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An assessment of exceedance probabilities of envelope curves

Richard M. Vogel,[1] Nicholas C. Matalas,[2] John F. England Jr.,[3] and Attilio Castellarin[4]

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[1] Envelope curves are often used to provide summary accounts of our flood experience, but their operational use has been limited by our previous inability to assign to them an exceedance probability "EP." General expressions are derived for the EP of an envelope curve at a particular site in a heterogeneous region, as well as measures of central tendency of EP across sites. Analytic results are reported for the case when floods follow a Gumbel or generalized extreme value distribution, and these results are contrasted with those of previous studies that sought to estimate the exceedance probability of extraordinary floods such as the flood of record (FOR) and the probable maximum flood (PMF). A case study involving FOR and PMF discharges for 226 rivers across the U.S.A. indicates that relatively consistent estimates of the average exceedance probability associated with both FOR and PMF envelope curves can be obtained using the theoretical approach introduced here.

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1. Introduction

[2] Estimates of extraordinary flood magnitudes and their associated probability are needed in a wide range of hydrologic engineering studies, particularly for situations in which there are either severe hazards and/or potential loss of human life such as in dam safety risk analyses. Here we define an extraordinary flood as one whose magnitude is equal to or greater than the probable maximum flood (PMF). Although such floods are likely to have catastrophic consequences and their recurrence times are likely to be measured in geologic time, we do not consider these issues here. Deterministic and stochastic approaches are available for estimation of extraordinary flood magnitudes [*National Research Council* (*NRC*), 1988]. Much greater attention has been given to the development of deterministic methods, than stochastic methods, for estimating extraordinary flood magnitudes including the PMF and the probable maximum precipitation (PMP) [*Cudworth*, 1989]. These widely used deterministic design procedures developed over the past 50 years are mature, used for the design and operation of large dams, and are considered as state of the practice. The PMF is derived from a PMP, where the PMP is defined as "theoretically the greatest depth of precipitation for a given duration that is physically possible over a given size storm area at a particular geographic location during a certain time of year." To many hydrometeorologists, implicit in this definition is the concept that the PMP describes the level of precipitation that has "virtually no risk of being exceeded" [*Myers*, 1969]. Technical assumptions in deriving PMF estimates from PMP estimates depend on a number of factors including antecedent moisture, loss rates, unit hydrograph shapes, and initial reservoir levels. These assumptions are not consistently considered among the different federal water agencies [*Newton*, 1983]. *Barker et al.* [1997] demonstrated the factors that affect PMF estimates when the PMP is held constant and documents that a unique value of PMF does not exist. The PMF is still used widely for assessing the capability of hydraulic structures to withstand extraordinary events in spite of the fact that, heretofore, there has not been an accepted measure of its exceedance probability [*Interagency Advisory Committee on Water Date* (*IACWD*), 1986; *American Society of Civil Engineers* (*ASCE*), 1988; *NRC*, 1985, 1988, 1999; *Resendiz-Carrillo and Lave*, 1987, 1990; *Shalaby*, 1994].

[3] The *IACWD* [1986] concluded that "no procedure proposed to date is capable of assigning an exceedance probability to the PMF or to near-PMF floods in a reliable, consistent, and credible manner." Their primary criticism relates to the large errors associated with the extrapolation of flood frequency curves beyond the length of record over which they are based, as well as the inherent sensitivity of extrapolations of extreme floods to the assumed form of the flood frequency distribution. Since 1986, there has been significant progress in the field of flood frequency analysis in arriving at a consensus as to the choice of an appropriate distribution. For example, there is now an increasingly global consensus that the generalized extreme value (GEV) distribution introduced by *Jenkinson* [1955] provides a meaningful distribution for floods based on both empirical [*Vogel and Wilson*, 1996; *Robson and Reed*, 1999; *Castellarin et al.*, 2001] and theoretical evidence [*Beirlant et al.*, 2005]. In addition to the GEV distribution, the log Pearson type III distribution (LP3) and three-parameter lognormal (LN3) distributions have received widespread (global) support in flood frequency analysis [see *Vogel and Wilson*, 1996, Tables 3 and 4] as well as the Gumbel distribution, which is a special case of the GEV distribution.

[1]Department of Civil and Environmental Engineering, Tufts University, Medford, Massachusetts, USA.
[2]Vienna, Virginia, USA.
[3]U.S. Department of the Interior, Bureau of Reclamation, Denver, Colorado, USA.
[4]DISTART, Università di Bologna, Bologna, Italy.

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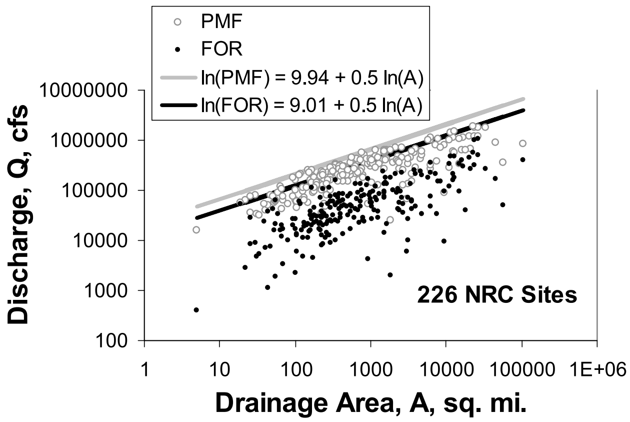

Figure 1. Envelope curves of flood of record (FOR) and the probable maximum flood (PMF) for the United States of America (1 mi^2 = 2.59 km^2; 1 cfs = 0.0283 m^3/s).

[4] Envelope curves are simple empirical relationships between the maximum peak flow experienced in a region and drainage area. The primary objective of this study is a determination of the exceedance probability associated with an envelope curve. Normally, an envelope curve is a curve drawn to envelope our flood experience based on the observed floods of record (FOR) in a region. However, one can also draw envelope curves based on estimates of PMFs in a region. An example of both types of envelopes are given in Figure 1 for 226 basins located across the U.S.A. The 226 basins upon which Figure 1 is based are a subset of 561 basins for which PMF estimates were summarized by the U.S. Nuclear Regulatory Commission [*USNRC*, 1977]. Further details regarding these basins and the methodology for estimating the envelopes are summarized below in a case study.

[5] Our goal is the development and application of a methodology for determining the exceedance probability associated with either of the two envelope curves drawn in Figure 1. We begin by reviewing previous research relating to a probabilistic assessment of envelope curves. Those discussions are followed by a theoretical treatment of the problem, which in turn is followed by a case study that provides a probabilistic assessment of both envelope curves in Figure 1.

