Probabilistic behavior of a regional envelope curve

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[1] A regional envelope curve (REC) summarizes the current bound on our experience of extreme floods in a region. Although RECs are available for many regions of the world, their traditional deterministic interpretation limits the use of the curves for design purposes, as magnitude, but not frequency of extreme flood events, can be quantified. A probabilistic interpretation of a REC is introduced via an estimate of its exceedance probability. The effect of intersite correlation on the exceedance probability is discussed. Monte Carlo experiments are performed to assess the performance of the estimate of the REC's exceedance probability. Generalized curves are presented for assessing the impact of regional intersite cross correlation on the likelihood of exceeding an observed REC. We also discuss how future developments in the theory of records may improve our ability to quantify the information content of a REC.

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1. Introduction

[2] A regional envelope curve (REC) represents the bound on our current experience of extreme floods in a region. The bound on our experience of extreme floods gained up to the present through systematic observation of flood discharges in a region is defined in terms of the largest floods observed at all gauging stations in a region. Herein, the largest flood is termed the record flood, and gauging stations are referred to as sites. An example of a REC is illustrated in Figure 1 which plots, for each site, the normalized record flood, defined as the logarithm of the ratio of the record flood to its basin area, versus the logarithm of the basin area. The REC is the line drawn on Figure 1 which provides an upper bound on all the normalized record floods at present.

[3] The idea of bounding our flood experience dates back to *Jarvis* [1925], who presented a REC based on record floods at 888 sites in the conterminous United States. Roughly 50 years later, *Crippen and Bue* [1977] and *Crippen* [1982] updated the study by *Jarvis* [1925] by creating 17 different RECs, each for a different hydrologic region within the United States, based on a total of 883 sites. *Matalas* [1997] and *Vogel et al.* [2001] document that the RECs identified by *Crippen and Bue* [1977] and *Crippen* [1982] still bound our flood experience gained from 1977–1994 at 740 of the 883 sites compiled by Crippen and Bue. *Enzel et al.* [1993] examine the REC bounding the historical flood experience for the Colorado river basin and show that the same REC also bounds the paleoflood discharge estimates available for the basin.

[4] The development of RECs is not confined to the United States; they have been developed for Italy [*Marchetti*, 1955], western Greece [*Mimikou*, 1984], Japan [*Kadoya*, 1992], and elsewhere. RECs have been used to compare record flood experience in the United States, China, and the world by *Costa* [1987] and, more recently, by *Herschy* [2002].

[5] The REC provides an effective summary of our regional flood experience. The pioneering work of *Hazen* [1914], who formalized flood frequency analysis, a formalism still in use, and who was among the first to suggest a method for improving information at a site through the transfer of information from other sites (i.e., substitution of space for time), has tempered the use of a flood magnitude as a design flood without an accompanying probability statement. Our objective is to provide a probabilistic interpretation of the REC. In the almost 80 years since *Jarvis* [1925] introduced the envelope curve, a probabilistic interpretation of a REC has never been seriously addressed. RECs have continued to be constructed and viewed mainly as summary accounts of record floods, rather than as meaningful tools for the design of measures to protect against "catastrophic" floods.

[6] It has been suggested that there is no obvious way to assign a probabilistic statement to a REC [see, e.g., *Crippen and Bue*, 1977; *Crippen*, 1982; *Vogel et al.*, 2001]. *Water Science and Technology Board, Commission on Geosciences, Environment and Resources* [1999] argued that the

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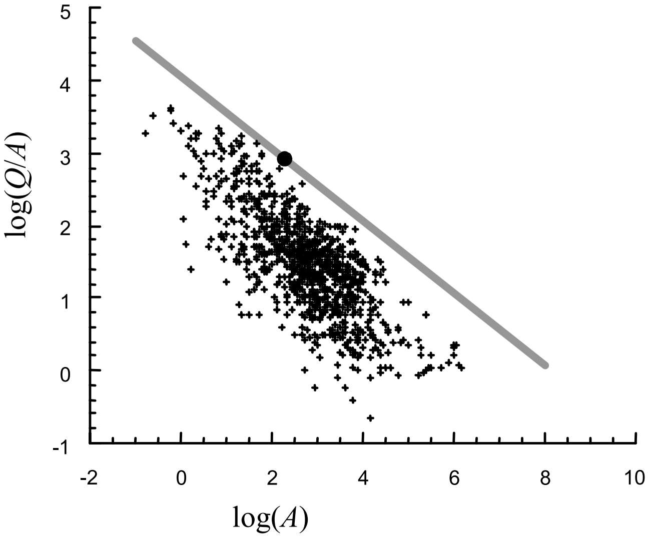

Figure 1. Flood experience accrued prior to 1925, Q in feet^3/s (1 foot = 0.3048 m) and A in miles^2 (1 mile = 1.609 km) [*Jarvis*, 1925]: elements of experience (pluses) and element of experience (circle) defining the intercept of the envelope curve (shaded line).

determination of the exceedance probability of a REC is difficult due to the impact of intersite correlation. As a consequence, RECs are assumed to have little utility beyond the suggestion of the *U.S. Interagency Advisory Committee on Water Data* [1986] that they are useful for "displaying and summarizing data on the actual occurrence of extreme floods." A probabilistic interpretation of the REC offers opportunities for several engineering applications which seek to exploit regional flood information to augment the effective record length associated with design flood estimates. A potential advantage of assigning a probabilistic statement to a REC is that this approach avoids the need to extrapolate an assumed at-site flood frequency distribution when estimating a design event.

[7] Our primary goals are to provide a probabilistic interpretation of the REC, to approximate its exceedance probability, and to quantify the effect of intersite correlation on estimates of the exceedance probability. Our secondary goal is to place the problem of estimating the exceedance probability of a REC within the context of the theory of records [*Arnold et al.*, 1998] in the hope that future developments in that theory will be extended to this problem.

2. A Probabilistic Interpretation of an Envelope Curve

[8] It is common practice to construct a REC, as in Figure 1, which plots the logarithm of the ratio of the record flood to the drainage area, $\ln(Q/A)$ versus $\ln(A)$. *Jarvis* [1925] suggested modeling the REC for the United States using,

$$\ln\left(\frac{Q}{A}\right) = a + b\ln(A). \tag{1}$$

with $a = 9.37$ (or $a = 4.07$ if log is used in (1) instead of ln) and $b = -0.50$, where Q and A are in cubic feet per second and square miles, respectively. Together with *Jarvis* [1925], other empirical studies showed that b is negative and greater than $-2/3$ for various portions of the world [*Linsley et al.*, 1949; *Marchetti*, 1955; *Crippen and Bue*, 1977; *Matalas*, 1997; *Herschy*, 2002].

[9] Assuming a fixed value of b, the intercept a in (1) may be estimated by forcing the REC to bound all record floods to the present, say up to the year n. Let X_j^i denote the annual maximum flood in year $i = 1, 2, \ldots n$ at site $j = 1, 2, \ldots M$, where M is the number of sites in the region. Let $X_j^{(i)}$ denote the flood flow of rank (i) at site j, where ranking is from smallest (1) to largest (n). The REC's intercept up to the year n can then be expressed as

$$a^{(n)} = \max_{j=1,\ldots,M}\left\{\ln\left(\frac{X_j^{(n)}}{A_j}\right) - b\ln\left(A_j\right)\right\} \tag{2}$$

where A_j is the area of site $j = 1, 2, \ldots M$.

2.1. Theoretical Assumptions

[10] We propose a probabilistic interpretation of a REC based on two fundamental assumptions. The implications of these simplifying assumptions are discussed in the discussion section (i.e., section 4.1), together with other possible limitations of our probabilistic interpretation. The two assumptions read as follows.

[11] 1. The study region is homogeneous in the sense of the index flood hypothesis [see, e.g., *Dalrymple*, 1960] and therefore the probability distribution of standardized annual maximum peak flows is the same for all sites in the region. Here, region has its broadest possible meaning, including the geographical sense (i.e., traditional index flood hypothesis), and also a pooling group of sites, which are climatically and hydrologically similar without being necessarily geographically contiguous [e.g., *Burn*, 1990; *Castellarin et al.*, 2001]. The standardized annual maximum peak flow, X', is defined for a given site as the annual maximum peak flow, X, divided by a site-dependent scale factor, μ_X (i.e., the index flood), assumed in this study to be equal to the at-site mean of X. Under this assumption, the flood quantile with exceedance probability p, x_p, is

$$x_p = \mu_X x'_p \tag{3}$$

where x'_p is the regional dimensionless flood quantile with exceedance probability p.

[12] 2. The relationship between the index flood μ_X and basin area A is of the form,

$$\mu_X = CA^{b+1} \tag{4}$$

where C is a constant and b is the same as in (1) and (2).

