

FLOOD-FLOW FREQUENCY MODEL SELECTION IN SOUTHWESTERN UNITED STATES

By Richard M. Vogel,¹ Member, ASCE, Wilbert O. Thomas Jr.,² and Thomas A. McMahon,³ Member, ASCE

ABSTRACT: Uniform flood frequency guidelines in the United States recommend the use of the log Pearson type 3 (LP3) distribution in flood frequency investigations. Many investigators have suggested alternate models such as the generalized extreme value (GEV) distribution as an improvement over the LP3 distribution. Using flood-flow data at 383 sites in the southwestern United States, we explore the suitability of various flood frequency models using *L*-moment diagrams. We also repeat the experiment performed in the original Water Resource Council report (*Bulletin 17B*, issued in 1982), which led to the LP3 mandate. All our evaluations consistently reveal that the LP3, GEV, and the two- and three-parameter lognormal models (LN2 and LN3) provide a good approximation to flood-flow data in this region. Other models such as the normal, Pearson, and Gumbel distributions are shown to perform poorly. Recent research indicates that regional index-flood procedures should be more accurate and more robust than the type of at-site procedures evaluated here. Nevertheless, this study reveals that index-flood procedures need not be restricted to the GEV distribution because the LN2, LN3, and LP3 distributions appear to be suitable alternatives, at least in the southwestern United States.

INTRODUCTION

Many innovations in the field of flood frequency analysis have occurred since the decision of the U.S. Water Resources Council (1967) to recommend the use of the log Pearson type 3 (LP3) distribution for flood-flow investigations in the United States. The state of the art of selecting a regional flood frequency distribution at the time of the LP3 mandate was considerably different from the current situation. For example, in describing the U.S. Water Resource Council (WRC) work group study ("Guidelines" 1967), Benson (1968) argued that "no single method of testing [alternative hypotheses] was acceptable to all those on the Work Group, and the statistical consultants could not offer a mathematically rigorous method," leading to the conclusion that "there are no rigorous statistical criteria on which to base a choice of method."

More recently, *L*-moment diagrams and associated goodness-of-fit procedures (Hosking and Wallis 1987; Wallis 1988; Cunnane 1989; Hosking 1990; Chowdhury et al. 1991; Vogel and Fennessey 1993; Vogel et al. 1993) were advocated for evaluating the suitability of selecting various distributional alternatives in a region. For example, Hosking and Wallis (1987) found *L*-moment diagrams useful for selecting the generalized extreme value distribution (GEV) over the Gamma distribution for modeling annual maximum hourly rainfall data. Similarly, Wallis [(1988), Fig. 3] found an *L*-moment diagram useful for rejecting Jain and Singh's (1987) conclusion that annual maximum flood flows at 44 sites were well approximated by a Gumbel distribution and for suggesting a GEV distribution instead.

¹Assoc. Prof., Dept. of Civ. Engrg., Tufts Univ., Medford, MA 02155.

²Hydro., U.S. Geol. Survey, 12201 Sunrise Valley Dr., Reston, VA 22092.

³Prof., Univ. of Melbourne, Parkville, Victoria 3052, Australia.

Note. Discussion open until October 1, 1993. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on April 8, 1992. This paper is part of the *Journal of Water Resources Planning and Management*, Vol. 119, No. 3, May/June, 1993. ©ASCE, ISSN 0733-9496/93/0003-0353/\$1.00 + \$.15 per page. Paper No. 3787.

Another approach for evaluating the fit of alternate probability models and associated parameter estimation schemes is nonparametric experiments of the type performed by Beard (1974) and summarized by the Interagency Advisory Committee on Water Data (IACWD) ["Guideline" (1982), Appendix 14]. Using 300 stations distributed across the United States, Beard counted the number of stations for which the estimated 1,000-yr flood flow was exceeded in the historical record. Eight independent methods were employed for estimating the 1,000-yr flood at each site; results are reproduced in Table 1. Beard argued that with a total of $n = 14,200$ station-years of data across the 300 sites, one would expect approximately 14 exceedances of the true 1,000-yr flood flow. Only the LP3 and LN2 distributions came close to reproducing the 14 expected exceedances. Beard (1974) performed many other tests, but it was this test that convinced hydrologists that both the LP3 and LN2 models approximate the distribution of observed flood-flow data throughout the entire United States.

