0.15	RE-PR	0.000	1.178	1.247	−1.609	6.932	6.780	0.066	0.67 (0.54)	0.37 (0.36)
		$^{(2)}x_6$			P-PR	0.000	0.996	1.281	−1.500	6.571	6.455	0.072	0.63 (0.45)	0.36 (0.36)
		$^{(3)}x_9$			ST-TR	0.000	0.680	1.197	−1.355	5.174	5.060	0.451	0.62 (0.41)	
		$^{(4)}x_{19}$			FE-PR		1.191					0.066		0.37 (0.36)
		$^{(5)}x_{20}$			M-BL		1.145							0.37 (0.37)
P_h	Q_{50}	$^{(1)}RF_t$	13	0.11	RE-PR	0.000	1.911	0.551	3.204	0.201		0.099	0.54 (0.16)	0.64 (0.63)
		$^{(2)}x_2$			P-PR	0.000	1.434	0.454	2.518	0.153		0.092	0.72 (0.40)	0.62 (0.62)
		$^{(3)}x_{10}$			ST-TR	0.000	1.299	0.414	2.193	0.108		0.607	0.72 (0.07)	
		$^{(4)}x_{21}$			FE-PR		1.938					0.097		0.63 (0.63)
					M-BL		1.761							0.64 (0.64)
	Q_{70}	$^{(1)}RF_t$	10	0.09	RE-PR	−32.613	1.955	0.997	−1.852	−0.264	0.298	0.088	0.89 (<0)	0.53 (0.52)
		$^{(2)}x_2$			P-PR	−27.673	1.644	0.874	−1.493	−0.242	0.270	0.120	0.88 (<0)	0.53 (0.53)
		$^{(3)}x_{10}$			ST-TR	−25.048	1.577	0.824	−1.369	−0.209	0.223	0.383	0.88 (0.79)	
		$^{(4)}x_7$			FE-PR		2.026					1.112		0.52 (0.52)
		$^{(5)}x_{19}$			M-BL		1.858							0.53 (0.53)
	Q_{90}	$^{(1)}RF_t$	10	0.09	RE-PR	−36.759	1.799	0.992	−2.594	−0.391	0.419	0.214	0.83 (<0)	0.24 (0.22)
		$^{(2)}x_2$			P-PR	−32.156	1.506	0.861	−2.272	−0.369	0.393	0.144	0.75 (0.68)	0.24 (0.23)
		$^{(3)}x_{10}$			ST-TR	−27.22	1.270	0.637	−2.110	−0.201	0.161	0.369	0.90 (<0)	
		$^{(4)}x_7$			FE-PR		1.979					0.185		0.23 (0.21)
		$^{(5)}x_{19}$			M-BL		2.150							0.21 (0.21)

[a] P_l, perennial stream class with low elasticity; P_m, perennial stream class with medium elasticity; P_h, perennial stream class with high elasticity; RE-PR, random-effects panel regression; P-PR, pooled panel regression; FE-PR, fixed-effects panel regression; M-BL, median baseline elasticity model; ST-TR, space-for-time traditional regression; RF_t, annual rainfall (for ST-T average annual rainfall during the period of record is used); x_j, model explanatory variables j (see Table 1); VIF_{RF}, variance inflation factor of average annual rainfall; β_i, multiple linear regression model coefficient for explanatory variable i; E_1, standard error of regression coefficient β_1; NSE, Nash Sutcliffe efficiency (value in parenthesis is the result from the leave-one-out approach); subscript "s" refers to spatial; subscript a refers to "a" annual temporal.

those coefficients, and documents their increasing variability for basins with lower rainfall elasticity. These results indicate that RE-PR is most important for P_l streams and that MLRs for P_l streams may be most prone to omitted-variable bias. The magnitude of skew in individual effects for each flow-duration percentile was greatest for the P_h stream class (Figure 4) and therefore the heterogeneity in low flows for P_h streams may not be considered a normally distributed random variable and indicates that FE-PR may be more appropriate than RE-PR for P_h streams, although rainfall elasticity estimates from MLRs using RE-PR and FE-PR were about equal.

3.4. Comparison of Rainfall Elasticity Estimates

Of primary importance in this study is a comparative assessment of rainfall elasticity to determine if PR provides more reliable estimates than TR. We use the median rainfall elasticity values for each stream class (ε_{x_m})

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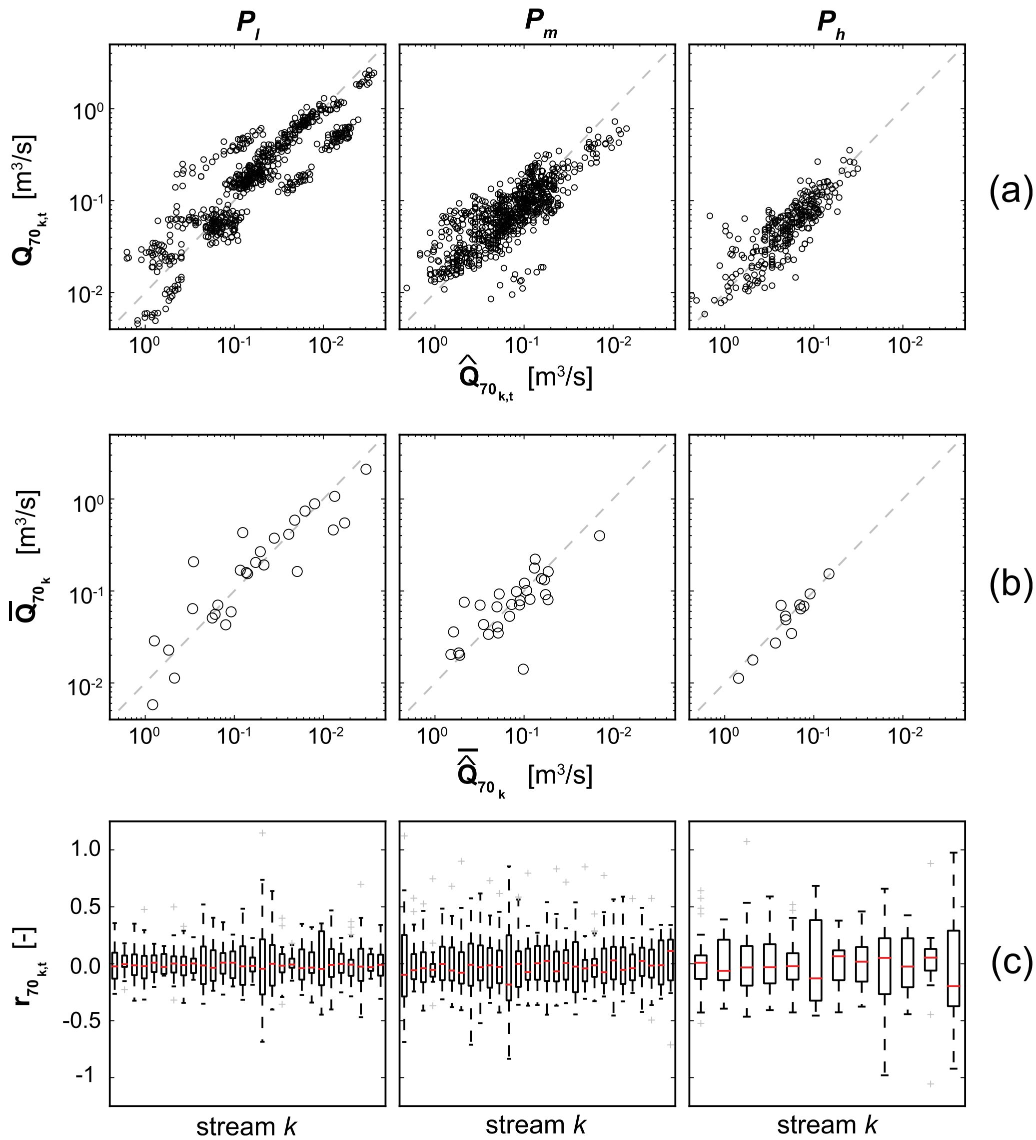