[13] Combining (3) and (4) leads to a relation between $\ln(x_p/A)$ and $\ln(A)$ that is analogous to (1),

$$\ln\left(\frac{x_p}{A}\right) = \ln\left(\frac{\mu_X x'_p}{A}\right) = \ln\left(Cx'_p\right) + b\ln(A). \tag{5}$$

The formal analogy between (1) and (5) originates from our simplifying assumptions and implies that, if the index flow scales with the drainage area, then the slope of the REC for a region can be identified from this scaling relationship. More importantly, we show below that the assumptions also imply that (1) a probabilistic statement can be associated with the intercept $a^{(n)}$ of (1), which is determined from the largest standardized annual maximum peak flow observed in the region (here standardization is achieved via the flood index method using (4) to express the index flood), and (2) the exceedance probability (p value) of the REC is equal to the p value of the standardized maximum flood. Implications 1 and 2 can be shown by combining (2) and (4) as follows,

$$a^{(n)} = \max_{j=1,\ldots,M}\left\{\ln\left(\frac{CA_j^{b+1}X_j'^{(n)}}{A_j}\right) - b\ln\left(A_j\right)\right\} = \ln(C) + \ln\left(\max_{j=1,\ldots,M}\left\{X_j'^{(n)}\right\}\right), \quad (6)$$

where $X_j'^{(n)}$ is equal to $X_j^{(n)}/\mu_{X,j}$, with $\mu_{X,j}$ index flood of site $j = 1, 2, \ldots M$, and $\max_{j=1,\ldots,M}\{X_j'^{(n)}\}$ is the standardized maximum flood (hereafter referred to as regional record flood, Y'). Under these assumptions, the original problem of estimating the p value of the REC reduces to estimating the p value of the regional record flood, the maximum of $n{\cdot}M$ standardized annual maximum peak flows. If the regional data do not exhibit either spatial or temporal correlation, an unbiased estimate of the exceedance probability p of the regional record flood can be computed using the Weibull plotting position by setting the overall sample length equal to the total number of sample years of data ($n{\cdot}M$) in the region [*Stedinger et al.*, 1993]. Although annual maximum flood series are often assumed to be serially uncorrelated, intersite correlation among flood flows observed at different sites in the same year should not be ignored [e.g., *Matalas and Langbein*, 1962; *Stedinger*, 1983; *Hosking and Wallis*, 1988; *Vogel et al.*, 2001]. Below we consider the impact of cross correlation on the REC using the concept of equivalent (or effective) sample years of independent data.

2.2. Approximation to Variance of Regional Record Flood for a Correlated Region

[14] It is well known that intersite correlation leads to increases in the variance of flood statistics [see, e.g., *Hosking and Wallis*, 1988]. For the case of M normally distributed and spatially correlated flood sequences with constant mean and variance, each with record length n, *Yule* [1945] and *Matalas and Langbein* [1962, equation (16)] document that the variance of a regional mean is inflated by a factor which depends upon the average correlation among the sites $\bar{\rho}$,

$$\mathrm{Var}[\bar{x}|\bar{\rho}] = \frac{\sigma_X^2}{Mn}[1 + \bar{\rho}(M-1)], \quad (7)$$

where $\bar{x}$ indicates the regional sample mean and σ_X^2 the population variance of the M sequences, each of length n; if $\bar{\rho} = 0$ the variance of $\bar{x}$ reduces to $\sigma_{X'}^2/(M\ n)$. *Matalas and Langbein* [1962] defined the relative information content, I, of the mean of M spatially correlated flood series, each of length n, as the ratio,

$$I = \frac{\mathrm{Var}[\bar{x}]}{\mathrm{Var}[\bar{x}|\bar{\rho}]} = [1 + \bar{\rho}(M-1)]^{-1} \quad (8)$$

The information content of the mean, I, in (8), is measured relative to the variance of the mean associated with spatially and serially uncorrelated flows. Hence $I = 1$ when $\bar{\rho} = 0$ and $I < 1$ when $\bar{\rho} > 0$. Values of $I < 1$ reflect the fact that intersite correlation reduces the overall information content of the regional sample. The effective number of regional samples associated with estimation of the regional mean $M_{\bar{x}}$ is then

$$M_{\bar{x}} = M\ I \quad (9)$$

Similarly, *Stedinger* [1983, equation (35)] derived the variance of an estimate of the regional variance of M cross-correlated sequences of length n as

$$\mathrm{Var}\left[\overline{s_X^2}\middle|\overline{\rho^2}\right] = \frac{2\sigma_X^4}{M(n-1)}\left[1 + \overline{\rho^2}(M-1)\right] = \mathrm{Var}\left[\overline{s_X^2}\right]\left[1 + \overline{\rho^2}(M-1)\right], \quad (10)$$

where $\overline{s_X^2}$ stands for the estimator of the regional variance that, for samples of different length, is a weighted average in which each sample variance is weighted proportionally to the record length of the corresponding site, and $\overline{\rho^2}$ is the average squared correlation of concurrent flows. Analogous to the effective number of regional samples (sites) for estimation of the regional mean given in (9), the effective number of regional samples (sites) for the estimation of the regional variance, $M_{s_x^2}$, can be computed as follows:

$$M_{s_x^2} = M\left[1 + \bar{\rho}^2(M-1)\right]^{-1}. \quad (11)$$

In general, (11) yields a number of effective sites that is higher than the result produced by (9).

[15] Our study focuses on the estimation of the exceedance probability, p, of the regional record flood, which under the assumptions adopted in this study, coincides with the p value of the REC bounding our regional experience of extreme floods. This problem is equivalent to determining the number of effective sites associated with estimation of the regional record flood which in turn defines the REC. We show below that neither (9) nor (11) are suitable for this problem, because they both markedly underestimate the number of effective sites. Consequently, we consider a theoretical first-order approximation to guide us to an empirical relation which can approximate the regional information content associated with a REC.

[16] If we assume that a Gumbel distribution [e.g., *Stedinger et al.*, 1993], with population mean $\mu_{X'}$ and variance $\sigma_{X'}^2$, is a suitable regional parent distribution for the M spatially uncorrelated sequences of length n of standardized annual maximum peak flows, then the regional dimensionless flood quantile associated with a given value of the exceedance probability p, reads,

$$x_p' = \mu_{X'} + K_p\sqrt{\sigma_{X'}^2}, \text{with } K_p = -\frac{\sqrt{6}}{\pi}[\gamma + \ln[-\ln[1-p]]] \quad (12)$$

where K_p is termed the frequency factor and depends on p, whereas $\gamma = 0.5772$ is Euler's number [see, e.g., *Stedinger et al.*, 1993]. Under the same assumptions, it can be shown that the regional record flood, $Y' = \max_{j=1,\ldots,M}\{X_j'^{(n)}\}$, also follows a Gumbel distribution [*Ang and Tang*, 1990], with

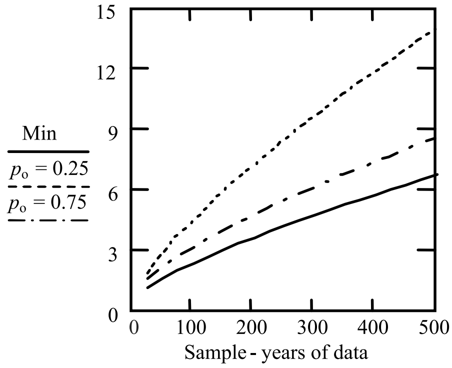

Figure 2. Equation (15): ratio of Θ to the sum of the first three terms within square brackets for various sample years of data, $M{\cdot}n$, and exceedance probability p_o.

population variance $\sigma^2_{X'}$. The quantile y'_p associated with a given exceedance probability p is

$$y'_p = \mu_{X'} + K_{M \cdot n, p}\sqrt{\sigma^2_{X'}}, \tag{13a}$$

with

$$K_{M \cdot n, p} = \sqrt{6}(\ln(M \cdot n) - \ln(-\ln(1-p)) - \gamma)\pi^{-1}, \tag{13b}$$

where frequency factor $K_{M \cdot n,p}$ depends on $M{\cdot}n$ and p. By substituting the sample moments, $\bar{x}'$ and $s^2_{x'}$, for their corresponding population moments, $\mu_{X'}$ and $\sigma^2_{X'}$ in (13), we obtain a sample estimate of the regional record flood quantile, $\hat{y}'_p$. Employing a first-order Taylor series approximation to the variance of $\hat{y}'_p$, we obtain

$$\mathrm{Var}\left[\hat{y}'_p\right] \approx \left(\frac{\partial \hat{y}'_p}{\partial \bar{x}'}\right)^2 \mathrm{Var}[\bar{x}'] + \left(\frac{\partial \hat{y}'_p}{\partial s^2_{x'}}\right)^2 \mathrm{Var}\left[s^2_{x'}\right] + 2\left(\frac{\partial \hat{y}'_p}{\partial \bar{x}'}\right)\left(\frac{\partial \hat{y}'_p}{\partial s^2_{x'}}\right)\mathrm{Cov}\left[\bar{x}', s^2_{x'}\right] + \left(\frac{\partial \hat{y}'_p}{\partial p}\right)^2 \mathrm{Var}[p] \tag{14}$$

where p is a random variable across flood samples and we assume that $\mathrm{Cov}[\bar{x}', p] = \mathrm{Cov}[s^2_{x'}, p] = 0$.