2. Previous Studies

[6] In contrast to the deterministic PMP and PMF methods, research on probabilistic approaches to dealing with extraordinary floods is in its infancy. On the basis of recommendations by *NRC* [1985] and *ASCE* [1988], in the mid-1990s, the Bureau of Reclamation began efforts to estimate probabilities of the PMF and PMP. While the *NRC* [1985] estimated PMF peak flow probabilities in the range of 10^{-4} to 10^{-6}, subsequent efforts by the Bureau of Reclamation led to a range of 10^{-6} to 10^{-12}, depending on storm type and subjective geographic criteria (Reclamation Flood Hydrology Group, 1996, unpublished notes). *Shalaby* [1994] found a similarly wide range of probabilities corresponding to PMF estimates for the mid-Atlantic region depending on which probability distribution was extrapolated. *Shalaby* [1994] extrapolated several probability distributions fitted to observed flood series at 46 watersheds in which PMF estimates were available. *Resendiz-Carrillo and Lave* [1987, 1990] followed the same approach as the work of *Shalaby* [1994] but for fewer sites and found similarly wide discrepancies in the exceedance probabilities associated with observed PMFs depending on which probability distribution was considered. The studies by *Resendiz-Carrillo and Lave* [1987, 1990] and *Shalaby* [1994] all estimated the exceedance probability of observed PMFs by the simple extrapolation of "at-site" flood frequency curves based on relatively short records. Each of these studies was concerned with the estimation of the exceedance probability of a PMF at a particular site, unlike this study, which formulates the problem in the context of envelope curves. Most importantly, none of the previous studies employed a theoretical probabilistic interpretation of an envelope curve as is introduced in the next section and which is later documented to yield significantly different results than a simple extrapolation of the flood frequency curve. Furthermore, none of the previous studies employed the generalized extreme value (GEV) distribution, which has found widespread acceptance in flood frequency analysis over the past few decades [see *Vogel and Wilson*, 1996, Table 4] due in part to recent innovations in our ability to evaluate alternative distributional hypotheses [*Hosking and Wallis*, 1997] as well as the fact that the GEV model arises as the limiting distribution for extreme value processes such as floods.

[7] *Bullard* [1986] reported PMF estimates along with estimates of the flood of record (FOR) for 61 watersheds distributed roughly uniformly across the United States of America. Since none of the FOR values exceeded the PMF for over 18,000 station-years of data included in the analysis of *Bullard* [1986], *Lave et al.* [1990] argued that the probability of a PMF is less than 1/18,000 if one assumes that the PMFs were estimated uniformly and that flood series at the 61 watersheds are spatially independent of one another. *Baldewicz* [1984] found no evidence of exceedance of a PMF in more than 600,000 dam-years of observation, leading *Lave et al.* [1990] to conclude an annual likelihood of a PMF at a site to be 1/600,000 or less.

[8] In the report "Improving American River Flood Frequency Analyses" [*NRC*, 1999], the NRC committee said, in reference to envelope curves, "This combination of strong heterogeneity and spatial correlation makes it difficult to estimate probabilities of exceeding envelope discharges." The statement implies that an exceedance probability can be attached to an envelope curve, but that the value of the probability is difficult to determine owing to regional heterogeneity and spatial correlation. The *NRC* [1999] further states that "PMF estimates for the American River [and for any other river] provide some information about the upper tail of the flood distribution," and empirical data indicate that (presumably for the American River) the exceedance probability of the PMF "should probably be smaller than 10^{-4} and almost surely less than 10^{-3}".

[9] Our study is not the first to estimate the exceedance probability associated with an envelope curve. Envelope curves are often based upon observed floods of record (FOR) in a region, and *Douglas and Vogel* [2006] documented some of the theoretical properties of the FOR. *Castellarin et al.* [2005] proposed a probabilistic interpretation of a regional envelope curve (REC) derived from

FORs generated by the index flood assumption and a simple power law scaling relationship between drainage area and the index flood. Under those assumptions, *Castellarin et al.* [2005] showed that the REC probability is simply the exceedance probability associated with the largest standardized annual maximum peak flow in the region. Importantly, their approach accounts for the influence of the spatial correlation of flood series. *Castellarin* [2007] introduced innovations for handling unequal record lengths, an unbiased plotting position for the GEV distribution, and a suitable approach for modeling the spatial correlation of flood records. *Castellarin et al.* [2007] extended the work of *Castellarin et al.* [2005] to a multivariate framework where both the index flood and the REC depend on multivariate basin attributes. This study approaches the same problem using a more general probabilistic formulation which does not depend upon the index flood assumption.

[10] Until this study, and the other recent studies cited above, the consensus view was that it is simply not possible to assign an exceedance probability to an envelope curve. For example, *Linsley et al.* [1949, p. 572], in their book "Applied Hydrology," suggested that "the appropriate frequency for values determined from the enveloping curve and the related formulas cannot be defined." More recently, in reference to the method of envelope curves, the "Handbook of Hydrology" [*Pilgrim and Cordery*, 1993, chapter 9 p. 37] suggested that "Probabilities of floods cannot be estimated objectively by this method."

3. Distribution of the Flood of Record

[11] Assume that a region has m sites, at each of which observations of annual floods are concurrent, spanning a period of n years $\{x_{i,t}: i = 1,\ldots, m; t = 1,\ldots, n\}$. At each site, the observations are independent and identically distributed (iid) as follows:

$$F[X_{i,t}] = F[X_i] = \Pr[X_i < x] \forall i, t \tag{1}$$

In general, the floods at one site are dependent upon those at the other sites, and therefore

$$F[X_1, X_2, \ldots, X_m] \neq F[X_1]F[X_2]\cdots F[X_m] \tag{2}$$

[12] The structure of dependence may be represented by the correlation matrix $\boldsymbol{R}$ as follows:

$$\boldsymbol{R} = \begin{bmatrix} 1 & r_{1,2} & r_{1,3} & r_{1,4} & r_{1,m} \\ r_{2,1} & 1 & r_{2,3} & r_{2,4} & r_{2,m} \\ r_{3,1} & r_{3,2} & 1 & r_{3,4} & r_{3,m} \\ r_{4,1} & r_{4,2} & r_{4,3} & 1 & r_{4,m} \\ r_{m,1} & r_{m,2} & r_{m,3} & r_{m,4} & 1 \end{bmatrix} \tag{3}$$

As is well known, $\boldsymbol{R}$ is a symmetric matrix: $r_{j,k} = r_{k,j}\ \forall_{j,k}$. For each sequence, the flows may be ordered from smallest to largest: $\{x_i^{(u)}: i = 1,\ldots, m; u = 1,\ldots,n\}$ where u denotes rank: $u = 1$, smallest, to $u = n$, largest; the lowest value is the lower record value, and the largest value is the upper record value. The flow $x_i^{(n)}$ is the flood of record (FOR) at site i. Given that, at each site, the observations are iid, the distribution of the FOR in a sequence of length n is

$$G_{(n)}[Y_i] = F^n[X_i] = \Pr[Y_i < y_i] \tag{4}$$

where $x_i^{(n)} = y_i \leq \infty$. The correlation matrix for the largest observations is

$$\boldsymbol{W} = \begin{bmatrix} 1 & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,m} \\ w_{2,1} & 1 & w_{2,3} & w_{2,4} & w_{2,m} \\ w_{3,1} & w_{3,2} & 1 & w_{3,4} & w_{3,m} \\ w_{4,1} & w_{4,2} & w_{4,3} & 1 & w_{4,m} \\ w_{m,1} & w_{m,2} & w_{m,3} & w_{m,4} & 1 \end{bmatrix} \tag{5}$$

where

$$\lim_{n\to\infty} \boldsymbol{W} \to \boldsymbol{I} \tag{6}$$

where $\boldsymbol{I}$ denotes the identity matrix; all off-diagonal terms are equal to zero. The correlation between record values is asymptotically zero [*Sibuya*, 1960; *Husler and Reiss*, 1989].