Figure 3. Goodness-of-fit for multiple linear regressions, using random-effects panel regression methods, of annual low-flow duration discharges $Q_{70_{k,t}}$ at perennial streams with low (P_l), medium (P_m), and high (P_h) rainfall elasticity. (a) Overall performance comparing observed ($Q_{70_{k,t}}$) and modeled ($\hat{Q}_{70_{k,t}}$) annual low-flow duration discharge. (b) Spatial performance comparing observed ($\overline{Q_{70_k}}$) and modeled ($\overline{\hat{Q}_{70_k}}$) averaged low-flow discharges for each stream class. (c) Ranges of normalized model residuals $\left(r_{70_{k,t}} = \frac{\hat{Q}_{70_{k,t}} - Q_{70_{k,t}}}{Q_{70_{k,t}}}\right)$ for each stream k. Boxes indicate the interquartile range. Whiskers indicate the most extreme value within 1.5 times the interquartile range and values beyond this range are plotted by a marker. The median value is marked by a red line.

as a baseline reference, which is independent of the MLRs. Rainfall elasticity determined for the MLRs may be affected by multicollinearity and/or omitted-variable bias if rainfall elasticity diverges from ε_{X_m} and therefore not consistent with the local rainfall elasticities at the gaged streams within the stream class. Rainfall elasticity values, including ε_{X_m} associated with the M-BL model, and model goodness-of-fit metrics among the tested model structures are summarized in Table 2. The values of rainfall elasticity determined from FE-PR and RE-PR were about equal to ε_{X_m}. The absolute differences in rainfall elasticity determined from RE-PR and FE-PR were generally negligible indicating that rainfall elasticity resulting from RE-PR and FE-PR were unlikely negatively affected by multicollinearity and omitted-variable bias. Rainfall elasticity determined from P-PR and ST-TR was underestimated relative to ε_{X_m} except for the MLR estimating $Q_{50_{k,t}}$ for the P_l stream class. The absolute differences between ε_{X_m} and rainfall elasticity ranged from 0.09 to 0.48 for rainfall elasticity determined from P-PR, and ranged from 0.21 to 0.61 for rainfall elasticity determined from ST-TR.

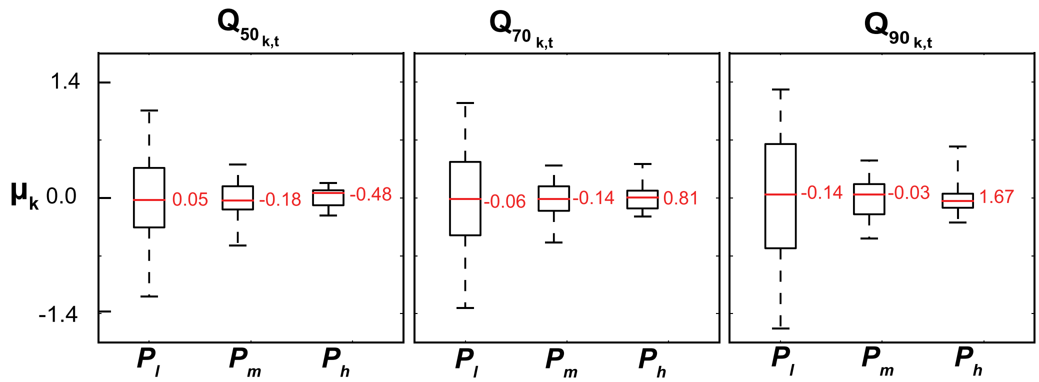

Figure 4. Ranges of fixed effects (μ_k) for perennial streams k with low (P_l), medium (P_m), and high (P_h) rainfall elasticity for $Q_{50_{k,t}}$, $Q_{70_{k,t}}$, and $Q_{90_{k,t}}$ flow-duration discharges. Box plot indicate minimum, maximum, and interquartile ranges. The median value is marked by a red line and red labels report the skew of the μ_k within a stream class and flow-duration discharge.

Rainfall elasticity resulting from P-PR and ST-TR may be negatively affected by multicollinearity and omitted-variable bias. Differences between ε_{x_m} and rainfall elasticity for all model structures were generally greatest for the P_l streams and the P_l stream class shows evidence of being the most negatively affected by multicollinearity and omitted-variable bias. This observation is consistent with the high VIF value for rainfall and the analysis of the individual effects. Differences between ε_{x_m} and rainfall elasticity for all model structures were generally greater for models estimating $Q_{90_{k,t}}$ than models estimating $Q_{50_{k,t}}$ and $Q_{70_{k,t}}$ indicating that lower flow characteristics are more affected by omitted-variable bias than higher flow characteristics. This is expected because the factors driving the variability in low-flow duration discharge are generally better observed for higher flows ($Q_{50_{k,t}}$) that are controlled more by surface processes than lower flows ($Q_{90_{k,t}}$) that are generally controlled by groundwater discharge into the stream.

For all PR methods, the standard error in rainfall elasticity ranged from 3% to 12%, with the exception of the MLR estimating $Q_{70_{k,t}}$ for P_h streams using FE-PR. The standard errors in rainfall elasticity from FE-PR were about equal to those of RE-PR, were smaller than those for P-PR for P_l and P_m streams, and were greater than those for P-PR for P_h streams. The standard errors in rainfall elasticity for ST-TR were between 2 and 13 times greater than the standard errors for RE-PR and ranged from 34 to 61%.