[17] The presence of intersite correlation inflates $\mathrm{Var}[\bar{x}']$ and $\mathrm{Var}[s^2_{x'}]$ in a way which is analogous to what was reported in (7) and (10), and may produce effects on $\mathrm{Cov}[\bar{x}', s^2_{x'}]$ and $\mathrm{Var}[p]$ as well. For the case of M spatially and temporally independent flood series, each with record length n, expanding (14) about the values $\bar{x}' = \mu_{X'}$, $s^2_{x'} = \sigma^2_{X'}$ and $p = p_o$, that is a given value of exceedance probability, expressing $\mathrm{Var}[\bar{x}']$, $\mathrm{Var}[s^2_{x'}]$ and $\mathrm{Cov}[\bar{x}', s^2_{x'}]$ as functions of $\sigma_{X'}$ and the third and fourth central moments through the relationships proposed by *Kendall and Stuart* [1963], and noting that the variable p is uniformly distributed with $\mathrm{Var}[p] = 1/12$, leads to

$$\mathrm{Var}\left[\hat{y}'_{p_o}\right] = \frac{\sigma^2_{X'}}{Mn}\left[1 + 1.1K^2_{M \cdot n, p_o} + 1.14K_{M \cdot n, p_o} + \Theta(M, n, p_o)\right] \tag{15}$$

where $K_{M \cdot n, p_o}$ is obtained by substituting p_o for p in (13b) and

$$\Theta(M, n, p_o) = \frac{Mn}{2(\pi(1-p_o)\ln(1-p_o))^2}. \tag{16}$$

[18] The first three terms within square brackets in (15) correspond in this order to $\mathrm{Var}[\bar{x}']$, $\mathrm{Var}[s^2_{x'}]$ and $\mathrm{Cov}[\bar{x}', s^2_{x'}]$, whereas the function Θ corresponds to $\mathrm{Var}[p]$. The presence of the $M{\cdot}n$ sample years of data in the numerator of Θ suggests that this term, may dominate $\mathrm{Var}[\hat{y}'_{p_o}]$. Figure 2 reports the ratio of $\Theta(M, n, p_o)$ to the sum of the first three terms of (15) as a function of the sample years of data, $M{\cdot}n$. The solid line in Figure 2 depicts the minimum value of this ratio as a function of $M{\cdot}n$, whereas the dashed and the dot-dashed lines report the values of the ratio for p_o equal to 0.25 and 0.75, respectively. Figure 2 shows that Θ dominates the sum of the first three terms of (15) because the minimum value of the ratio is larger than 2 for $M{\cdot}n$ = 100 and increases rapidly as $M{\cdot}n$ increases.

[19] Given the considerable influence of $\mathrm{Var}[p]$ in (15) our attention focuses on the impact of intersite correlation on this term using Monte Carlo experiments. The experiments used the simulation algorithm described in Appendix A to generate 20,000 samples of M = 2, 5, 10, 20, 50, 100, 200 and 500 random sequences of multivariate normal deviates with zero mean and unit variance, each with length n = 2, 5 10, 20, 50, 100 and 200, with mean correlation coefficient among the sequences $\bar{\rho}$ = 0.0, 0.2, 0.4, 0.6 and 0.8. We refer throughout the text to a mean coefficient $\bar{\rho}$ for simplicity, even though the Monte Carlo experiments utilize a realistic spread of the synthetic correlation coefficients between sequences (see Appendix A, step 1, and Table 1). For each replicate s = 1, 2, . . .20,000, the true exceedance probability of the largest among the M largest deviates of all synthetic series, $z_s^{\max}$, was computed using,

$$p_{M,s} = 1 - \Phi\left(z_s^{\max}\right)^n \tag{17}$$

where $\Phi(\cdot)$ stands for the normal cumulative distribution function (cdf) with zero mean, and $\Phi(\cdot)^n$ is the cdf of the maximum of a series of n independent variates with cdf

Table 1. Average Values of Correlation Coefficients to the Power of 1, 2, 3, and 4 for the Generated M-by-M Correlation Matrix **P**, With M = 2, 20, and 200[a]

M	$\bar{\rho}$	$\overline{\rho^2}$	$\overline{\rho^3}$	$\overline{\rho^4}$
2	0.200	0.040	0.008	0.002
2	0.400	0.160	0.064	0.026
2	0.600	0.360	0.216	0.130
2	0.800	0.640	0.512	0.410
20	0.200	0.165	0.134	0.122
20	0.400	0.271	0.194	0.156
20	0.600	0.445	0.334	0.267
20	0.800	0.680	0.573	0.490
200	0.200	0.083	0.047	0.032
200	0.400	0.204	0.121	0.081
200	0.600	0.389	0.266	0.192
200	0.800	0.651	0.535	0.444

[a]See Appendix A, step 1.

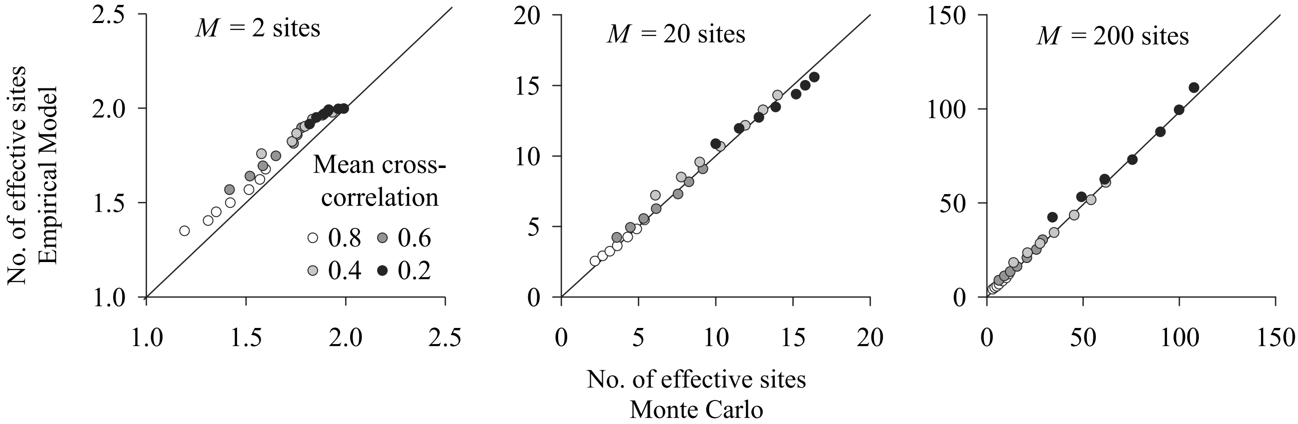

Figure 3. Values of the number of effective sites: values obtained from the Monte Carlo experiments versus values estimated with the empirical equation (19).

$\Phi(\cdot)$. The variance of the 20,000 $p_{M,s}$ values obtained for each set [M, n, $\bar{\rho}$] was then calculated. For the spatially uncorrelated case (i.e., $\bar{\rho}$ = 0), the values of Var[$p_{M,s}$] obtained from the Monte Carlo experiments were practically coincident (i.e., differences always smaller than 3.5%) with the theoretical result [e.g., *Loucks et al.*, 1981, equation (3.55)],

$$\mathrm{Var}[p_M] = \frac{M}{(M+2)(M+1)^2}, \tag{18}$$

which is simply the variance of an estimate of the exceedance probability associated with the largest of M observations.

[20] The Var[$p_{M,s}$] values obtained from the Monte Carlo simulations increase as $\bar{\rho}$ increases and as n decreases. The observed increase of Var[$p_{M,s}$] with $\bar{\rho}$ was expected, as intersite correlation inflates the variance of p_M reducing the overall information content; as the sample length n increases, though, the effect of $\bar{\rho}$ on Var[p_M] diminishes.

[21] An estimate of the number of effective sequences (equivalently, sites) associated with an estimate of the exceedance probability of the regional record flood is denoted as M_{EC} because it represents the effective number of sites associated with the REC. M_{EC} was computed by finding the M value that satisfies (18) for each value of Var[$p_{M,s}$] resulting from the Monte Carlo experiment for a particular set of [M, n, $\bar{\rho}$]. Among several relations tested, the expression

$$\hat{M}_{EC} = \frac{M}{1+\overline{\rho^{\beta}}(M-1)}, \text{ with } \beta := 1.4\frac{(nM)^{0.176}}{\overline{(1-\rho)^{0.376}}} \tag{19}$$

produces satisfactory estimates of the M_{EC} values obtained through Monte Carlo simulation. The empirical model (19) is characterized by an overall *Nash and Sutcliffe* [1970] efficiency measure E = 0.996, with $E \in [-\infty, 1]$ and E = 1 for the perfect fit. Figure 3 compares the M_{EC} values obtained from the Monte Carlo experiments versus the corresponding values estimated from (19). It is important to stress that the bar over $\overline{\rho^{\beta}}$ and $\overline{(1-\rho)^{0.376}}$ in (19) implies that these terms are average values of the corresponding functions of the correlation coefficients (i.e., $\overline{\rho^{\beta}}$ is the average of the $M(M-1)/2$ values of $\rho_{k,j}^{\beta}$, where $\rho_{k,j}$ is the correlation coefficient between sites k and j, with $1 \le k < j \le M$). This feature of the empirical model takes into account the actual distribution of correlation coefficients for a particular study region and makes the empirical model itself suitable for different spatial structures of the cross-correlation model.