[13] For nonnormal observations, the elements of $\boldsymbol{R}$ and $\boldsymbol{W}$ may not provide a meaningful measure of the linear association between the sequences and the record values, respectively, particularly if association is measured by product moment correlations. The effect of nonnormality on the elements of $\boldsymbol{R}$ and $\boldsymbol{W}$ is not assessed herein.

4. Envelope Curve Definitions and Probabilistic Framework

4.1. Envelope Curves as a Summary of Flood Experience

[14] Envelope curves are simple empirical relationships between the maximum peak flow experienced in a region and drainage area. The variables x_i, y_i, and z_i represent three different random variables equal to the annual maximum flood at site i, the flood of record at site i, and the ordinate of the point on the envelope curve corresponding to site i, respectively. Let A_i denote the drainage area at site i. The scatter diagram of $\ln(y_i)$ versus $\ln(A_i)$, where $y_i \equiv x_i^{(n)}$, is an expression of our flood experience over the period $t = 1$ to $t = n$. The experience may be bound by an enveloping line; that is, a line below which all our experience, expressed in terms of the FORs and their relation to drainage area, lies (see the black line in Figure 1) To give meaning to the enveloping line, the line is set with a slope b and passed through that point, such that all other points lie below the line, hence the name, envelope line. The envelope line is defined as

$$\ln(z_i) = a + b\ln(A_i) \tag{7}$$

The slope b may be specified in a number of ways. For example, the value of b may be taken to be equal to the slope of the line of regression of $\ln(y)$ on $\ln(A)$. Another specification might be $b = (b^* + 1)$, where b^* is the slope of the enveloping line of the scatter diagram formed by $\ln(y_i/A_i)$ versus $\ln(A_i)$. The Myer slope is $b^* = -1/2$ [*Jarvis*, 1926], whereby for the scatter diagram $\ln(y_i)$ versus $\ln(A_i)$, the corresponding slope is $b = 1/2$.

[15] One may also estimate the values of a and b by solving the following linear optimization problem:

$$Min \sum_{i=1}^{m} [\ln(y_i) - a - b\ln(A_i)] \tag{8a}$$

subject to

$$\ln(y_i) - a - b\ln(A_i) \leq 0 \quad \forall i \tag{8b}$$

The optimization problem given in equations (8a) and (8b) is easily solved for the model parameters a and b using a linear programming algorithm such as the simplex algorithm which is widely available. For the remainder of this study, we assume $b = 0.5$.

[16] Assume that $\boldsymbol{W} = \boldsymbol{I}$. Consider a region of m sites. The size of the region is denoted as A_e:

$$A_e \leq \sum_{i=1}^{m} A_i \tag{9}$$

[17] Because one site may be upstream (downstream) of another site, A_e is equal to or less than the sum of the drainage areas of the m sites; A_e is the size of the effective area of the region. Given two regions, the assumption $\boldsymbol{W} = \boldsymbol{I}$ is operationally more viable for the larger of the two regions. In the larger region, the average distance between sites is greater; correlation between flood sequences at two sites tends to diminish with increasing distance between the sites. In a region of pronounced orographic features, the off-diagonal elements of $\boldsymbol{R}$ are, in absolute value, smaller than they otherwise would be in the absence of the orographic features. In turn, the off-diagonal elements of $\boldsymbol{W}$ are, in absolute value, smaller than they otherwise would be.

4.2. Estimating Envelope Curve Probabilities

[18] Here we derive the probability of exceeding the envelope curve at a particular site i at time $t = n + 1$, where perhaps, at the site, a water project is envisaged. Consider a hypothetical region consisting of m sites where the sequence length n at each site is sufficiently long [to the limit $n \rightarrow \infty$, see equation (6)] such that the matrix of the correlations between record values may be represented by an identity matrix, $\boldsymbol{W} \approx \boldsymbol{I}$ [*Sibuya*, 1960; *Husler and Reiss*, 1989]. At time $t = n + 1$, the flood at site i will be a flood of record at that site, R_i with probability as follows:

$$P(R_i) = (n+1)^{-1};\ \forall i \tag{10}$$

Given that the flood of record event R_i occurs at site i, the record flood will exceed the envelope value at that site, E_i with probability given by

$$P(E_i|R_i) = \int_{z_i}^{\infty} \mathrm{d}G_{(n)}(Y_i) \tag{11}$$

where $\boldsymbol{z_i}$ denotes the ordinate of the point on the envelope line given in equation (7) corresponding to the abscissa $\ln(A_i)$.

[19] Of interest here is the occurrence of both events E_i and R_i at time $t = n + 1$, that is, having observed at time $t = n + 1$ a record flood at site i, that also exceeded the envelope. This particular event, which we term E_iR_i, will occur with exceedance probability given by

$$\begin{aligned} P(E_iR_i) &= P(R_i)P(E_i|R_i) \\ &= (n+1)^{-1}\int_{z_i}^{\infty} \mathrm{d}G_{(n)}(Y_i) \end{aligned} \tag{12}$$

Note that all sites are not equal in our analysis, because site $i = i'$ is the site that identifies the current ($t = n$) envelope and for which the record flood, $z_{i'} = x_{i'}^{(n)}$, falls on the envelope curve. All future record floods at that site will exceed the envelope line, so that

$$P(E_{i'}|R_{i'}) = \int_{z_i = x_{i'}^{(n)}}^{\infty} \mathrm{d}G_{(n)}(y_{i'}) = 1 \tag{13}$$

whereby

$$\begin{aligned} P(E_{i'}R_{i'}) &= P(R_{i'})P(E_{i'}|R_{i'}) \\ &= P(R_{i'}) \\ &= (n+1)^{-1} \end{aligned} \tag{14}$$

[20] If a water project is contemplated at site i, then of particular interest at that site is the local exceedance probability in year $(n + 1)$ of the envelope line set in year n, i.e., the probability that the flow in year $(n + 1)$ at site i will exceed the envelope line set in year n:

$$\begin{aligned} \Phi_i(z_i) &= \begin{cases} P(E_{i'}R_{i'}); & \text{if } i = i' \\ P(E_iR_i); & \text{if } i \neq i' \end{cases} \\ &= \begin{cases} (1+n)^{-1}; & \text{if } i = i' \\ (1+n)^{-1}\displaystyle\int_{z_i}^{\infty} \mathrm{d}G_{(n)}(Y_i); & \text{if } i \neq i'. \end{cases} \end{aligned} \tag{15}$$