3.5. Model Goodness of Fit

The regional model spatial goodness-of-fit values were slightly better for RE-PR than P-PR and ST-TR to estimate low flows in P_m streams, and slightly better for P-PR and ST-TR than RE-PR to estimate low flows in P_l and P_h streams. Under cross-validation, RE-PR performs slightly better than ST-TR except for MLRs to estimate low flows in P_h streams. Temporal goodness-of-fit measures were similar among MLR models developed with RE-PR and FE-PR and the M-BL model, and these models had slightly better fits relative to models developed with P-PR. Figure 5 is a comparison of model temporal goodness-of-fit at each stream k (NSE_a^k) and indicates that generally the median NSE_a^k within a stream class is greater for M-BL (ε_{x_m}) and MLRs developed using RE-PR and FE-PR than for MLRs developed using P-PR. A greater number of streams have NSE_a^k values greater than 0.5 for MLRs developed using RE-PR and FE-PR than for MLRs developed using P-PR. ST-TR was not included in Figure 5 because this method does not provide a temporal result, although it is applied to estimate moving 30 year average results in Figure 6b. Figure 6 is an effort to evaluate the model performance in estimating year-to-year (Figure 6a) and long-term (Figure 6b) changes in low flows. The NSE_a^k for regional values of rainfall elasticity determined from RE-PR was generally superior to NSE_a^k for rainfall elasticity determined from P-PR (Figure 6a) and ST-TR (Figure 6b). The robustness of rainfall elasticity between the 1978–2007 reference period and the 1921–1977 validation period was greater for RE-PR than P-PR.

4. Application in Ungaged Streams to Estimate Effects of Changes in Rainfall on Low Flows and Stream Habitat

RE models developed in this study were applied to ungaged basins on the island of Maui for which downscaled climate projections were available to estimate the effects of projected changes in rainfall on low flows and usable habitat for native stream fauna. Drainage basins for the island of Maui were defined by a

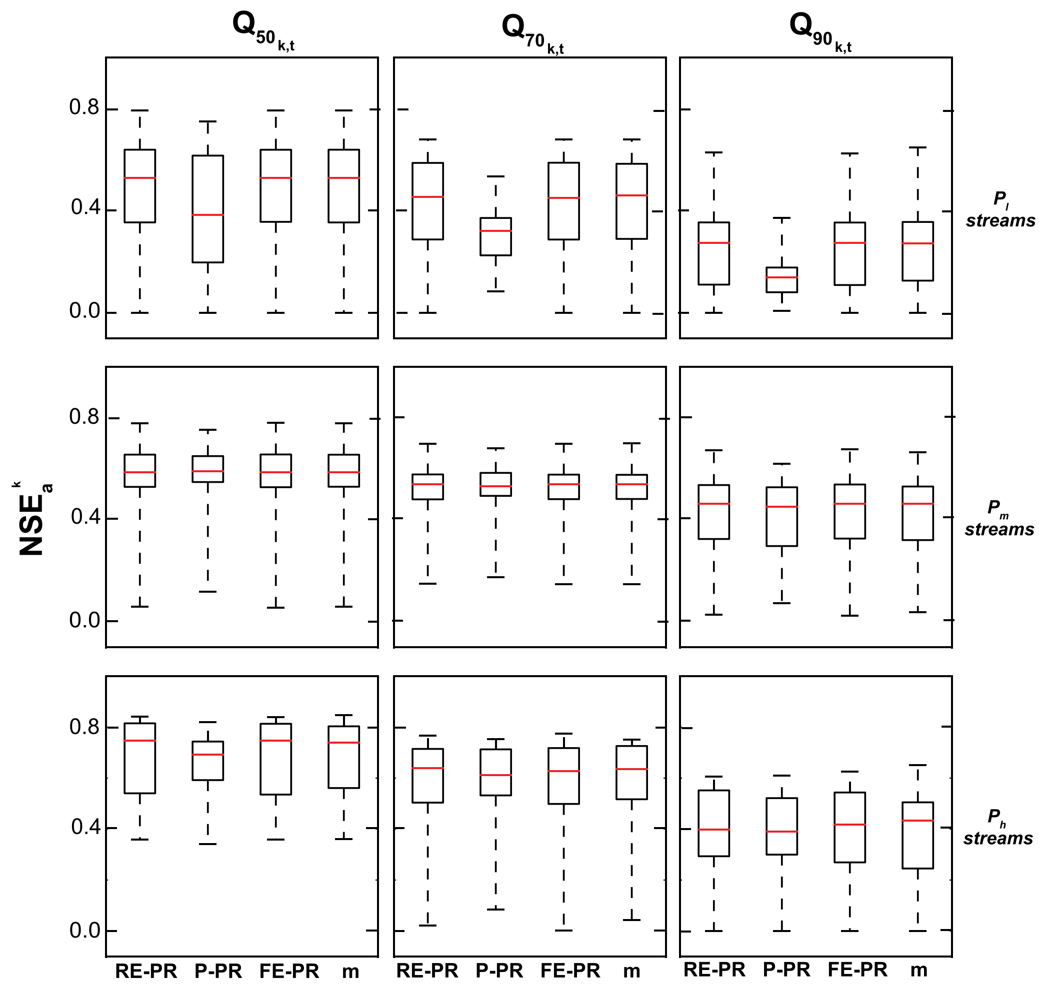

Figure 5. Local temporal goodness-of-fit for multiple linear regressions to estimate $Q_{50_{k,t}}$, $Q_{70_{k,t}}$, and $Q_{90_{k,t}}$ at each perennial stream k with low (P_l), medium (P_m), and high (P_h) rainfall elasticity using random-effects (RE-PR), pooled (P-PR), and fixed-effects (FE-PR) panel regression methods and the median (m) elasticity values observed at gaged streams within a class. Each box plot represents the range of local Nash-Sutcliffe efficiency (NSE_a^k) for each stream and low-flow duration discharge. Box plot indicate minimum, maximum, and interquartile ranges. The median value is marked by a red line.

stream network on a 20,000 cell threshold developed by *Rea and Skinner* [2012] using a USGS 10 m DEM. For each ungaged drainage basin on Maui, physical characteristics were determined with the same data and methods as the gaged drainage basins (section 2.1.1), and streams were classified using the linear decision function resulting from the discriminant analysis in section 3.1 The classification of gaged and ungaged basins on Maui is visualized in Figure 7. The authors recognize that some ungaged streams may be misclassified by the model, although on an island-wide scale the classifications generally are consistent with available information.