[22] Even though I_{EC} was developed empirically, (1) it reproduces the dependence of I_{EC} on the overall sample years of data, $n{\cdot}M$, through the exponent β in (19) and (2) it amplifies this dependence for a higher degree of intersite correlation through the term $\frac{1}{\overline{(1-\rho)^{0.376}}}$ in (19). Most importantly, I_{EC} tends to the theoretical values 1 and M^{-1}, when $\bar{\rho}$ tends to 0 and 1, respectively.

2.3. Exceedance Probability of the Envelope Curve

[23] As in the work of *Matalas and Langbein* [1962], spatially and serially uncorrelated flow sequences are adopted as the reference basis of information content. For uncorrelated sequences, an approximate estimate of the variance of the regional record flood is given by (15). It was shown that (15) is dominated by the term Θ. In terms of the number of effective sites, the empirical relation in (19) expresses the reduction of the regional information content due to cross correlation, ignoring all terms except for those associated with Var[p]. Given the significant role played by Θ on (15), (19) is a better approximation than either (9) or (11) of the number of effective sites in a region with cross-correlated floods. Under the assumptions adopted here, we recommend the use of (19) for estimation of the exceedance probability, $\hat{p}_{EC}$, of the REC.

[24] Through a series of Monte Carlo experiments (see Appendix A), the accuracy of the estimator

$$\hat{p}_{EC} = \frac{0.56}{\hat{M}_{EC}\, n + 0.12}, \tag{20}$$

is assessed. This estimator uses the *Gringorten* [1963] plotting position and sets the regional sample years of data to $\hat{M}_{EC} \cdot n$, with $\hat{M}_{EC}$ estimated using (19); the Gringorten plotting position provides unbiased quantiles for the Gumbel distribution [see, e.g., *Stedinger et al.*, 1993].

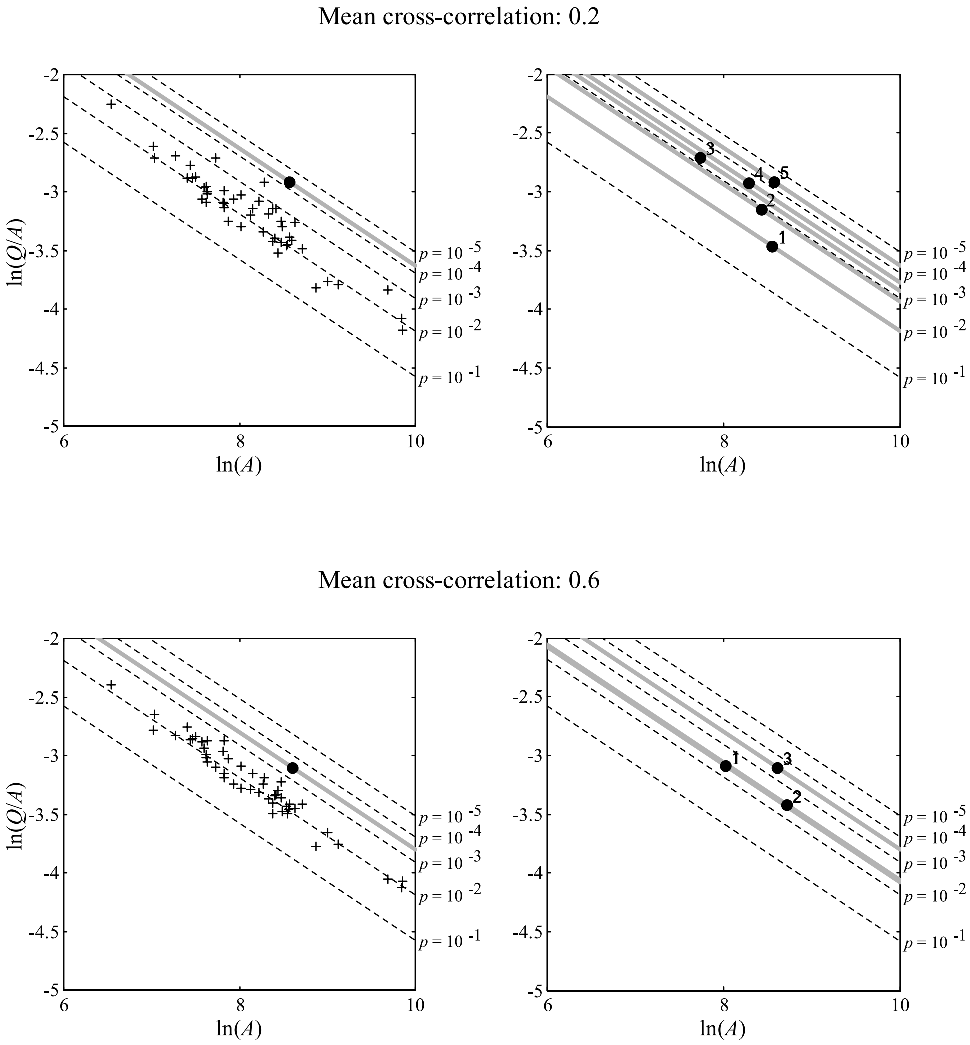

Figure 4. Synthetic index flood regions and dynamics of the envelope: M = 50 annual flood series of length n = 100 years and mean correlation coefficients, $\overline{\rho}$ = 0.2 and $\overline{\rho}$ = 0.6.

[25] Figure 4 illustrates the progression of regional record flood experience using RECs for two different cross-correlated synthetic regions generated through the simulation algorithm, each region having M = 50 sites and each site having n = 200 years of data. Figure 4 (left) reports the record flood experiences (pluses) and the largest regional flood (circle), which defines the intercept of the REC (shaded line). Figure 4 (right) shows the progression in time of the REC from n = 1 to 200 years (shaded lines), numbering the gains in regional flood experience. Figure 4 also reports the traces of the theoretical RECs with a given exceedance probability p (for spatially independent region, dashed lines), obtained as the envelope of the flood quantiles with exceedance probability p. The range of $\overline{\rho}$ = 0.1–0.6 in Figure 4 is typical of flood flows [e.g., *Benson*, 1965; *Stedinger*, 1983].

[26] The average intercept $\overline{a}^{(i)}$ resulting from the 20,000 Monte Carlo simulations for time step i (see Appendix A, step 4) yields the expected REC for a region of given characteristics (i.e., the REC that on average is expected to bound the flood experience for a group of M sites with record length i, and mean intersite correlation $\overline{\rho}$). An estimate of the *p value* associated with the expected REC, or equivalently $\overline{a}^{(i)}$, can be obtained from (20), and then converted into the corresponding Gumbel reduced variate $-\ln(-\ln(1 - \hat{p}_{EC}))$. The Gumbel reduced variate of the average intercept $\overline{a}^{(i)}$ can also be computed as,

$$-\ln\left(-\ln\left(1-p\left(\overline{a}^{(i)}\right)\right)\right) = \gamma + \frac{\pi\left(\exp\left(\overline{a}^{(i)}\right) - \mu_{X'}\right)}{\sqrt{6\sigma_{X'}^2}}. \quad (21)$$

Figure 5 uses solid lines to compare the Gumbel reduced variates of the average intercept $\overline{a}^{(i)}$ estimated using (20) and (21), that is, using the proposed empirical model and Monte Carlo experiments, respectively. We express p in terms of the Gumbel reduced variate to improve the readability of Figure 5 for high values of p. Overall, the agreement is good though it is somewhat better for small degrees of cross correlation, and tends to deteriorate as cross correlation increases. Still, (20) produces rather accurate estimates of the exceedance probability of a REC, and it captures its overall behavior over a broad range of M and $\overline{\rho}$ values. All scenarios that do not appear in Figure 5 (i.e.,

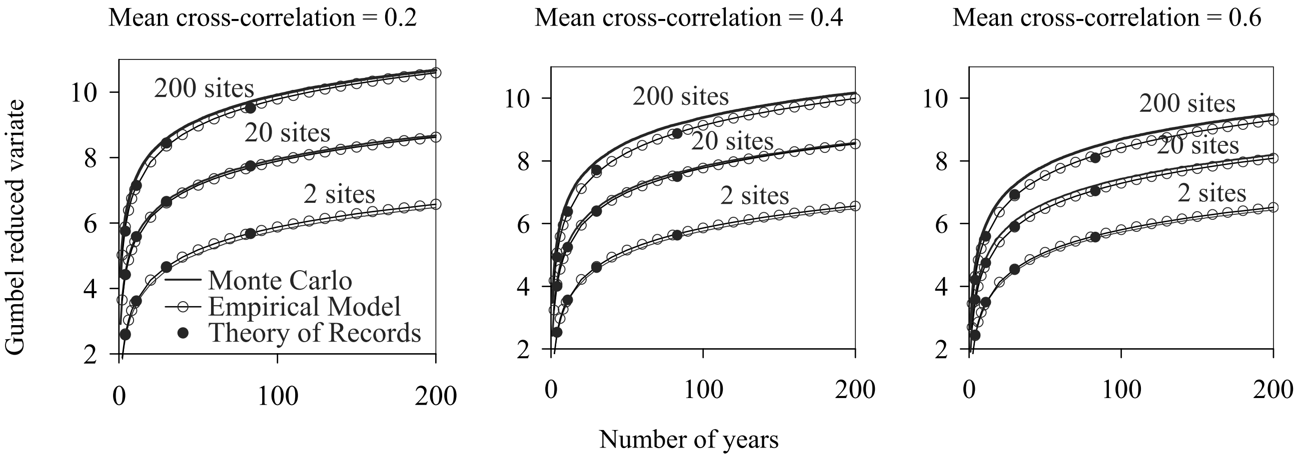

Figure 5. Nonexceedance probability, expressed in terms of Gumbel reduced variate, of the expected REC for a cross-correlated region with 2, 20, and 200 sites as the sample length increases from 1 to 200 years: estimates resulting from (21) (Monte Carlo) and obtained through (20) by evaluating M_{EC} with (19) (empirical model) or (24) (theory of records).