A third approach to evaluating the fit of alternative probability models to a regional data base is to employ probability plots and associated probability plot correlation coefficient (PPCC) tests (Vogel 1986; Vogel and Kroll 1989; Vogel and McMartin 1991; Chowdhury et al. 1991). Such tests are useful, simple, and powerful for most two-parameter distributional alternatives, (Vogel 1986; Vogel and Kroll 1989). However, Vogel and McMartin (1991) show that a PPCC test for the LP3 distribution exhibits remarkably low power in discriminating against similar distributional hypotheses. Chowdhury et al. (1991) arrived at similar conclusions for the GEVPPCC test. We elected not to perform PPCC hypothesis tests here due to the likely ambiguity of such test results for the three-parameter alternatives considered.

In the following sections, we employ *L*-moment diagrams and Beard's nonparametric test to annual maximum flood-flow data in the southwestern United States. Our goal is to both assess the adequacy of the existing LP3 flood frequency procedures and to choose other plausible procedures for approximating the underlying distribution of flood flows in this region.

STUDY REGION

The annual maximum flood-flow data employed in this study include 383 U.S. Geological Survey gaging stations with unregulated streamflow record lengths of 30 or more years. These stations are located in the 10-state region

TABLE 1. Number of Stations Where One or More Observed Flood Events Exceeds 1,000-Yr Flood Flow

Method (1)	Number of exceedances (2)
Log Pearson type 3 (LP3)	14
Lognormal (LN2)	18
Gumbel (MLE estimators)	77
Log Gumbel	1
Gamma	68
Pearson type 3 (P3)	56
Regional log Pearson type 3	20
Gumbel (best linear invariant estimator)	253

Note: From Beard (1974) and IACWD ["Guidelines" (1982), Table 14-2].



FIG. 1. Ten-State Study Region in Southwestern United States

of the southwestern United States shown in Fig. 1. The 383 stations are a subset of the 1,059 stations employed by Thomas et al. (1992) for developing regional hydrologic equations for estimating floodflow quantiles in this 10-state region. Our data base is considerably smaller than the one used by Thomas et al. (1992) because we only considered sites with records longer than 30 years; and we rejected sites that contained annual maximum flood flows equal to zero. Sites that contained annual maximum flood flows equal to zero were rejected so that our goodness-of-fit evaluations would not be confounded by methods for the treatment of zeros, which would have been required had we included these sites. Fig. 2 illustrates the distribution of record lengths, with an average length of 50 yr.

The basins with longer records in the southwestern United States tend to be operated for water supply purposes, hence such basins contain higher base flow runoff than short-record sites. Since we dropped sites with zero annual maximum flood flows, and since we only used sites with longer records ($n \geq 30$ yr), the data base employed here consists of mostly larger, less arid basins in the arid southwestern United States. We would have liked to include the smaller, more arid basins since such basins exhibit the most variable streamflow and pose the greatest hydrologic modeling challenge.



FIG. 1. Ten-State Study Region in Southwestern United States

of the southwestern United States shown in Fig. 1. The 383 stations are a subset of the 1,059 stations employed by Thomas et al. (1992) for developing regional hydrologic equations for estimating floodflow quantiles in this 10-state region. Our data base is considerably smaller than the one used by Thomas et al. (1992) because we only considered sites with records longer than 30 years; and we rejected sites that contained annual maximum flood flows equal to zero. Sites that contained annual maximum flood flows equal to zero were rejected so that our goodness-of-fit evaluations would not be confounded by methods for the treatment of zeros, which would have been required had we included these sites. Fig. 2 illustrates the distribution of record lengths, with an average length of 50 yr.

The basins with longer records in the southwestern United States tend to be operated for water supply purposes, hence such basins contain higher base flow runoff than short-record sites. Since we dropped sites with zero annual maximum flood flows, and since we only used sites with longer records ($n \geq 30$ yr), the data base employed here consists of mostly larger, less arid basins in the arid southwestern United States. We would have liked to include the smaller, more arid basins since such basins exhibit the most variable streamflow and pose the greatest hydrologic modeling challenge.

$$\tau_4 = \frac{\lambda_4}{\lambda_2} \equiv L\text{-kurtosis} \dots\dots\dots (1c)$$

where λ_r , $r = 1, \dots, 4$ = first four L -moments; and τ_2 , τ_3 , and τ_4 = L -coefficient of variation (L - C_v), L -skewness, and L -kurtosis, respectively. The first L -moment, λ_1 , is equal to the mean μ , hence it is a location parameter. Hosking (1990) shows that λ_2 , τ_3 , and τ_4 can be thought of as measures of a distributions scale, skewness, and kurtosis, respectively, analogous to the ordinary moments σ , γ , and κ , respectively.