Known hydrogeological features can be related to regions of low-flow sensitivity to rainfall revealed by the application of the discriminant analysis to ungaged basins on Maui (Figure 7). Low flows in West Maui streams were least sensitive to changes in rainfall. The headwaters of these streams originate in high-rainfall and groundwater-recharge regions [*Johnson et al*., 2014] where groundwater is impounded to high levels in dike compartments and provides a persistent groundwater source [*Takasaki and Mink*, 1985; *Izuka et al*., 2016]. Low flows in East Maui streams were more sensitive to changes in rainfall. Base flow in these streams is dependent on perched aquifers and other vertically extensive freshwater lenses where volcanic rocks have low permeability [*Gingerich*, 1999]. Regions with streams classified as nonperennial are in areas that do not receive persistent orographic rainfall caused by the northeasterly trade winds and have low

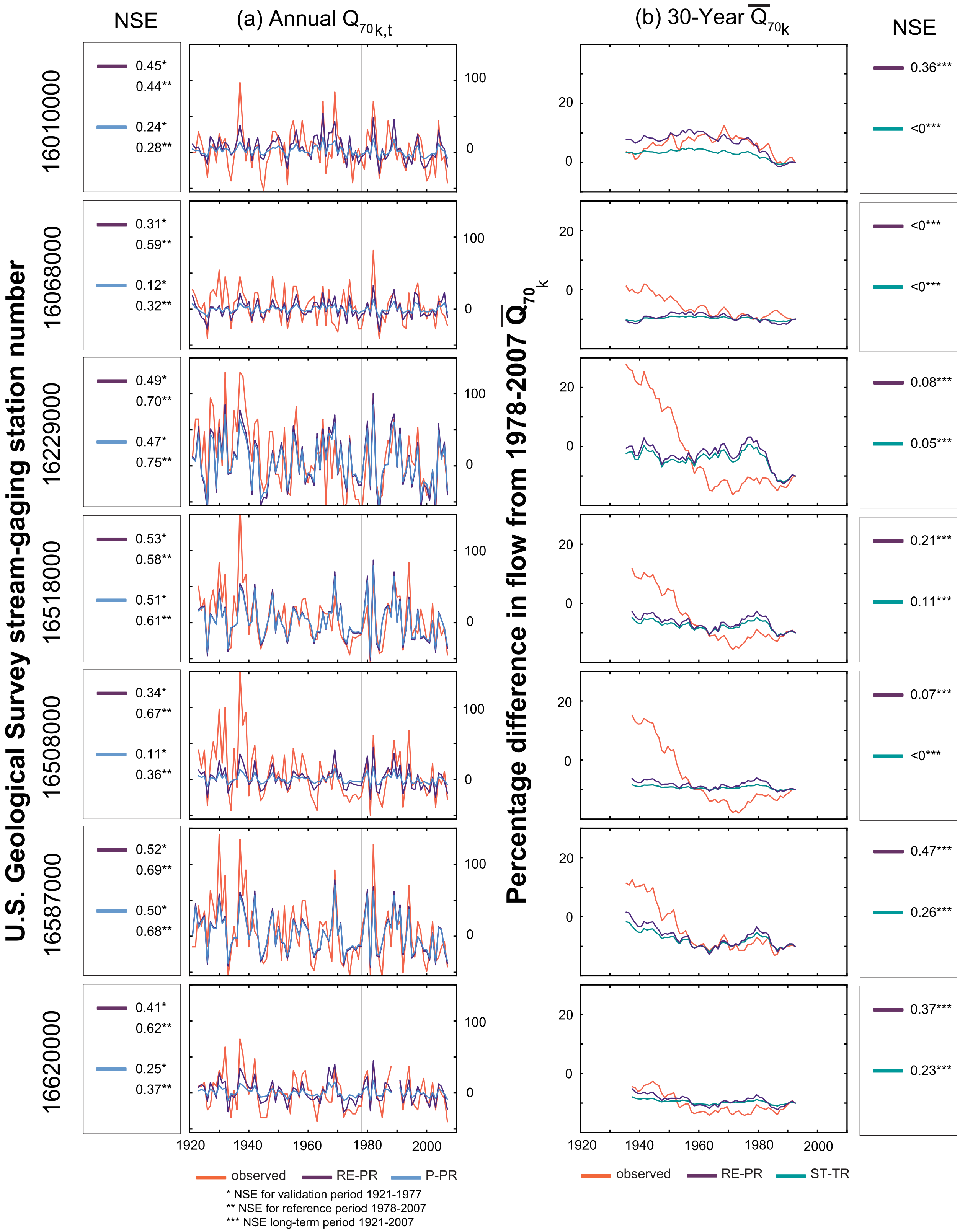

Figure 6. (a) Annual and (b) 30 year moving average annual time series of observed and modeled $Q_{70_{k,t}}$ at selected long-term U.S. Geological Survey gaging stations included in this study. Grey line separates the validation period from the current period used in the regression analysis (1978–2007). Values are expressed as a fraction of the mean annual Q_{70} during 1978–2007. Nash-Sutcliffe efficiency (NSE) values are reported in each subfigure legend (*1921–1977 annual values, **1978–2007 annual values, ***1921–2007 30 year mean annual values).