M = 5, 10, 50, 100 and 500 for $\bar{\rho}$ = 0.2, 0.4 and 0.6, and all M values for $\bar{\rho}$ = 0.8) yielded similar agreement.

3. A Different Perspective: Envelope Curve and Records

[27] A record event is defined as an event whose magnitude either exceeds or is exceeded by all previously observed events. The REC provides an upper bound on record-breaking flood experiences to date [*Vogel et al.*, 2001] and therefore is closely connected with the theory of records, which, as it is shown in this section, may offer a promising framework for representing the probabilistic behavior of RECs.

[28] Despite the rich literature on the mathematics of record events, from the pioneering work of *Chandler* [1952] to a recent textbook on the subject [*Arnold et al.*, 1998], there are only a few water resources investigations which have employed the theory of records. *Vogel et al.* [2001] used the theory of records for identifying nonstationarities in flood series, taking into account the presence of spatial correlation among the series. *Nagaraja et al.* [2002] analyzed the statistical properties of univariate and bivariate series of flood records and used the results to predict the number of record-breaking annual floods at two sites along the Missouri river during the next 50 years. N. C. Matalas and J. R. Olsen (Correlation between records from sequences derived from bivariate normal-Markov Processes, submitted to *Water Resources Research*, 2003) use Monte Carlo experiments to assess the relation between the correlation of record events and the correlation of the sequences from which the record events attain, using bivariate normal and bivariate lognormal distributions for representing the population of annual floods.

[29] The main goals of this section are (1) to analyze the gains in regional flood experience summarized by the REC in the context of record breaking events and (2) to use Monte Carlo simulations to evaluate the behavior of sequences of regional record floods for cross-correlated regions, and (3) to illustrate an alternative approach to identify the regional information content associated with a REC.

[30] Assume that (1) annual floods are independent and identically distributed (iid) random variables, (2) their probability distribution is a continuous distribution, which yields no unbreakable record floods, whether or not the distribution is bounded above, and (3) counting the first observation in a sequence as a (trivial) record, then the number of records in an annual flood series of length n, termed R, is a random variable with mean, μ_R, and variance, σ_R^2, defined as [*Glick*, 1978; *Vogel et al.*, 2001],

$$\mu_R = \sum_{i=1}^{n} \frac{1}{i}, \text{ and } \sigma_R^2 = \sum_{i=1}^{n} \frac{1}{i} - \sum_{i=1}^{n} \frac{1}{i^2}. \tag{22}$$

It is interesting to note that μ_R and σ_R^2, as well as the probability distribution of R, are independent of the distribution of floods.

[31] The regional gain in flood experience that causes an upward shift in the REC involves all M sites simultaneously "competing" to break the upper bound that forms the REC as was illustrated earlier in Figure 4. In a region with M sites a new record event (hereafter referred to as envelope record) occurs when at least one site experiences a record flood event and the magnitude of that flood also exceeds the upper bound identified by the current REC. When a new envelope record event is experienced, the REC is shifted upward to bound the new gain in regional flood experience.

[32] Under the hypotheses adopted here, the temporal dynamics of the REC coincides with the temporal dynamics of the record breaking process of the series of maxima of the M standardized annual floods, which is a univariate iid sequence even in the presence of intersite correlation.

[33] Figure 6 compares the theoretical average number of records μ_R, for a univariate iid sequence, based on (22), with the average number of envelope records obtained from our Monte Carlo experiments. Figure 6 considers regions with M = 2 to 200 sites each with sample lengths n = 1 to 200 years both with and without cross correlation. All curves reported in Figure 6 are nearly coincident, and analogous outcomes were obtained for the variance of the number of

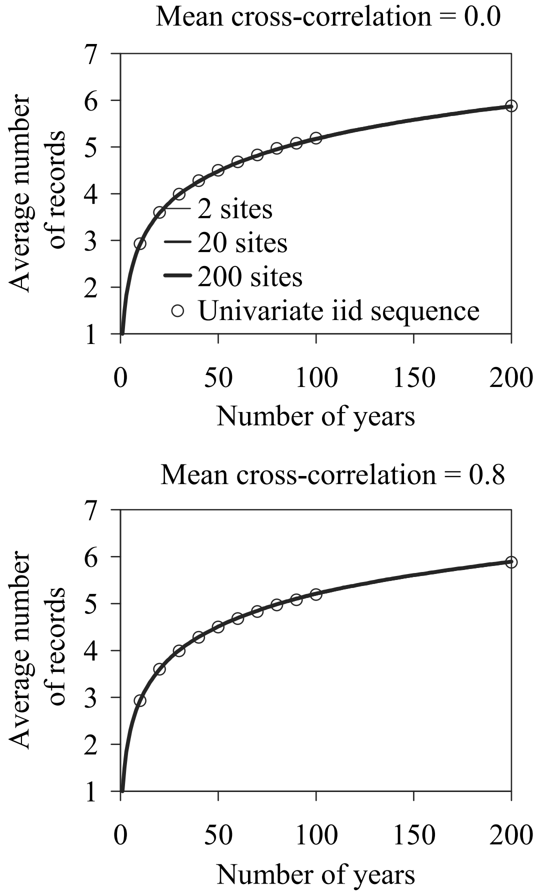

Figure 6. Values of μ_R for a univariate iid sequence of record-breaking events and average number of envelope records obtained through Monte Carlo experiments for different cross-correlated regions.

record events, showing the equivalence between the temporal dynamics of the record series for a realization of an iid sequence of random variables and of a REC. Unfortunately, these results do not assist us in understanding how cross correlation affects the exceedance probability of the expected REC as discussed below.

[34] In the analysis of the M series of flood flows, the first set of M observations is defined to be a (trivial) envelope record event, experienced concurrently at all M sites in the region. In order to have an envelope record in the year i with $i = 2, 3 \ldots n$, the following condition must be met,

$$M^* = \sum_{j=1}^{M} I\left(\ln\left(\frac{X_j^i}{A_j}\right) > a^{(i-1)} + b\ln\left(A_j\right)\right) \geq 1$$

$$\text{with } I(S) = \begin{vmatrix} 1 \text{ if } S \text{ is true} \\ 0 \text{ if } S \text{ is false} \end{vmatrix}, \tag{23}$$

in which M^* is the number of sites simultaneously experiencing an envelope record and X_j^i represents the annual flood observed at time $i = 1, 2, \ldots n$ for site $j = 1, 2, \ldots M$. If an envelope record is experienced, the new upper bound (i.e., the intercept $a^{(i)}$) is created for the entire region by referring to the site associated with the largest gain in flood experience.

[35] In year 1, $M^* = M$ by definition (trivial envelope record). If an envelope record is experienced at a later year, $i = 2, 3 \ldots n$, then $1 \leq M^* \leq M$. In general, for a cross-correlated region experiencing an envelope record we expect to have, on average, higher M^* values than for an uncorrelated region. Figure 7 illustrates the estimates of $E[M^*]$ for the first six gains in flood experience (i.e., REC updates) obtained from the Monte Carlo experiments for a range of cross correlations and regions. The first gain reported in Figure 7 is the trivial envelope record in year 1, and the sixth gain is the last reported in Figure 7 because the average number of records μ_R is approximately 6 for sequences of length equal to the maximum length considered in the simulation experiments (i.e., $\mu_R = 5.88$ for $n = 200$). Figure 7 illustrates for a range of M values and degrees of cross correlation that aside from the trivial initial record which is experienced by all M sites, the expectation of M^* assumes values close to one, starting from the second gain in flood experience if $\bar{\rho} = 0$ and tends to be higher for higher $\bar{\rho}$ values. If the M sequences are perfectly correlated with one another ($\bar{\rho} = 1$), then $M^* = M$ for every gain.