[21] Equation (15) yields an exceedance probability corresponding to the ordinate of the point on the envelope line z_i corresponding to the abscissa, $\ln(A_i)$, based on m samples, each of length n, hence the probability $\Phi_i(z_i)$ is a random variable with a distribution and moments which depend upon the distributional properties of both the ordinate of the envelope line z_i as well as the flood series at site i. We consider two summary measures of $\Phi_i(z_i)$, (1) its expectation $E[\Phi_i(z_i)]$ which we term the expected exceedance probability of an envelope (EEPE) and (2) $\Phi_i(E[z_i])$ which we term the exceedance probability of the expected envelope (EPEE). The EEPE is defined by

$$E[\Phi_i(z_i)] = \begin{cases} (1+n)^{-1}; & \text{if } i = i' \\ \displaystyle\int_0^{\infty}\left[(1+n)^{-1}\int_{z_i}^{\infty} G_{(n)}(Y_i)dz\right]g_{(mn)}(z_i)dz; & \text{if } i \neq i' \end{cases} \tag{16}$$

where $g_{(mn)}(z_i) = \frac{dG_{(mn)}(z_i)}{\mathrm{d}z}$ and $\Phi_i(z_i)$ is given in equation (15). Here $g_{(mn)}(z_i)$ represents the pdf associated with the value of the envelope curve at a particular site i. Since the envelope is defined by flood series at m-independent sites, each of length n, the record length, associated with the pdf of z, $g_{(mn)}(z_i)$, is equal to mn. Similarly, the EPEE is defined by

$$\Phi_i(E[z_i]) = \begin{cases} (1+n)^{-1}; & \text{if } i = i' \\ (1+n)^{-1}\displaystyle\int_{\mu_z}^{\infty} dG_{(n)}(Y_i); & \text{if } i \neq i' \end{cases} \tag{17}$$

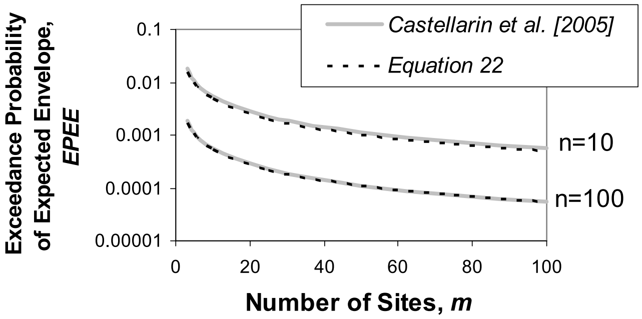

Figure 2. Comparison of EPEE estimated from this study [equation (22)] and the work of *Castellarin et al.* [2005] for Gumbel case.

where μ_z denotes the expectation of z. Since the case for $i = i'$ is somewhat trivial, we ignore that case in our remaining analysis.

[22] The summary measures EPEE and EEPE represent two different probabilistic statements regarding an envelope curve. Previous efforts to identify the exceedance probability of an envelope by *Castellarin et al.* [2005] have focused on EPEE, whereas we are unaware of anyone who has computed EEPE. If ones concern is with making a probabilistic statement regarding the single envelope based on historical observations, then EEPE is an appropriate summary measure, whereas if ones concern is with making a probabilistic statement regarding the expected envelope, then EPEE is an appropriate summary measure.

5. Envelope Exceedance Probabilities: Gumbel Case

[23] Consider the case in which flood series x arise from a Gumbel distribution with cumulative distribution function

$$F_X(x) = \exp\left\{-\exp\left(-\frac{(x-\xi)}{\alpha}\right)\right\} \tag{18}$$

defined for $-\infty \le x \le \infty$ with mean and variance of x given by $\mu_x = \xi + \gamma\alpha$ and $\sigma_x^2 = (\pi\alpha)^2/6$, respectively, where $\gamma = 0.5772\ldots$ is the Euler constant. *Gumbel* [1958, p. 201] showed that the expected record flood μ_y from a Gumbel series of length n is given by

$$\mu_y = \xi + \alpha(\gamma + \ln(n)) \tag{19}$$

Since the envelope curve is based on mn iid Gumbel observations, the expectation of the envelope curve is given by

$$\mu_z = \xi + \alpha(\gamma + \ln(mn)) \tag{20}$$

[24] Substitution of equation (18) into equation (4) yields the cdf of the record flood at site i denoted as $G_{(n)}(Y_i)$ so that the exceedance probability of the envelope in equation (15) becomes

$$\Phi_i(z_i) = \frac{1 - G_{(n)}(z_i)}{n+1} = \frac{1 - \exp\left(-n\exp\left(-\frac{z_i-\xi}{\alpha}\right)\right)}{n+1}; \quad \text{if} \quad i \ne i' \tag{21}$$

The EPEE is obtained by substitution of $z_i = \mu_z$ from equation (20) to equation (21), which after subsequent algebra, leads to

$$\Phi_i(\mu_z) = \text{EPEE} = \frac{1 - \exp\left(-\frac{\exp(-\gamma)}{m}\right)}{n+1}; \quad \text{if} \quad i \ne i' \tag{22}$$

where $\Phi_i\,(\mu_z)$ denotes the exceedance probability associated with the expected envelope curve μ_z, at site i, when flows are iid Gumbel. *Castellarin et al.* [2005, equations (19) and (20)] used Monte Carlo simulation to show that the exceedance probability associated with the expected envelope (EPEE) for independent Gumbel floods can be expressed through the plotting position of *Gringorten* [1963]. Their result

$$\Phi_i^{\text{Castellarin}}(\mu_z) = 1 - \frac{mn-\lambda}{mn+1-2\lambda}, \text{ where } \lambda = 0.44, \text{ and therefore} \tag{23a}$$

$$\Phi_i^{\text{Castellarin}}(\mu_z) = \frac{0.56}{mn + 0.12} \tag{23b}$$

is nearly identical to equation (22) as shown in Figure 2.

[25] The expected exceedance probability of the envelope EEPE for the Gumbel case is obtained by the substitution of $\Phi_i(z_i)$ given by equation (21) and $g_{(mn)}\,(z_i) = \frac{\mathrm{d}G_{(mn)}(z_i)}{\mathrm{d}z}$ into equation (16) which leads to

$$\begin{aligned} E[\Phi_i(z_i)] &= \text{EEPE} \\ &= \int_0^\infty \frac{1 - \exp\left(-n\cdot\exp\left(-\frac{z-\xi}{\alpha}\right)\right)}{n+1}\,\frac{dG_{(mn)}(z_i)}{dz}dz \ \text{ if } i \ne i' \\ &= \frac{1}{n+1}\left[1 - \frac{m}{m+1}\left(1 - \exp\left(-n(m+1)\right.\right.\right. \\ &\qquad \left.\left.\left.\cdot\exp\left(\frac{\pi}{C_v\sqrt{6}} - \gamma\right)\right)\right)\right] \\ &\approx \frac{1}{(n+1)(m+1)} \end{aligned} \tag{24}$$

where C_v is the coefficient of variation of the annual maximum flood flows x. Figure 3 illustrates the ratio of the exact and approximate expressions for EEPE given in equation (24) as a function of the product mn and C_v. Figure 3 illustrates that the approximation is generally excellent whenever the product $mn > 3$, regardless of the value of C_v.

[26] Figure 4 provides a comparison of the values of EPEE and EEPE for the Gumbel case and illustrates that the values of EEPE are always greater than EPEE.