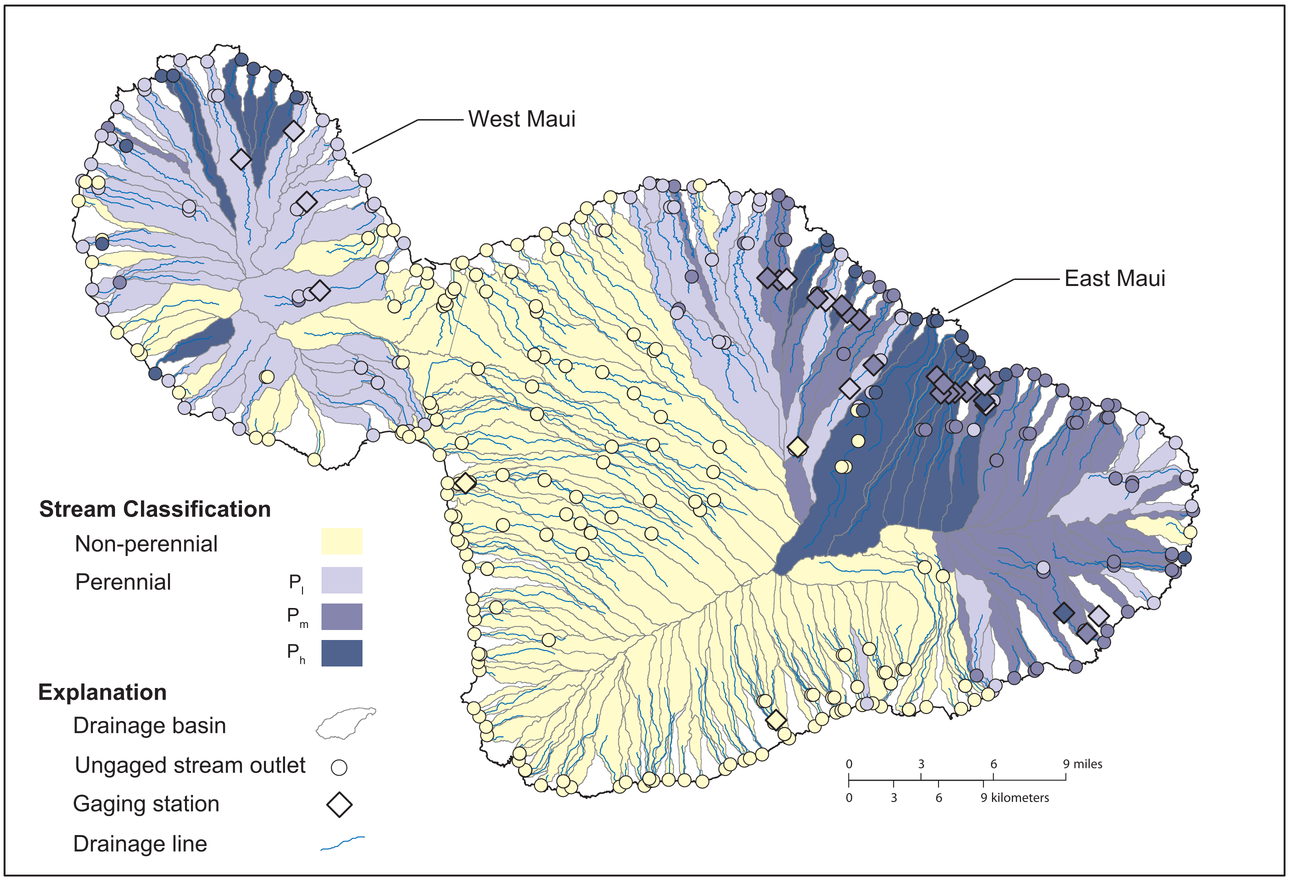

Figure 7. Classification of streams on the island of Maui estimated as a function of basin characteristics and annual rainfall for recent conditions using a linear decision function and classification of gaged stream (diamond) derived from streamflow and rainfall observations. Some ungaged streams may be misclassified. Shading of basins corresponds to the classification at the most downstream estimation point. Inland stream estimation points (circle) correspond to confluences of a stream network based on a 10 m DEM and a 20,000 cell threshold.

annual rainfall, low runoff-to-rainfall ratios, and low recharge [*Johnson et al*. 2014]. Much of the rainfall infiltrates the surface, groundwater levels are low, and base flow generally is absent in these basins.

Small-scale topographic features, such as the Hawaiian Islands, are not represented by General Circulation Models (GCMs) used to project future climate [*Timm and Diaz*, 2009]. Two sets (A and B) of high-resolution downscaled rainfall anomalies for the mid and late-21st century relative to reference rainfall during 1978–2007 [*Giambelluca et al*., 2013] were applied to the MLRs developed using RE-PR in this case study to estimate changes in $\overline{Q_{50_k}}$, $\overline{Q_{70_k}}$, and $\overline{Q_{90_k}}$ and usable stream habitat for native aquatic species. Set A was derived from Coupled Model Intercomparison Project 5 (CMIP5) and the RCP8.5 scenario statistically downscaled rainfall projections developed by *Timm et al*. [2015] and represents average annual rainfall anomalies for 2071–2099 (Figure 8a). Set B was derived from CMIP3 and the A1B emission scenario dynamically downscaled climate projections developed by *Asia-Pacific Data-Research Center* (*APDRC*) [2016] and represents average annual rainfall anomalies for 2090–2109 (Figure 8b). The two sets of annual rainfall anomalies are generally different in sign and provide a range of scenarios to analyze hydrologic changes. A regional relation developed for West Maui [*Oki et al*. 2010]-related usable habitat area for selected native aquatic fauna to low flows:

$$\mathrm{HA} = \left(100^{1-0.2413}\right) Q^{0.2413} \tag{7}$$

where *H* is usable habitat area in percentage of habitat at natural $\overline{Q_{70_k}}$, and Q is discharge, as a percentage of $\overline{Q_{70_k}}$ during current natural conditions. This formula is applied to estimate potential climate-change impacts on stream habitat in Maui streams. The average annual changes in $\overline{Q_{50_k}}$, $\overline{Q_{70_k}}$, $\overline{Q_{90_k}}$, and *H* for each set of rainfall projections are summarized in Table 3. Average annual changes in $\overline{Q_{70_k}}$ resulting from projected rainfall sets A and B are shown in Figures 8c and 8d, respectively. Results of the case study on Maui