[36] For a region with M concurrent sequences of annual flood flows of length n and mean cross correlation equal to $\bar{\rho}$, one may argue that there exists a relationship between the

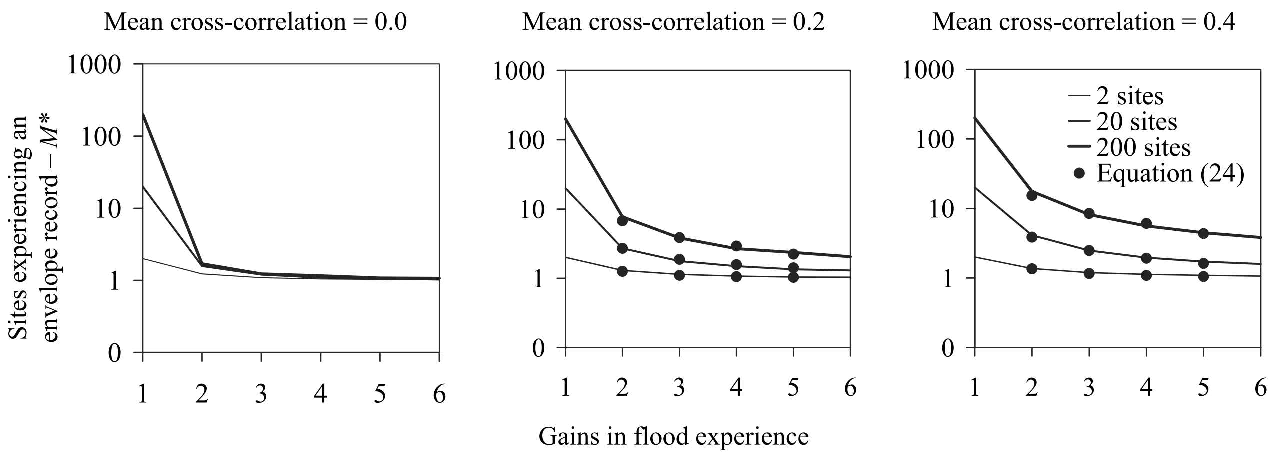

Figure 7. Average number of sites experiencing envelope records as a function of the number of gains in flood experience: results of the Monte Carlo experiments and estimates computed by (24).

expectation of M^* and the number of effective sites M_{EC} for which an approximate formula is provided by (19) (e.g., if the cross correlation increases then M_{EC} decreases and the expectation of M^* increases; if $\bar{\rho} = 1$, then $M_{EC} = 1$ and $M^* = M$). Moreover, one may formulate such a relation assuming the following simple proportion holds

$$\frac{\mathrm{E}[M^*|\rho = 0]}{\mathrm{E}[M^*|\rho]} = I_{EC} = \frac{M_{EC}}{M}. \qquad (24)$$

The validity of (24) was evaluated in two ways. First, estimates of $\mathrm{E}[M^*|\bar{\rho}]$ were computed by substituting into (24) the values of $\mathrm{E}[M^*|\rho = 0]$ obtained from the Monte Carlo experiments and estimates of M_{EC} obtained from (19) for different degrees of cross correlation and M values and for n values equal to 4, 11, 30 and 83; according to (23), the four n values, in this order, are the sample lengths associated with μ_R values that are as close to 2, 3, 4 and 5 as possible. Figure 7 reports estimates of $\mathrm{E}[M^*|\bar{\rho}]$ obtained for different M and $\bar{\rho}$ values and the n values indicated before (solid circles), showing fairly good agreement with the $\mathrm{E}[M^*|\bar{\rho}]$ values resulting from the simulation experiments.

[37] Also, for n values equal to 4, 11, 30, and 83, M_{EC} was expressed by (24) in terms of $\mathrm{E}[M^*|\bar{\rho}]$, $\mathrm{E}[M^*|\rho = 0]$ and M, and estimates of M_{EC} were computed by using the values of M^* obtained from the simulation experiments. Such M_{EC} values were then used in (20) to estimate the nonexceedance probability of the average intercept of the RECs obtained from the series of Monte Carlo experiments, $1 - \hat{p}_{EC}$. Figure 5 also reports these estimates (theory of records) in terms of Gumbel reduced variate, using solid circles for various M and $\bar{\rho}$ values and n = 4, 11, 30, and 83. Again, the agreement shown in Figure 5 is rather good.

[38] The above experiments illustrate that the number of sites in a region experiencing simultaneous envelope record breaking events, M^*, depends significantly upon the degree of cross correlation among the flood sequences. It was also shown that M^* can be quite useful for describing the regional information content associated with a REC and hence could be useful for estimating the exceedance (or nonexceedance) probability of a REC (see Figure 5). It is our hope that future developments in the theory of record-breaking processes will focus on the relationship between M^* and M, n and $\bar{\rho}$, which could replace our need to resort to (empirical) Monte Carlo experiments for describing the probabilistic behavior of a REC.

4. Discussion and Recommendations

4.1. Applicability of the Key Assumptions

[39] Our goal was to formulate a probabilistic interpretation of the REC in order to develop an estimator of the exceedance probability associated with a REC. For this purpose, it was necessary to adopt a number of simplifying assumptions that necessarily affect the applicability of our results. The fundamental assumptions were (1) flood sequences are iid, (2) flood sequences are of equal and concurrent length, (3) annual maximum floods follow a Gumbel distribution, (4) the procedure proposed by *Hosking and Wallis* [1988] can be used to model the covariance among flood series, (5) the index flood hypothesis applies to the study region, and (6) the index flood is determined by drainage area alone. These simplifying assumptions enabled the development of a probabilistic interpretation of the envelope curve. Relaxing any or all of the assumptions would in no way negate the existence of a probability statement. Relaxation would simply change the form of statement, but the statement would still be a probabilistic one.

[40] Assumption 1 is a classical assumption in regional flood frequency analysis [e.g., *Stedinger et al.*, 1993]. Although assumption 2 is adopted in several studies dealing with regional flood frequency analysis and intersite correlation [e.g., *Matalas and Langbein*, 1962; *Stedinger*, 1983], future analyses should incorporate a more realistic temporal distribution of data within the region. Assumption 3 is also a common hypothesis in flood frequency analysis [see, e.g., *Morrison and Smith*, 2002] and enabled the empirical identification of (19) (see section 2.2). Although some record properties of floods are independent of the distributional assumption [e.g., *Arnold et al.*, 1998], future research should investigate whether the parent distribution of the standardized annual floods, F'_X, affects the applicability of our results. Regarding assumption 4, the empirical model (19), for estimating the effective number of independent sites M_{EC} implicitly considers the actual distribution of correlation coefficients among series by referring to the average value of functions of the correlation coefficients (e.g., correlation coefficient to the power of β). Nevertheless, the impact of the intersite correlation model on the regional information content of a REC needs further research.

[41] The existence of homogeneous regions (assumption 5) in the strict sense is probably questionable aside from some trivial cases (e.g., very small regions in terms of geographic extent and number of sites). Nevertheless, several studies [see, e.g., *Lettenmaier et al.*, 1987; *Stedinger and Lu*, 1995] evaluated the index flood assumption showing that limited degrees of regional heterogeneity do not significantly affect the reliability of resulting regional estimates of a flood quantile. We hypothesize that the index flood assumption is also a convenient working hypothesis for analyzing the probabilistic behavior of RECs constructed for slightly heterogeneous regions. Furthermore, we expect the practical utility of RECs derived for highly heterogeneous regions to be limited, because a few of the discordant sites, i.e., sites for which the distributions of floods have very thick tails, are likely to dominate the REC.

[42] A simple simulation experiment was performed to better clarify this point, even though we are persuaded that this particular topic needs to be addressed by additional and more focused investigations. Using Monte Carlo techniques we repeatedly (i.e., 20,000 times) generated a set of synthetic regions with a range of regional heterogeneity. Each region consists of 20 spatially uncorrelated flood series of length n. The experiment neglects the effects of heterogeneity in the spatially correlated case, which is left for future research. The Gumbel distribution with theoretical mean equal to one and Cv equal to 0.4, EV1(1,0.4), was selected as the parent distribution for the generation of all series but one (i.e., discordant series); the discordant series was generated from a EV1(1,Cv^*), with Cv^* = 0.1, 0.2, ...1.0. Figure 8 illustrates how the expected regional record flood (i.e. average value of the 20,000 regional maxima) varies as a function of Cv^* for n equal to 10 and 50. Figure 8 clearly shows that a single discordant site can exert a strong control