L -Moment Diagrams

An L -moment diagram compares sample estimates of the dimensionless ratios, τ_2 , τ_3 , τ_4 , with their population counterparts for a range of assumed distributions. An advantage of L -moment diagrams is that one can compare the fit of several distributions using a single graphical instrument. Fig. 3 compares the relationship between nearly unbiased sample estimates of τ_4 and τ_3 (using open circles) and their population values. Here, sample estimates of τ_2 , τ_3 , and τ_4 are obtained using the unbiased probability-weighted moment estimators of λ_1 , λ_2 , λ_3 , and λ_4 as recommended by Stedinger et al. (1993), when constructing L -moment diagrams. Unbiased probability weighted moment estimators are equivalent to the unbiased L -moment estimators introduced by Hosking (1990). However, use of unbiased estimators of the sample L -moment λ_r , $r = 1, \dots, 4$, does not imply that the L -moment ratio estimators τ_i , $i = 1, 2$, and 3 , are unbiased. Fig. 3(a) was constructed using annual maximum flood flow data for the 383 sites described previously. The true relationships between L -kurtosis and L -skewness corresponding to the Pearson type 3 (P3), LN3, GEV, Gumbel, and normal distributions are shown for comparison in Fig. 3. These theoretical relationships are summarized in Hosking (1991a) and Stedinger et al. (1993) hence we only report the theoretical relationships for normal ($\tau_3 = 0$; $\tau_4 = 0.1226$) and Gumbel ($\tau_3 = 0.1699$; $\tau_4 = 0.1504$) distributions since their coordinates are difficult to discern amidst the scatter of points in Fig. 3.

Fig. 3(a) reveals that of the models tested, only the LN3 and GEV distributions appear consistent with this regional sample, and of those two distributions, it appears that the LN3 model provides the better fit to the data. Roughly half the observations are above the LN3 line and half are below; there are fewer than half the observations above the GEV line.

Unfortunately, a theoretical relationship between L -kurtosis and L -skewness is unavailable for the LP3 distribution. Since such a theoretical relationship only exists for the P3 distribution, we synthetically generated LP3, LN3, GEV, normal, and P3 samples for comparison in Fig. 3(b-f), respectively. We generated 383 independent samples of LP3, LN3, GEV, normal, and P3 data with record lengths equal to record lengths of the observed flood-flow data in Fig. 3(a). In generating the synthetic samples at each site, we assumed that population values of each model parameter equaled the sample estimates obtained at each site. The purpose of using different population parameters at each site was to attempt to capture the hydrologic heterogeneity of flood flows across the region. The character of generated LP3, LN3, and GEV samples displayed in Fig. 3(b-d) appears to be very similar to the observed floodflow samples in Fig. 3(a), yet the character of generated normal and P3 samples is quite different from the observed flood-flow samples in Fig. 3(a).