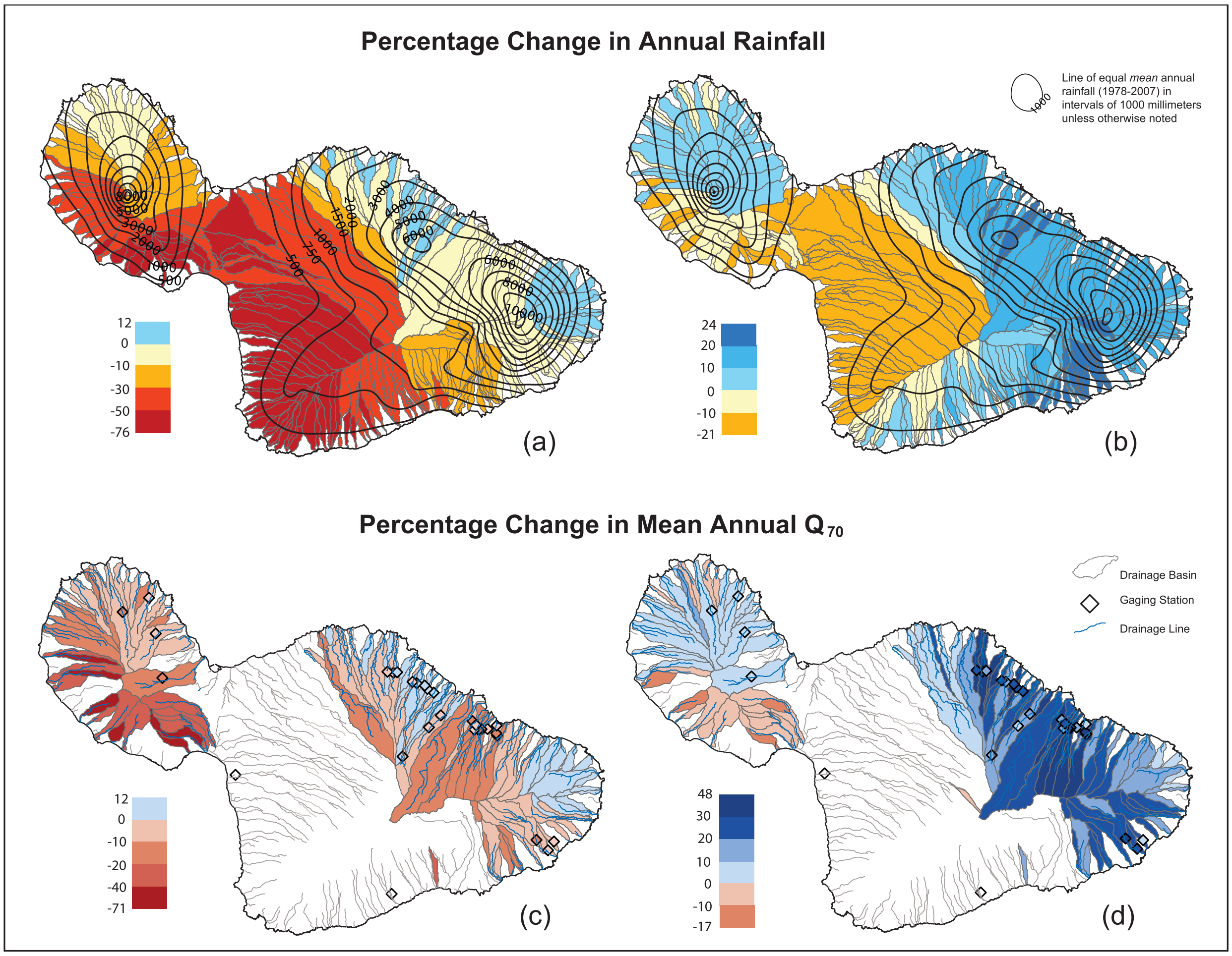

Figure 8. Case study of projected changes in rainfall on the Island of Maui, Hawaii, and associated estimates of changes in low flows. (a) Percentage change in average annual rainfall from statistical downscaling models [*Timm et al*., 2015] at ungaged basins between 1978–2007 and 2071–2099. (b) Percentage change in average annual rainfall from dynamical downscaling models [*Zhang et al*., 2012; *APDRC*, 2016] at ungaged basins between 1978–2007 and 2090–2109, Maui Hawaii. (c) Percentage changes in $\overline{Q_{70_k}}$ associated with rainfall A computed using rainfall elasticity (ε_x) values from random-effects panel regressions for ungaged basins and from streamflow records for gaging stations. (d) Percentage changes in $\overline{Q_{70_k}}$ associated with rainfall B computed using ε_x values from fixed effects panel regressions for ungaged basins and from streamflow records for gaged basins. Basins without an estimate of changes in low flow are classified as nonperennial and streams in these basins do not have natural low flows for recent conditions. Some ungaged streams may be misclassified.

indicate that some West Maui streams are likely to see declines in low flows and habitat for both projected rainfall sets.

5. Discussion

5.1. The Value of PR

MLRs developed in this study using RE-PR provided annual and average low-flow duration discharge estimates of acceptable accuracy for regional water-resources assessments for three perennial stream classes in Hawaii (Table 2 and Figure 3). Comparison of rainfall elasticity estimates and goodness-of-fit results from MLRs using different model structures demonstrated PR is able to robustly estimate rainfall elasticity because the proper assumptions on model structure are met. The BPLM test results indicated that the assumption of spatial homogeneity in MLRs is not justified in this study and demonstrated the requirement for regression methods that distinguish the spatial (between-) and temporal (within-) variability in the data, therefore P-PR is generally not appropriate. For this reason, the rainfall elasticity estimates in MLRs were generally improved using RE-PR and FE-PR and these methods provided more robust regional rainfall elasticity values than P-PR and ST-TR. In particular for the P_I stream class, for which heterogeneity between

Table 3. Median Percentage Change in Rainfall From *Timm et al.* [2015] (A) and from *Zhang et al.* [2012]; *APDRC* [2016] (B), Average Annual Low-Flow Duration Discharges ($\overline{Q_{50_k}}$, $\overline{Q_{70_k}}$, $\overline{Q_{90_k}}$) and Usable Stream Habitat Area for Native Aquatic Species (HA) in Ungaged Streams on Maui[a]

Stream Class	N	Percentage Change in Rainfall A	Percentage Change in Rainfall B	$\overline{Q_{50_k}}$ A	$\overline{Q_{50_k}}$ B	$\overline{Q_{70_k}}$ A	$\overline{Q_{70_k}}$ B	$\overline{Q_{90_k}}$ A	$\overline{Q_{90_k}}$ B	HA A	HA B
				Percentage Change Associated With Indicated Percentage Change in Rainfall							
P_l	91	−13 [−71;12]	8 [−21;23]	−11 [−57;10]	6 [−17;18]	−9 [−49;8]	5 [−14;16]	−7 [−38;7]	4 [−11;12]	−2 [−15;2]	1 [−4;4]
P_m	72	−1 [−33;9]	18 [0;23]	−2 [−62;11]	22 [−1;41]	−2 [−64;11]	22 [−1;42]	−2 [−58;11]	22 [−1;39]	0 [−22;3]	5 [0;9]
P_h	26	−6 [−36;4]	15 [−9;21]	−11 [−70;9]	29 [−17;40]	−11 [−71;9]	30 [−17;41]	−10 [−66;8]	27 [−16;37]	−3 [−26;2]	6 [−4;9]
NP	248	−48 [−76;8]	−11 [−18;24]	-	-	-	-	-	-	-	-

[a]P_l: perennial stream class with low rainfall elasticity; P_m: perennial stream class with medium rainfall elasticity; P_h: perennial stream class with high rainfall elasticity; *NP*: non-perennial stream class; n: number of ungaged stream points estimated; -: no low-flow and habitat-area estimates for non-perennial streams. Range of percentage change at ungaged streams is reported in brackets. Changes in low flows are computed using random-effects panel regressions.

streams was highest and the VIF values for rainfall were high (greater than 10), MLRs using ST-TR and P-PR may have been affected by multicollinearity and omitted-variable bias, resulting in rainfall elasticity estimates that were not consistent with the rainfall elasticity values observed at individual streams. Previous studies that have attempted to develop MLRs for estimating runoff characteristics in space and time using pooled methods (P-PR) [*Samaniego and Bárdossy*, 2005] acknowledge that strict conditions need to be met to develop models that are not affected by multicollinearity, in particular the careful selection of explanatory variables. Importantly, rainfall elasticity estimates from RE-PR and FE-PR were insensitive to the selection of static explanatory variables in the MLRs. This is in contrast with P-PR and ST-TR where rainfall elasticity estimates varied considerably depending on the selected static explanatory variables. Insensitivity of rainfall elasticity to the choice of explanatory variables is important because it demonstrates that the goodness of fit of the models is not affected by omitted explanatory variables or multicollinearity. We therefore conclude that RE-PR and FE-PR should be used to obtain reliable and consistent estimates of rainfall elasticity in MLRs and the MLRs developed for this study using RE-PR could be useful for regional climate-change assessments. Results indicated that:

1. Estimates of rainfall elasticity derived from RE-PR and FE-PR were consistent with ε_{x_m} and properly described the regional sensitivities of low flows to changes in rainfall within a stream class. Rainfall elasticity derived from ST-TR was the least accurate and was the most affected by the choice of explanatory variables.
2. The accuracy of rainfall elasticity estimated using P-PR and ST-TR compared to rainfall elasticity estimated using FE-PR and RE-PR was poor when the heterogeneity in low flows between the streams was greatest and the VIF value for rainfall was high (P_l stream class). This reflected the ability of FE-PR and RE-PR to account for both spatial and temporal variability and reduce bias resulting from omitted variables and multicollinearity.
3. Rainfall elasticity values derived from FE-PR were more consistent with ε_{x_m} than rainfall elasticity values derived from RE-PR when the heterogeneity between streams (μ_k) was skewed (P_h stream class). μ_k estimated at each site in FE-PR are not affected by omitted-variable bias and can account for the nonnormality in heterogeneity of the error term.
4. The model coefficients from RE-PR and FE-PR generally had smaller standard errors than coefficients from P-PR and ST-TR because RE-PR and FE-PR exploit the within-stream correlation and therefore the resulting rainfall elasticity values are estimated with greater accuracy using RE-PR and FE-PR. Overall rainfall elasticity values were more consistent among the model structures for stream classes with a strong relation between annual low-flow duration discharge and rainfall.

Models were compared for stream classes of different levels of heterogeneities in low flows. PR provided a formal framework to test the properties of these heterogeneities and determine whether factors controlling low flows are omitted from the models and if a predictive model using RE-PR is appropriate. Although the *p* values for the Hausman test did not strongly suggest that rainfall explanatory variables are exogenous, elasticity values from RE-PR and FE-PR were consistent, indicating that explanatory variables were appropriately selected to minimize the negative effects of multicollinearity and omitted-variable bias. For two (P_m and P_h streams) out of the three stream classes, all methods estimated coefficients of consistent values and therefore the space-for-time approach in the TR models may be valid. Conditions on RE-PR model structure assisted in selecting robust final models and the presented methods to select MLRs improves upon more complex and computationally intensive procedures to develop MLRs for space and time estimation of streamflow in ungaged basins [*Samaniego and Bárdossy*, 2005]. This study did not compare model selections using other methods and criteria. FE-PR does not require the selection of static basin characteristics, because coefficients for time-invariant explanatory variables are not estimated, and provides the simplest tool to assess effects of changes in climate on low flows within a stream class when assumptions for RE-PR cannot be met with available data.

5.2. Model Limitations for Climate-Change Applications

Models developed in this study were applied in a case study on Maui to evaluate the effects of rainfall changes on water availability for future climatic conditions. Estimated future low flows are uncertain because projected rainfall conditions are uncertain and outside the range of recorded past variability in many basins. The use of the models developed in this study therefore cannot be validated for long-term projections. Estimates of $Q_{70_{k,t}}$ using RE-PR and FE-PR were generally unbiased (Figure 3) and this is an essential condition if MLRs are to be used to estimate low flows in varying rainfall conditions. However, application of rainfall elasticity to historical time series (Figure 6) indicated that variability in rainfall alone was not always sufficient to estimate historical changes in low flows at long-term sites. Results of this study support assumptions that long-term changes in low flows may be associated with factors and coinciding trends other than rainfall, such as changes in land-surface processes and feedback with the atmosphere or changes in rainfall intensity [*Bassiouni and Oki*, 2013]. Uncertainty in rainfall [Frazier et al., 2015] during periods of limited observation may also complicate the interpretation of low flow-rainfall relations in certain basins. The regional rainfall elasticity estimates may have absorbed the generalized effect of other climatic variables not included in the model. The variability in these effects may be implicitly taken into account through the stream classification. Additional temporal climatic and land-cover data are needed to improve our understanding of past and future low-flow responses and reduce the uncertainty of the use of MLRs for long-term conditions. Others [*Fu et al*., 2007; *Andréassian et al*., 2015] have shown that multivariate estimators of local rainfall elasticity at each stream are preferred over bivariate estimators because temporal variability of low flows is not solely explained by rainfall. As time-series of other climatic variables and land-cover characteristics are developed for the Hawaiian Islands, MLRs including additional temporally variable explanatory variables can be developed and may improve our understanding of past and future low-flow responses.

Results of this study and previous investigations comparing rainfall elasticity to climatic variables such as evapotranspiration, rainfall, and humidity indexes [*Sankarasubramanian et al*., 2001; *Chiew et al*., 2006] suggest that rainfall elasticity may change in a changing climate. Also rainfall elasticity derived from annual data may not be equal to rainfall elasticity for long-term periods. *Vano and Lettenmeier* [2014] used a macro-scale hydrologic model to develop linear adjustments of annual rainfall elasticity as a function of changes in climatic variables from GCMs. These models indicated that rainfall elasticity increases with increasing changes in climate conditions and that seasonality affects rainfall elasticity. This study did not investigate the long-term temporal variability in rainfall elasticity because of limited long-term data and uncertainty in land-surface and climatic factors other than rainfall during historic conditions. Long-term data sets that incorporate a wide range of climate conditions are needed to better develop and validate long-term relations to estimate future low-flow conditions.

5.3. Patterns of Rainfall Elasticity Values in Hawaii Compared to Global Studies

This analysis pooled rainfall and streamflow data from 86 gaged sites on four islands in Hawaii. The stream classification provided insights into broad similarities and differences between low-flow sensitivity to changes in annual rainfall. Analysis of local rainfall elasticity (ε_{X_k}) at gaged streams indicated that rainfall elasticity ranged from 0.14 to 3 (Figure 2). Local rainfall elasticity values were generally inversely related to the magnitude of groundwater discharge into the stream.