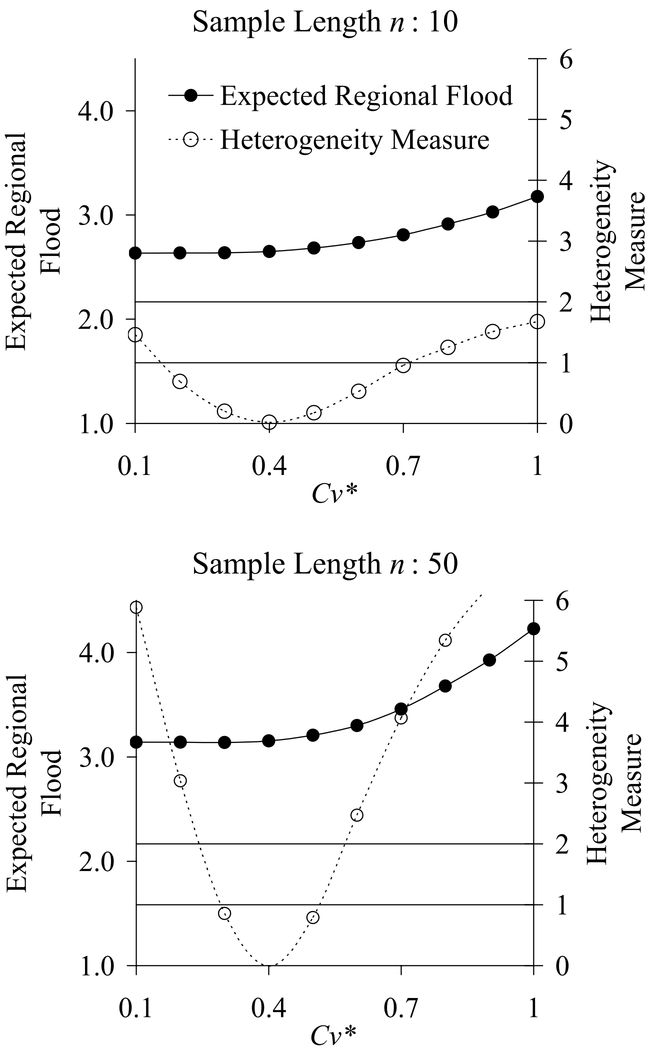

Figure 8. Expected regional flood obtained through Monte Carlo experiments for a heterogeneous group of 20 independent series of length n = 10 and 50. Nineteen series are generated from a Gumbel distribution with unit mean and Cv = 0.4, and one (discordant site) is generated with unit mean and Cv = Cv^*. The heterogeneity is quantified by the *Hosking and Wallis* [1993] heterogeneity measure.

on the regional record flood (that is on the REC under the hypotheses adopted in our study), and the control gets stronger as the length of the series increases.

[43] We also measured the degree of heterogeneity of the synthetic regions using H_1, the statistic that quantifies the heterogeneity of a group of series by comparing the regional variability of the sample L coefficient of variation, L-Cv, to the variation that would be expected in a homogeneous group [see *Hosking and Wallis*, 1993]. *Hosking and Wallis* [1993] suggest that a group of sites may be regarded as "acceptably homogeneous" if $H_1 < 1$, "possibly heterogeneous" if $1 \leq H_1 < 2$, and "definitely heterogeneous" if $H_1 \geq 2$. We referred to the average of 20,000 H_1 values obtained for each set of simulations (i.e., each value of Cv^*). On the one hand, it is interesting to observe that for short series (n = 10) heterogeneity may have a small influence on the REC, but it is also hard to detect (i.e., average $H_1 < 2$ for $Cv^* = 1$ in Figure 8). On the other hand, heterogeneity may have a stronger control on the REC for longer samples (n = 50), though this case is easier to diagnose (i.e., average $H_1 \approx 2.5$ for $Cv^* = 0.6$ in Figure 8).

[44] Although there is still the tendency to compile RECs for large regions without a critical evaluation of the degree of hydrological homogeneity associated with the pooled sites [see, e.g., *Herschy*, 2002], we maintain that the practical utilization of a REC should attempt to group river basins with similar hydrological behavior as done by *Crippen and Bue* [1977], who partitioned their data set into 17 hydrologic regions. Analogous to focused pooling techniques for regional flood frequency analysis [*Reed et al.*, 1999], RECs should be compiled for pooling groups of sites that have strong hydrological similarity, without necessarily belonging to the same geographical region. Recent literature reports several reliable techniques for assessing the hydrologic homogeneity of a region [e.g., *Hosking and Wallis*, 1993] and shows that for the regional frequency analysis of extreme floods it is important to pool information from hydrologically similar river basins [e.g., *Castellarin et al.*, 2001].

[45] The power law model reported in (4) (Assumption 6) can limit the applicability of the analysis because it uses drainage area as the only explanatory variable and, thereby, leaves a substantial portion of the variance of μ_X unexplained. Climatic and geomorphic information have been shown to improve the accuracy of multiple regression models for estimating μ_X [e.g., *Brath et al.*, 2001]. However, the basin area A normally explains at least 60% of the variability of μ_X [e.g., *Brath et al.*, 2001], and the significance of A becomes higher for flood events with lower exceedance probabilities, as proven by the limited scattering of record floods bounded by the envelope curves compiled by *Crippen and Bue* [1977]. Furthermore, (4) allows for a direct connection to the slope of envelope curve (see section 2.1). While we could relate μ_X to drainage area and other factors (e.g., basin slope, forest cover, etc.) it is not clear just how this relation could be connected to the envelope curve that takes account of only a single factor, namely, drainage area. This is an open question for future consideration.

[46] Future analyses should also address the investigation of the effects of considering a fixed value of b in (1) and (4), which is indeed a strong assumption and may significantly impact the results of the analysis. If b is assumed to be a value other than -0.5, our analysis would not be affected in any significant way. However, if we were to introduce a nonlinear slope or if we were to assume a linear or nonlinear slope to be a random realization of a "true" underlying slope, then our discussions would be significantly affected. Nevertheless, the assumption of a linear slope has empirical support, and several studies document for different regions around the world a limited variability of b values around -0.5 [e.g., *Jarvis*, 1925; *Marchetti*, 1955; *Herschy*, 2002]. Therefore (4) with a fixed $b = -0.5$ value is an adequate assumption for the scope of this initial study.

4.2. Graphical Representation of the Exceedance Probability of a REC

[47] The overall result of this study in (19) enables us to assess the impact of record length n, size of region M, and

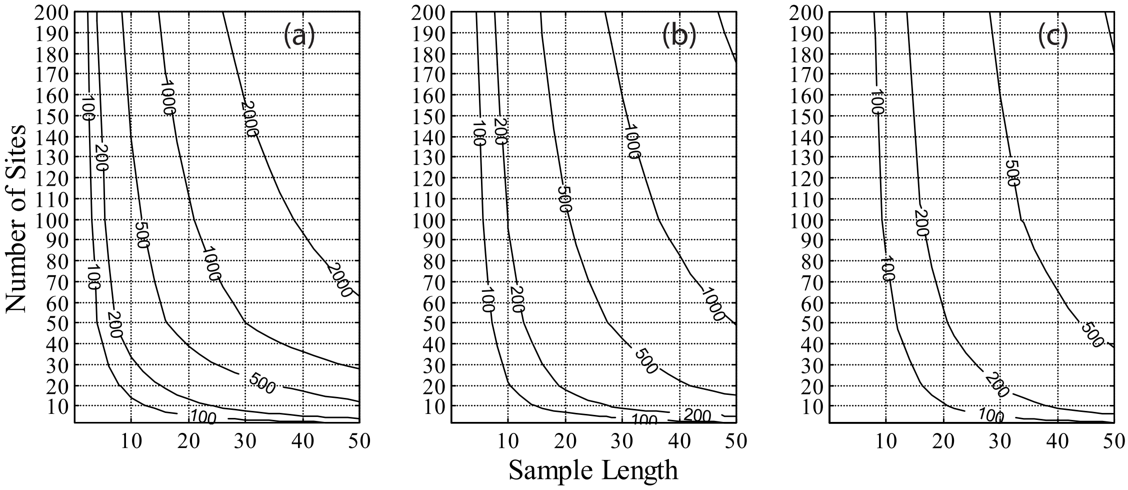

Figure 9. Recurrence interval (years), i.e., inverse of the exceedance probability, of the expected REC bounding the flood experience for $M \in [2, 200]$ sequences of annual floods, with sample length $n \in [1, 50]$ and different degrees of cross correlation summarized by $\overline{\rho}$, with $\overline{\rho}$ equal to (a) 0.2, (b) 0.4, and (c) 0.6.

cross correlation on the exceedance probability of the expected REC. A recurrence interval can be computed as the inverse of the expected REC exceedance probability, and plotted against n and M for different degrees of cross correlation. It is important to highlight that such a recurrence interval refers to the expected envelope curve and does not represent the expected recurrence interval of a REC. The frequency distribution of the time to a new record does not have moments, and therefore the mean number of years one must wait until the exceedance of $X_j^{(n)}$, does not exist, where $X_j^{(n)}$ is the largest flood observed at site j over an n-year period. This paradoxical result was proven by *Wilks* [1959] and *Gumbel* [1961] but has received little if any attention in the water resources literature.

[48] Figure 9 reports the return period of the expected REC as a function of sample length n and number of sites M for three different degrees of cross correlation, summarized in Figure 9 with the average values $\overline{\rho} = 0.2$, 0.4, and 0.6. The recurrence intervals reported in Figure 9 were computed by estimating the number of effective sites, M_{EC}, using (19) which in turn was used to estimate the exceedance probability of the intercept of the expected envelope curve, $\hat{p}_{EC}$, by using the Weibull plotting position,

$$\hat{p}_{EC} = \frac{1}{\hat{M}_{EC}\; n + 1}, \tag{25}$$

which is designed to give unbiased estimates of exceedance probabilities, for all distributions [e.g., *Stedinger et al.*, 1993].