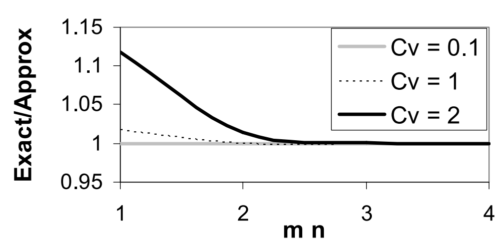

Figure 3. Comparison of the exact and approximate expressions for EEPE given in equation (24) as a function of mn, the product of the number of sites m and record length n, in years.

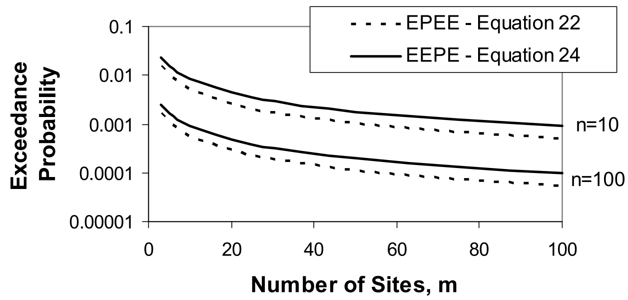

Figure 4. Comparison of EPEE and EEPE for the Gumbel case.

[27] Figure 5 illustrates that, over the range of values of m considered, the value of the ratio EEPE/EPEE increases as m increases in a manner strongly indicating that the ratio converges to a value equal to approximately 1.781.

5.1. Monte Carlo Experiments: Gumbel Case

[28] We conduct experiments to confirm that the analytic expressions for EPEE and EEPE are correct for the iid Gumbel case. Since *Castellarin et al.* [2005] used Monte Carlo experiments to arrive at the estimate of EPEE in equations (23a) and (23b) and since equations (22), (23a), and (23b) are in such good agreement, it is unnecessary to provide further confirmation of equation (22). Instead, we evaluate the expression for EEPE in equation (24) using a Monte Carlo experiment in which we generate m series of floods, each of length n, from a region defined by the index flood assumption

$$\mu_x^{(i)} = CA_i^{b+1} \tag{25}$$

where $\mu_x^{(i)}$ is the mean annual flow at site i, C is a constant, and $b = 1/2$. For the index flood assumption, the coefficient of variation is assumed constant in the region, and drainage areas are assumed to be uniformly distributed over the range (1,1000). The Monte Carlo experiment can be described as follows:

[29] 1. Generate m series of Gumbel floods each of length n, $x_{i,t}$, $i = 1,\ldots, m$; $t = 1,\ldots, n$.

[30] 2. Estimate the record flood at each site $y_i = \operatorname*{Max}_t\left(x_{i,t}\right)$ and estimate the envelope intercept a using $a = \operatorname*{Max}_i(\ln(y_i/A_i) - b\ln(A_i))$.

[31] 3. Generate an additional $M_1 = 10{,}000$ flows at each of m sites and count the total number of flows which exceed the envelope curve at each site denoted Count_i and compute the average exceedance probability $\Phi_i^{\mathrm{Count}} = \sum_{i=1}^{M_1} \mathrm{Count}_i / M_1 m$. An average estimate of Φ_i^{Count} is obtained by repeating the above experiment $M_2 = 1{,}000$ times and reporting its average value $\bar{\Phi}_i^{\mathrm{Count}} = \sum_{i=1}^{M_2} \Phi_i^{\mathrm{Count}}/M_2$. The estimate $\bar{\Phi}_i^{\mathrm{Count}}$ is termed a Monte Carlo counting estimate and provides an estimate of the average exceedance probability associated with many envelope curves (EEPE).

[32] 4. For each of the $M_1 = 10{,}000$ flow sequences generated in step 3, also estimate the probability of exceeding the envelope at each site using $\Phi_i^{\mathrm{extrap}} = F_x(z_i)$, where $F_x(z_i)$ is given in equation (18). Then report the average exceedance probability as $\bar{\Phi}_i^{\mathrm{extrap}} = \sum_{i=1}^{M_1} \Phi_i/M_1$ which is termed a Monte Carlo extrapolation estimate, because it involves extrapolation of the flood frequency curve.

[33] The results of the above Monte Carlo experiments are reported in Figure 6. What we observe is excellent agreement between the analytic exceedance probability EEPE given in equation (24) and both of the Monte Carlo counting and extrapolation estimates $\bar{\Phi}_i^{\mathrm{extrap}}$ and $\bar{\Phi}_i^{\mathrm{Count}}$.

5.2. Discussion: Gumbel Case

[34] The theoretical expression introduced in this study for the exceedance probability of an envelope curve $\Phi_i(z_i)$ given by equation (15) differs considerably from the approach taken in previous studies by *Resendiz-Carrillo and Lave* [1987, 1990] and *Shalaby* [1994]. Previous studies simply extrapolated the fitted frequency curve so that their estimate of the exceedance probability of the envelope was obtained as the at-site exceedance probability given by $\bar{F}_i(z_i) = 1 - \mathrm{F_i}(z_i)$, which for the Gumbel case reduces to

$$\bar{F}_{\mathrm{i}}(\mathrm{z}_i) = 1 - \exp\left(-\exp\left(-\frac{z_i - \xi}{\alpha}\right)\right) = 1 - \exp(-\exp(-\mathrm{w_i})) \tag{26}$$

where $w_i = (z_i - \xi)/\alpha$ is termed a reduced variate. The correct exceedance probability $\Phi_i(z_i)$, is given in equation (21) for the Gumbel case and is a function of n, the record length in years, unlike equation (26). We compare estimates of the two exceedance probabilities given by equations (21) and (26) in Figure 7. These results clearly show that, for the Gumbel case, the exceedance probabilities were always overestimated in those previous studies, though

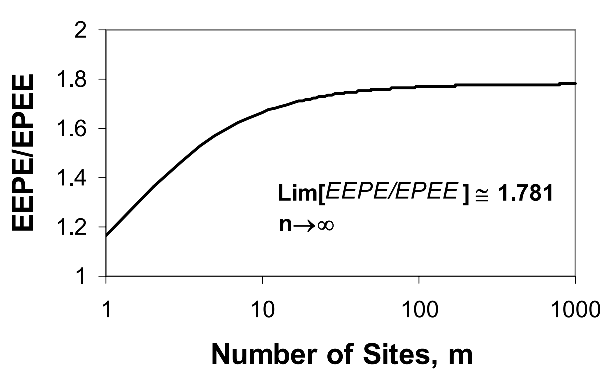

Figure 5. The ratio EEPE/EPEE as a function of m for the Gumbel case.

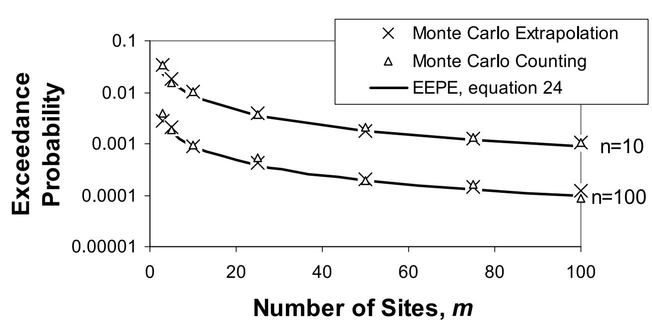

Figure 6. Comparison of Monte Carlo results with EEPE given by equation (24).