Many gaged sites in Hawaii with high values of low flows had local rainfall elasticity values less than 1 and low flows at these sites do not appear to be sensitive to annual changes in rainfall. Local rainfall elasticity for higher flow-duration discharges ($Q_{50_{k,t}}$) were generally higher than for lower flow-duration discharges ($Q_{70_{k,t}}$, $Q_{90_{k,t}}$) and this is consistent with other studies that compared rainfall elasticity for fast and low-streamflow components [*Harman et al*., 2011], although for some streams in regions disconnected from groundwater and where low-low discharges are more controlled by rainfall, ε_{90_k} was greater than ε_{50_k}. Previous studies in the continental United States [*Sankarasubramanian et al*., 2001; *Sankarasubramanian and Vogel*, 2003] and elsewhere [*Chiew*, 2006; *Chiew et al*., 2006; *Tang and Lettenmaier*, 2012] reported that rainfall elasticity values relative to total streamflow values are generally greater than those observed in Hawaii for low-flow duration discharges, and some of the lowest rainfall elasticity values occurred in tropical catchments with high streamflow and annual rainfall. Variation in rainfall elasticity is largely associated with differences in aridity [*Budyko*, 1974; *Schaake*, 1990] and differences in the phasing of water and energy inputs [*Sankarasubramanian et al*., 2001], and depends on recharge and hydrogeologic factors that buffer variability in groundwater storage [*Sankarasubramanian and Vogel*, 2003]. In Hawaii, local rainfall elasticity values were not correlated with rainfall, although in Figure 1a. low rainfall elasticity values were generally associated with basins having the highest groundwater recharge [*Engott et al*., 2015; *Johnson et al*., 2014; *Engott*, 2011; *Izuka et al*., 2016] and local rainfall elasticity values greater than 1.5 were associated with the most arid basins. Low local rainfall elasticity also indicated a weaker relation between annual low-flow duration discharge and annual rainfall, which suggests that factors other than annual rainfall control and buffer the annual variability in low flows at these streams.

In Hawaii, the spatial variability in local rainfall elasticity can be qualitatively described by broad hydrologeologic regions [*Izuka et al*., 2016]. Local rainfall elasticity values were correlated with basin characteristics associated with surface-water/groundwater interactions such as wetland vegetation cover, valley incision, and stream length. The weak correlation between local rainfall elasticity and available basin characteristics limits our ability to estimate and develop reliable and detailed stream classification models to determine local rainfall elasticity for each ungaged stream. Nevertheless, 76% of the spatial variability in rainfall elasticity was described by available basin characteristics used in this study. The stream classification results for the island of Maui provide a useful tool for resource managers to identify, prior to collecting additional data or developing detailed site-specific models, broad regions of low-flow vulnerability to changes in rainfall using limited data.

This study only estimated regional values of rainfall elasticity for three stream classes. The assumption that rainfall elasticity remains constant within a stream class may be restrictive for certain applications. In this analysis, the use of stream classes reduced the spatial area or data considered for each MLR to ensure this assumption is met. More advanced approaches, such as hierarchical models, have been used for example in regional streamflow frequency analysis [*Renard*, 2011]. Hierarchical models compared to PR are beneficial because stream classification would not be necessary as a first step and rainfall elasticity can be continuously estimated at each ungaged stream. The primary limitation of applying hierarchical models to the study site is the poor availability of basin characteristics that adequately describe that spatial variability of rainfall elasticity, in particular continuous variables describing groundwater and hydrogeologic characteristics in each gaged basin. Hierarchical models have the potential to improve upon the limitation of models presented in this study and should be further explored in regional hydrologic studies.

6. Conclusions

There is an increasing need to determine the influence of changes in climate and a variety of other time-varying anthropogenic influences on the hydrologic response of watersheds. Traditional regional hydrologic regression methods for estimation of streamflow at ungaged sites have focused on the use of pooled methods of MLR. *Steinschneider et al*. [2013] recently introduced PR methods for the development of MLRs that incorporate both watershed-specific information as well as time-series information concerning streamflow and climatic characteristics, thus enabling the development of very general regional multivariate stochastic streamflow models. Such PR methods hold a great deal of promise for improving our ability to understand and estimate hydrologic characteristics under nonstationary conditions as well as for improving our ability to determine the sensitivity of watershed streamflow response to hydroclimatic conditions. Although this initial proof of concept only included one time-varying explanatory variable (rainfall), the presented framework can be expanded using multiple climatic variables that better describe the sensitivity of streamflow characteristics to climate change and to further evaluate the benefits of PR to address issues of multicollinearity and omitted-variable bias. This study compared a variety of methods of MLR to develop regional multivariate models of low-flow discharges for the islands of Hawaii that can be used to assess low-flow response to annual rainfall variability. Given the limited available data and uncertainty in climate projections, parsimonious models are appropriate to estimate low-flow duration discharges for future rainfall conditions. PR was proposed as a robust framework to select MLRs with coefficients that can be interpreted for regional-scale climate-change assessments.

There are numerous advantages of the use of PR over TR methods for developing MLRs and these advantages have been documented by our study. This study showed that if the spatial heterogeneity in low-flow discharges is high and that multicollinearity in a MLR with regards to rainfall was evident (as was the case for the P_I stream class), RE-PR provided estimates of rainfall elasticity that were more consistent with local rainfall elasticity at individual streams used to develop the MLR than traditional methods that substitute space for time (ST-TR) as well as pooled model structures (P-PR). RE-PR improved the regional estimates of rainfall elasticity, by imposing strict conditions on model residuals, which reduces the negative effects of omitted-variable bias and multicollinearity. PR was also able to distinguish between the spatial and temporal variability in the hydrologic response of watersheds and to simultaneously account directly for both. Estimated rainfall elasticity values were therefore not biased by the choice of explanatory variables. The PR framework required a number of conditions on model residuals to be validated and was shown to be very useful for selecting appropriate explanatory variables and developing MLRs in which heteroscedasticity is not present. We also document how PR can provide a means to utilize station data of different periods and lengths of record, which is advantageous for data-poor regions. PR offers beneficial properties to synthesize multidimensional data, which are particularly relevant when investigating streamflow responses in a changing climate.

Until uncertainty in downscaled climate projections is reduced, the application of the developed MLRs on the island of Maui can only provide a simple spatial understanding of the sensitivity of low flows to changes in rainfall. This understanding is critical to resource managers prioritizing the development of adaptive-management strategies. The case study presented here led to numerous detailed conclusions concerning the relation between the low-flow response of streams in Hawaii and rainfall variability. Our analysis reveals that a more spatially explicit understanding of groundwater/surface-water interactions and improved methods to classify streams in terms of their sensitivity to changes in climate are needed to refine the regions used in this study and to more objectively determine regions of similar rainfall elasticity. PR methods were shown to be useful for estimation of plausible ranges of future changes in surface-water availability for the island of Maui and the estimates are useful to identify areas of concern and help develop adaptive-management strategies for surface-water use and ecological considerations. Limited availability of long-term data that incorporates the ranges of projected changes in climate is a major challenge to developing robust empirical models to assess future hydrologic conditions.

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