[49] The applicability of the diagrams of Figure 9 requires the study region to be homogeneous in the sense of the index flood hypothesis. It is worth noting that Figure 9 refers to series of equal length n and to a particular distribution of correlation coefficients among series (see Appendix A). One must also keep in mind that Figure 9 was obtained from an approximation of the regional information content based on (19). Nevertheless, the accuracy of the empirical estimator was extensively analyzed using Monte Carlo experiments both within the context of the theory of extremes (see Figure 5) and the theory of record-breaking events (see Figure 7).

5. Conclusions

[50] The study investigates the probabilistic behavior of the regional envelope curve (REC) which bounds our experience of extreme floods in a region. Our primary objective was to formulate a probabilistic interpretation of the REC and to approximate its exceedance probability, p_{EC}, as a function of the available sample years of regional flood data. Primary attention was given to the impact of intersite correlation on the exceedance probability associated with a REC.

[51] First, our study introduced an empirical estimator of p_{EC} as follows: (1) it is shown that if the index flow hypothesis and a power law relationship between the drainage area and the index flow hold for the study region, then p_{EC} is the p value associated with the largest standardized annual maximum peak flow observed in the region; (2) by developing a first-order approximation to the variance of the estimated standardized regional flood quantile associated with a given p value, $\hat{y}_p$, it is shown that the variance of p controls the variance of $\hat{y}_p$; (3) by analyzing the variance of p resulting from a series of Monte Carlo experiments, an empirical relation was identified to estimate the equivalent number of independent series (effective number of sites) for a cross-correlated region as a function of the number of sequences M, their length n (equal for all series) and the mean spatial correlation among the flood series, $\overline{\rho}$; (4) the estimated number of effective sites is used for the evaluation of p_{EC} using a plotting position formula; (5) the reliability of the estimated p_{EC} is assessed through another series of Monte Carlo experiments by generating a large number of cross-correlated regions with different numbers of sites and degrees of cross correlation among the series.

[52] Second, by studying the temporal dynamics of the REC, that is, the progression in time of the gains in regional flood experience, we are able to show how this problem relates to the theory of record-breaking events. Under the hypotheses adopted, our analysis (1) showed that the

temporal dynamics of the REC coincides with the temporal dynamics of a serially uncorrelated univariate record-breaking process, and (2) indicated how future developments in the theory of records may provide useful analytical results to aid in the quantification of the information content of a REC.

[53] Finally, the overall results of this study are summarized in Figure 9, which illustrates the return period of the expected REC versus M, n, for three different degrees of cross correlation, assuming the index flood hypothesis to hold for the study region.

[54] On the one hand our results are approximate, yet this is only an initial effort at a probabilistic description of a REC. Further analyses should investigate the effects of the following issues on the information content of a REC: (1) the intersite correlation model, (2) a realistic variability of the series lengths within the region, (3) the applicability of the index flow hypothesis, and (4) the scaling relationship between the drainage area and the index flood. Assigning a probabilistic statement to a REC is a fundamental and necessary step if RECs are to be used as a design tool. Without an accompanying probabilistic statement, a REC is only a compilation of record floods in a region.

Appendix A: Simulation Algorithm

[55] The simulation algorithm introduced by *Hosking and Wallis* [1988] is used to generate a large number of cross-correlated synthetic regions. The algorithm assumes that if each site's flood frequency distribution were transformed to normality using the transformation F, then the joint distribution for all sites in the region would be multivariate normal. The simulation algorithm involves the following steps: (1) generation of the matrix $\mathbf{P}$ of intersite correlation coefficients, (2) generation of a multivariate vector $\mathbf{y}$ having a multivariate normal distribution with correlation matrix $\mathbf{P}$, and (3) application of the inverse transformation F^{-1} to obtain data with the required marginal distribution. A brief description of the simulation algorithm is given below.

[56] Assume a region with M sites having record length n, parent distribution of the regional standardized annual flood denoted as $F_{X'}$ and drainage area A_j for site j. Also assume the mean correlation coefficient among the normalized floods is $\overline{\rho}$.

[57] 1. Generate the M-by-M correlation matrix $\mathbf{P}$. To achieve a realistic spread of the $\rho_{i,j}$ values while ensuring that $\mathbf{P}$ is positive definite, the following procedure was adopted [e.g., *Hosking and Wallis*, 1988, p. 590]: (1) choose the M sites to be points uniformly distributed within the unit square; (2) set $\rho_{k,j}$ to be $\exp(-\alpha \cdot d_{k,j})$, where $d_{k,j}$ is the distance between sites k and j, whereas α is selected so that the mean of the $M(M-1)/2$ $\rho_{k,j}$ coefficients, for $1 \leq k < j \leq M$, is $\overline{\rho}$. Table 1 illustrates the actual spread of $\rho_{i,j}$ values generated by this procedure reporting the average values of $\rho_{k,j}^{\beta}$ for $1 \leq k < j \leq M$, with $M = 2$, 20 and 200 sites, and $\beta = 1$ ($\overline{\rho}$), $\beta = 2$ ($\overline{\rho^2}$), $\beta = 3$ ($\overline{\rho^3}$) and $\beta = 4$ ($\overline{\rho^4}$).

[58] 2. Generate the regional sample: (1) Generate a matrix $\mathbf{z} = [\mathbf{z_1}, \mathbf{z_2}, \ldots, \mathbf{z_M}]$ with M columns and n rows. Each column $\mathbf{z_j}$, $j = 1, 2, \ldots M$, contains n multivariate normal deviates with zero mean, unit variance, and covariance matrix $\mathbf{P}$. (2) Transform each element of the matrix $\mathbf{z}$ into a realization from the correct marginal distribution by setting $x_j'^i = F_X'^{-1}(\Phi(z_j^i))$, with $i = 1, 2, \ldots n$ and $j = 1, 2, \ldots M$ and where Φ is the cdf of the standard normal distribution. (3) Obtain M series of annual floods X_j^i by multiplying each element $x_j'^i$ by $C\, A_j^{1+b}$.

[59] 3. Analyze the dynamics of the REC bounding the experience of extreme floods for the generated region, in time. The first year of the synthetic sample is by definition, a gain in flood experience; accordingly, the intercept of the REC for this time step, $a^{(1)}$, is set to

$$a^{(i)} = \max_{j=1,\ldots,M} \left\{ \ln\left(\frac{X_j^i}{A_j}\right) - b \ln\left(A_j\right) \right\}, \qquad \text{(A1)}$$

where $i = 1$, and X_j^1 is the first annual flood generated for site j, a record flood by definition; for all time steps $i \in [2, n]$ the possible gain in regional flood experience is analyzed as follows: if the REC bounding the flood experience in year $i-1$ is exceeded at time step i (i.e., gain of new experience), then the REC is shifted upward by evaluating the intercept $a^{(i)}$ using (A1), where X_j^i is the flood generated for site j at time step i; otherwise (i.e., no gain in regional flood experience at time step i), the intercept $a^{(i)}$ is set equal to $a^{(i-1)}$.

[60] 4. Repeat steps 2 and 3 a large number of times and calculate the mean $\overline{a}^{(i)}$ of the series of $a^{(i)}$, for $i = 1, 2, \ldots n$. $\overline{a}^{(i)}$ identifies the expected REC, the envelope curve that, on average, is obtained for a region with M sites having record length i and a degree of cross correlation identified by $\overline{\rho}$ (see Table 1).

[61] The algorithm generated 20,000 synthetic regions for each pair (M, $\overline{\rho}$), where $M = 2$, 5, 10, 20, 50, 100, 200 and 500 sites, with maximum sample length $n = 200$ years, and $\overline{\rho} = 0.0$, 0.2, 0.4, 0.6, 0.8, and 1.0.

[62] A Gumbel distribution was used as parent distribution of the regional standardized annual floods, $F_{X'}$, with $Cv^* = 0.4$ (i.e., $\mu_{X'} = 1$ and $\sigma^2_{X'} = 0.16$). The same distribution was used for generating cross-correlated annual floods in homogeneous regions by *Hosking and Wallis* [1988, sect. 4, p. 591].

[63] The parameter b in (1), (2), (4), (5), and (A1) was set to -0.5. Although any negative value could have been used without modifying the results of the analysis, $b = -0.5$ is the value suggested by *Jarvis* [1925] and is consistent with the empirical values observed worldwide [e.g., *Herschy*, 2002]. Furthermore, slope values ranging from -0.45 to -0.50 can be obtained by analyzing the data set compiled by *Crippen and Bue* [1977] for the 17 regions proposed by the authors and collectively. With no loss of generality, the coefficient C of (4) and (5), is assumed to equal 1.

[64] To produce a realistic spread of the drainage areas in a region, even though this has no influence on the results of the analysis, the areas of all sites in the simulated region, A_j with $j = 1, 2, \ldots M$, are generated randomly by sampling from the empirical distribution of the basin areas contained in the Hydro-Climatic Data Network (HCDN), a data set compiled by *Slack et al.* [1993].

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