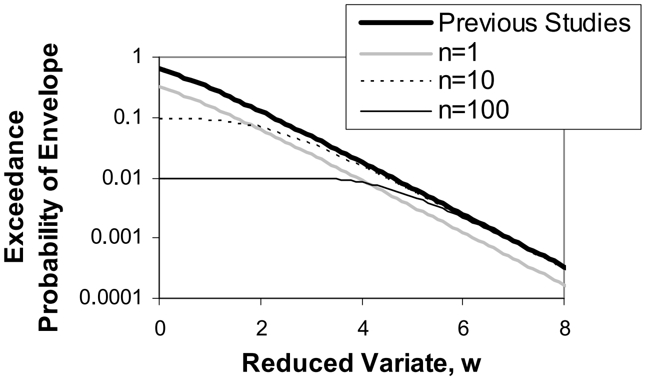

Figure 7. Comparison of exceedance probability of an envelope given in equation (26) and used in previous studies with the results derived here in equation (21) for the Gumbel case and for different record lengths n in years.

for very large values of the reduced variate w, equations (21) and (26) appear to converge for $n > 1$.

6. Envelope Probabilities: GEV Case

[35] Consider the case in which flood series x arise from a GEV distribution [*Jenkinson*, 1955] whose cumulative form is

$$F_X(x) = \exp\left[-\left(1 - \kappa\frac{(x-\xi)}{\alpha}\right)^{1/\kappa}\right] \tag{27}$$

where $\xi + (\alpha/\lambda) \leq x \leq \infty$ when $\kappa < 0$ and $-\infty \leq x \leq \xi + (\alpha/\kappa)$ for $\kappa > 0$, with mean and variance of x given by $\mu_x = \xi + \alpha[1 - \Gamma\,(1 + \kappa)]/\kappa$ and $\sigma_x^2 = \alpha^2\,\{\Gamma(1 + (2/\kappa)) - [\Gamma(1 + (1/\kappa))]^2\}$, respectively. Note that, when $\kappa = 0$, the GEV distribution reduces to the Gumbel distribution. *Stedinger et al.* [1993] and *Hosking and Wallis* [1997] provided further information on the GEV distribution. *Douglas and Vogel* [2006] showed that the expected record flood μ_y, from a GEV series of length n, is given by

$$\mu_y = \xi + \frac{\alpha}{\kappa}\left[1 - \frac{\Gamma(1+\kappa)}{n^\kappa}\right] \tag{28}$$

Since the envelope curve is based on mn iid GEV observations, the expectation of the envelope curve is given by

$$\mu_z = \xi + \frac{\alpha}{\kappa}\left[1 - \frac{\Gamma(1+\kappa)}{(mn)^\kappa}\right] \tag{29}$$

[36] Substitution of equation (27) into equation (4) yields the cdf of the record flood at site i denoted $G_{(n)}(Y_i)$ so that the exceedance probability of the envelope in equation (15) becomes

$$\Phi_i(z_i) = \frac{1 - G_{(n)}(z_i)}{n+1};\ \text{if } i \neq i' \tag{30}$$

The EPEE is obtained by substitution of $z_i = \mu_z$ from equation (29) into equation (30), which, after subsequent algebra, leads to

$$\Phi_i(\mu_z) = \text{EPEE} = \frac{1 - \exp\left[-\frac{(\Gamma(1+\kappa))^{1/\kappa}}{m}\right]}{n+1};\ \text{if } i \neq i' \tag{31}$$

We were unable to obtain a closed form solution for EEPE for the GEV case.

[37] As expected, EPEE for the GEV case in equation (31) reduces to EPEE for the Gumbel case [equation (22)] as κ approaches zero. *Castellarin* [2007, equations (5) and (9)] reports the exceedance probability associated with the expected envelope for independent GEV floods as

$$\Phi_i^C(\mu_z) = 1 - \frac{mn - \lambda}{mn + 1 - 2\lambda} \tag{32a}$$

$$\text{where } \lambda = \frac{\exp(\gamma) - 1}{\exp(\gamma)} - \frac{\pi^2\kappa}{12\exp(\gamma)} = 0.439 - 0.462\kappa \tag{32b}$$

which holds for κ values between -0.5 and 0.5 and returns the plotting position of *Gringorten* [1963] when $\kappa = 0$ (Gumbel distribution). EPEE in equations (32a) and (32b) turns out to be nearly the same as the EPEE in equation (31) as shown in Figure 8. In general, the agreement between equations (31), (32a), and (32b) increases as both m and n increase. This result was expected because equations (32a) and (32b) is an asymptotic formula; *Castellarin* [2007] recommended its use for samples larger than 10 sample-years of data (i.e., $mn > 10$).

7. Exceedance Probability of PMF Envelope Line

[38] A PMF envelope line may be set above and parallel to the FOR envelope line. In reference to the PMF envelope line, the exceedance probability for the line can be obtained in the manner outlined above for determining the exceedance probability of the FOR envelope line. Similarly, the exceedance probability can be obtained for any other line placed parallel to and even higher than the FOR envelope line. For a region where at each site values of the FORs and the PMFs are given, the exceedance probability of a line bounding the PMFs in relation to drainage area can be determined on the basis of the exceedance probability of the envelope line for the FORs. Assume that

[39] 1. The slope of the envelope line for the PMFs is the same as the slope of the envelope line for the FORs;

[40] 2. The PMFs follow the same distribution as the FORs; and

[41] 3. The PMF envelope curve overrides (is above) the FOR envelope curve.

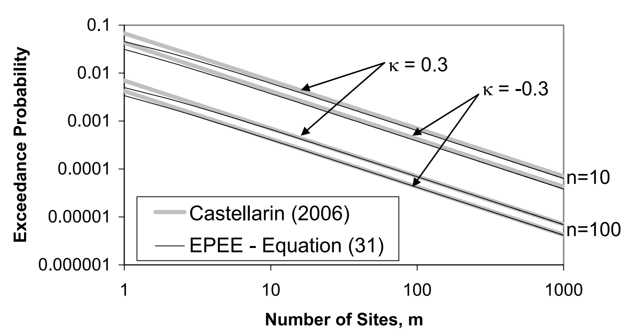

Figure 8. Comparison of EPEE from equation (31) and from the work of *Castellarin* [2007] for the GEV case for different shape parameters κ and record lengths n in years.

Figure 9. Location of 226 U.S. Geological Survey gauging stations used to construct envelope curves.

[42] These three assumptions need to be examined. Future research should determine whether empirical evidence provides reasonable support for assumptions (1) and (3). A more difficult question is whether or not assumption (2) is reasonable and supportable. The distributions of the floods themselves, $F(X)$, and of the FORs, $G(Y)$, are interrelated [see equation (4)]. It remains to be seen to what extent, if any, that the distribution of the PMFs is interrelated to $F(X)$ and $G(Y)$.

8. Case Study

[43] In this section we apply the theoretical approach introduced here for estimation of the average exceedance probability associated with the flood envelopes illustrated earlier in Figure 1. Previously, we showed that, for a heterogeneous region (defined as a region in which the matrix of the correlations between record values is approximated by the identity matrix $\boldsymbol{W} = \boldsymbol{I}$), the exceedance probability of the envelope at a particular site i, $\Phi_i(z_i)$, may be obtained from equation (15). The assumption $\boldsymbol{W} = \boldsymbol{I}$ is likely to be plausible if the sequence length n at each site is sufficiently long [for example, see equation (6)] and/or the region is extremely large so that orographic influences are pronounced and climatic influences are heterogeneous, leading to uncorrelated flood series. In addition, for each site i, a flood series of adequate length n_i is needed as well as an estimate of the PMF. The database described below attempts to satisfy, at least approximately, all of these conditions. Future work may benefit by separating flood series into snowmelt versus rainfall driven floods and frontal versus convective storms, because correlations among flood series are likely to be influenced by the physical mechanisms which give rise to those floods. On the one hand, one would like the envelope curves to be based on as many site-years of flood data as possible such as the recent data set derived from 14,815 U.S. Geological Survey gauging stations across the U.S.A. [*O'Connor and Costa*, 2004]. On the other hand, to assure that the assumption $\boldsymbol{W} = \boldsymbol{I}$ is operationally viable, the distance between sites should be large, leading to a smaller national data set, certainly much smaller than the one developed by *O'Connor and Costa* [2004]. To our knowledge, the largest national data set of PMFs was compiled by the U.S. Nuclear Regulatory Commission [*USNRC*, 1977] for 561 sites. We only considered basins listed in that report for which it was possible to identify a U.S. Geological Survey streamgauge at or near the project for which a PMF was computed and the U.S. Geological Survey gauging station had a record length of at least 50 years. The resulting 226 sites (see Figure 9 for location map) had flood sequences ranging in length from n = 50 to 132 years, with a mean sequence length of 72.2 years.

[44] A primary assumption underlying this case study is that the time series of floods upon which the envelopes are based are uncorrelated in space, so that $\boldsymbol{W} = \boldsymbol{I}$, at least approximately for these 226 sites, where $\boldsymbol{W}$ is given in equation (5). That assumption is tested in Figure 10a, which plots the cross correlation $r_{i,j}$ between each pair of sequences relative to the 226 sites and the distance between the sites i and j and in Figure 10b, which plots the empirical cumulative distribution of the values of $r_{i,j}$. Figure 10b shows that 90% of the intersite correlations are less than 0.37 and 95% of the intersite correlations are less than 0.49. We conclude from Figure 10a that, for some of the sites which are only a few hundred kilometers apart, the intersite

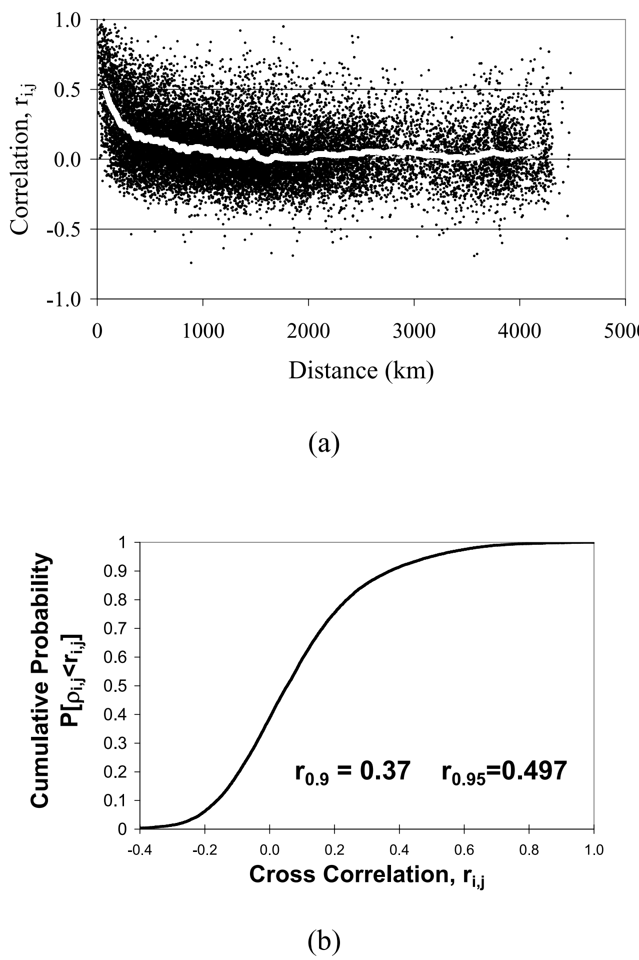

Figure 10. (a) Plot of cross correlation between site i and site j, $r_{i,j}$ versus the distance between sites i and j and (b) the cumulative probability distribution of estimates of cross correlation $r_{i,j}$.

Table 1. Comparison of the Mean Values of FOR and PMF Envelope Exceedance Probabilities $\bar{\Phi}$ Corresponding to the 226 Sites for Each of Four Distributions

Distribution	FOR $\bar{\Phi}$	PMF $\bar{\Phi}$
GEV	2.03×10^{-4}	7.13×10^{-5}
LN3	14.8×10^{-4}	74.9×10^{-5}
LP3	4.97×10^{-4}	10.9×10^{-5}
GL	2.59×10^{-4}	9.10×10^{-5}
Average	6.08×10^{-4}	25.5×10^{-5}

correlations can be significant (i.e., above 0.5). However, over 95% of the intersite correlations are less than 0.49, so that the assumption $\boldsymbol{W} = \boldsymbol{I}$, for this data set is at least approximated (keeping in mind that the elements of $\boldsymbol{W}$ will always be less than the elements of $\boldsymbol{R}$). Estimates of $\Phi_i(z_i)$ in equation (15) and its average value across the 226 sites are obtained for both the FOR and PMF flood envelope curves depicted in Figure 1. We consider the following most common three-parameter distributions now in use for flood frequency analysis: generalized extreme value (GEV), lognormal (LN3), log Pearson type III (LP3), and the generalized logistic (GL) distributions. L-moment ratio diagrams [see *Hosking and Wallis*, 1997], constructed for the 226 sites considered here, confirmed that the GEV, LN3, and GL pdfs all provide plausible flood frequency models. These results are consistent with the results of *Vogel and Wilson* [1996] who examined L-moments diagram for flood flows at 1455 sites in the U.S.A. Although the three-parameter distributions considered here provide a good fit to complete series of annual maximum floods, it remains an open question whether they provide a good probabilistic description of the extreme tail behavior of flood series as is the focus here. At-site estimates of $\Phi_i(z_i)$ are obtained at all 226 sites using L-moment estimates of distributional parameters, and the expectation of $\Phi_i(z_i)$ across sites (EEPE) is obtained using the record-length weighted estimator as follows:

$$\bar{\Phi} = \frac{\sum_{i=1}^{m} n_i \Phi_i(z_i)}{\sum_{i=1}^{m} n_i} \tag{33}$$

Table 1 summarizes estimates of EEPE given by $\bar{\Phi}$ in equation (33) for the FOR and PMF national envelope curves corresponding to each pdf. Figure 11 summarizes the significant variability in individual at-site estimates of $\log_{10}[\Phi_i(z_i)]$ for both the FOR and PMF envelopes using boxplots. The record-length-weighted average values of $\bar{\Phi}$ reported in Table 1 and depicted in Figure 11 (using circles with cross hairs) are remarkably similar for the four different pdfs considered here. This result is quite different from the results of *Shalaby* [1994], who found differences of several orders of magnitude across distributions, in average exceedance probabilities associated with PMF observations for 46 watersheds in the mid-Atlantic region of the U.S.A. The relatively consistent values of $\bar{\Phi}$ corresponding to the PMF envelope, across distributions, also differs considerably from the previous conclusions drawn by *Resendiz-Carrillo and Lave* [1987, 1990]. We did not consider the Gumbel model for the estimation of EEPE $= \bar{\Phi}$ in Table 1 and Figure 11 because previous research [*Vogel and Wilson*, 1996] and L-moment diagrams constructed for this 226-site data set both indicate that a Gumbel model is not a plausible model for the entire U.S.A. If floods were Gumbel then equation (24) implies that the FOR envelope would have an average exceedance probability of EEPE $\approx 1/\sum_{i=1}^{226} n_i = 6.15 \times 10^{-5}$ which is an order of magnitude smaller than the values of EEPE reported in Table 1 for more plausible distributions. The Gumbel distribution has a fixed value of skewness, whereas flood sequences yield a wide range of positively valued skews.

9. Conclusions

[45] Envelope curves representing the current bound on flood experience have limited use because of our inability to assign to them an exceedance probability. *Castellarin et al.* [2005] and *Castellarin* [2007] offered an approach to the estimation of the exceedance probability of the expected regional envelope curve. This study introduced a general expression for the exceedance probability of an envelope curve $\Phi_i(z_i)$ in equation (15) along with the following two summary statistics: the expected exceedance probability of the envelope (EEPE) in equation (16) and the exceedance probability of the expected envelope (EPEE) in equation (17). We document that, for the case of spatially independent Gumbel and GEV flood series, the expression for EPEE in equation (17) yields results nearly equal to the expressions given by *Castellarin et al.* [2005], although their approach involved a completely independent derivation.

[46] The expression for $\Phi_i(z_i)$ in equation (15) is for a heterogeneous region which we define as a region in which the matrix of the correlations between record flood values may be represented by the identity matrix $\boldsymbol{W} = \boldsymbol{I}$. In general, the matrix of correlations between record flood values $\boldsymbol{W}$ will have elements which are smaller than the corresponding elements of the matrix of correlations among the original flood series $\boldsymbol{R}$. The assumption $\boldsymbol{W} = \boldsymbol{I}$ is likely to be plausible if the sequence length at each site is sufficiently long and/or the region is extremely large so that orographic influences are pronounced and climatic influences are heterogeneous, leading to uncorrelated flood series. *Castellarin*

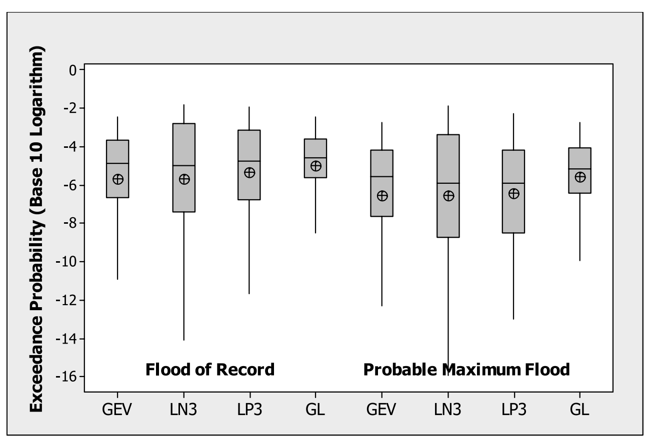

Figure 11. Boxplots of $\log_{10}[\Phi_i(z_i)]$ corresponding to z equal to the probable maximum flood (PMF) and flood of record (FOR) for 226 sites in the U.S.A.

et al. [2005] and *Castellarin* [2007] considered estimation of EPEE for the more challenging case in which $\boldsymbol{W} \neq \boldsymbol{I}$. Estimates of $\Phi_i(z_i)$ in equation (15) may be obtained for any distribution function via numerical integration. In addition, this study derived analytic results for the moments of $\Phi_i(z_i)$ for the Gumbel and GEV cases. For the Gumbel case, theoretical expressions are given for both EEPE [in equation (24)] and EPEE [in equation (22)]. EPEE is also given for the GEV case, but we were unable to obtain a closed form solution for EEPE for the GEV case.

[47] One of the key benefits of our approach is that it is easily extended to the estimation of the exceedance probability of an envelope curve on the basis of PMFs. A comparison with previous studies indicated that estimates of $\Phi_i(z_i)$ given by equation (15) differ considerably from a simple extrapolation of the flood frequency curve as performed by *Resendiz-Carrillo and Lave* [1987, 1990], *Shalaby* [1994], and others.

[48] A case study was implemented using historical flood series from 226 sites located across the U.S.A. (Figure 9) where estimates of the PMF were also available. Envelope curves were developed for both the FOR and PMF observations (Figure 1). Estimates of EEPE were obtained corresponding to four flood frequency distributions (GEV, LN3, LP3, and GL), and the resulting FOR and PMF envelopes had average exceedance probabilities of approximately 6.08×10^{-4} and 2.55×10^{-4}, respectively. Although there was a great deal of variability associated with individual estimates of envelope exceedance probabilities $\Phi_i(z_i)$, their record-length-weighted arithmetic average values were relatively constant across the four distributions considered. This result indicates that the approach introduced here offers significant promise for the estimation of exceedance probabilities associated with envelope curves for heterogeneous regions.

[49] Promising future areas of research include the following: relaxing the assumption that flood series are iid; developing expressions for EEPE and EPEE for other distributions such as GEV, LN3, and LP3; investigating the influence of the envelope curve slope parameter; and developing a probabilistic approach for including single observations of large floods at miscellaneous sites. Further improvements in the determination of the envelope of flood experience should result from the multivariate approach introduced by *Castellarin et al.* [2007], which relates the flood envelope to basin geomorphological and climatic characteristics in addition to basin area. We also recommend further evaluation of the shape of the flood envelope, because its mathematical structure remains an open question.

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A. Castellarin, DISTART, Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy. (attilio.castellarin@mail.ing.unibo.it)

J. F. England Jr., U.S. Department of the Interior, Bureau of Reclamation, Denver, CO 80225, USA. (jengland@do.usbr.gov)

N. C. Matalas, 709 Glyndon Street SE, Vienna, VA 22180, USA. (nmatalas@aol.com)

R. M. Vogel, Department of Civil and Environmental Engineering, Tufts University, Medford, MA 02155, USA. (richard.vogel@tufts.edu)