In Fig. 4 we plot estimates of L -kurtosis versus estimates of L -skewness

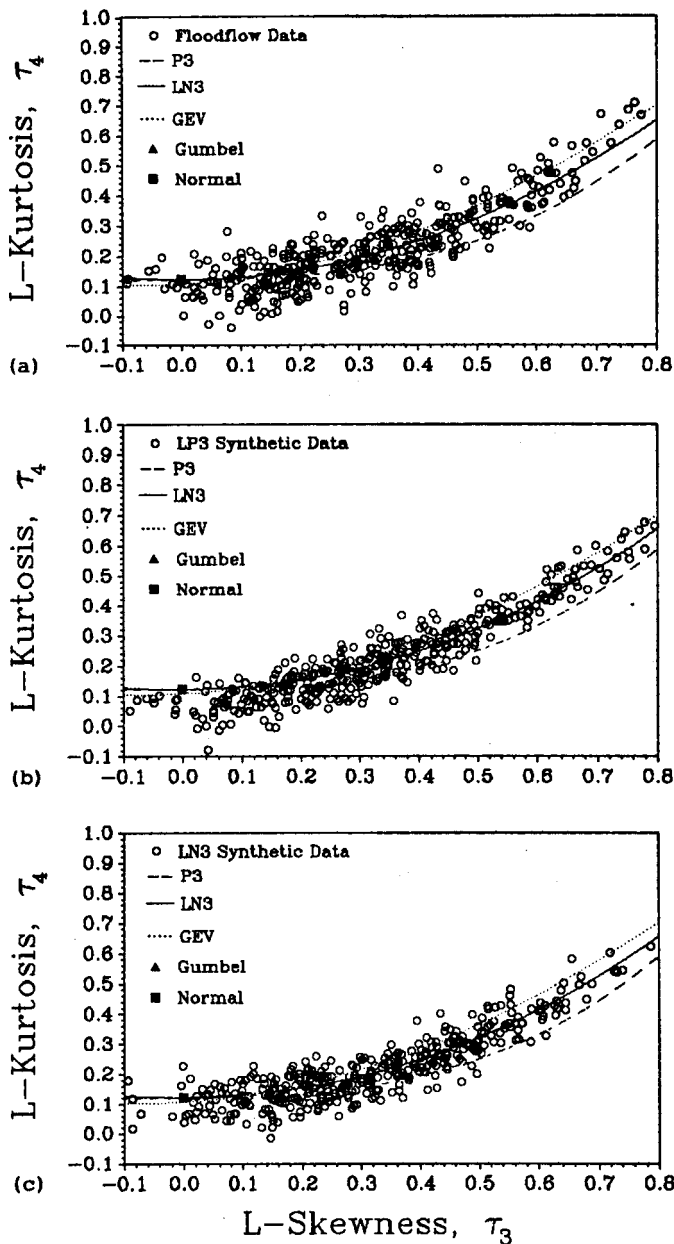


FIG. 3. *L*-Moment Diagrams of: (a) Observed Annual Maximum Floodflow Data; (b) Synthetic Log Pearson Type 3 (LP3) Data; (c) Synthetic Three-Parameter Log-normal (LN3) Data

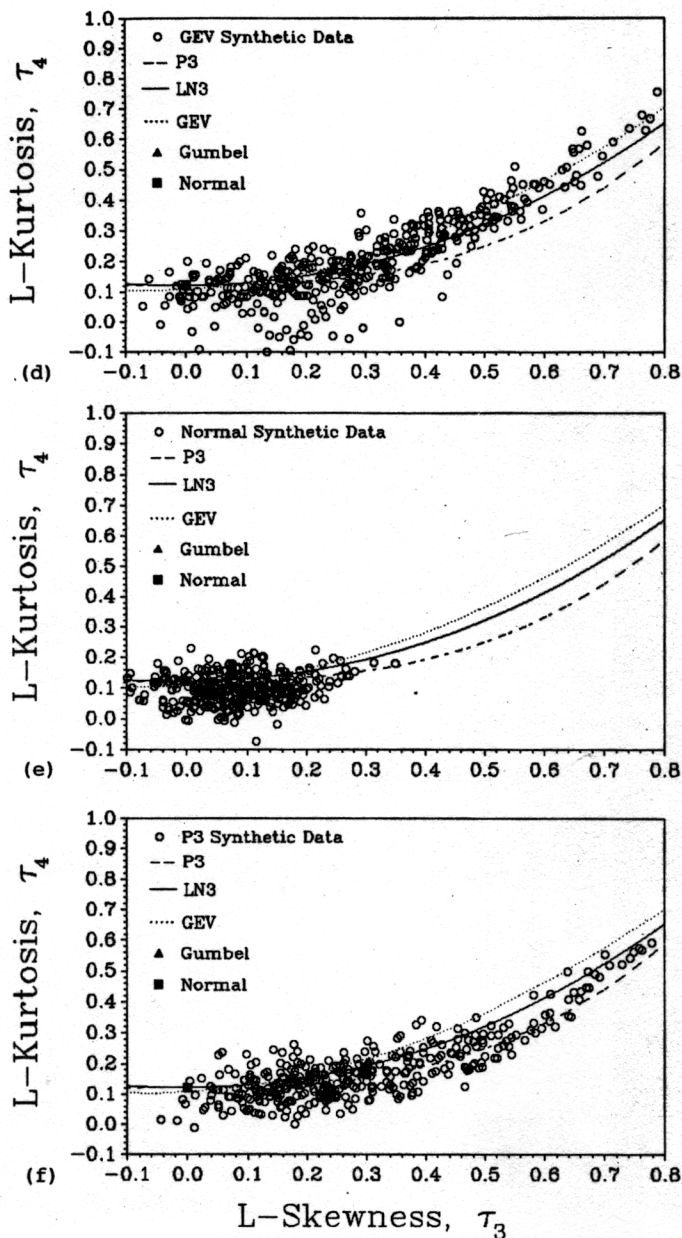


FIG. 3. L-Moment Diagrams of: (d) Synthetic Generalized Extreme Value (GEV) Data; (e) Synthetic Normal Data; and (f) Synthetic Pearson Type 3 (P3) Data

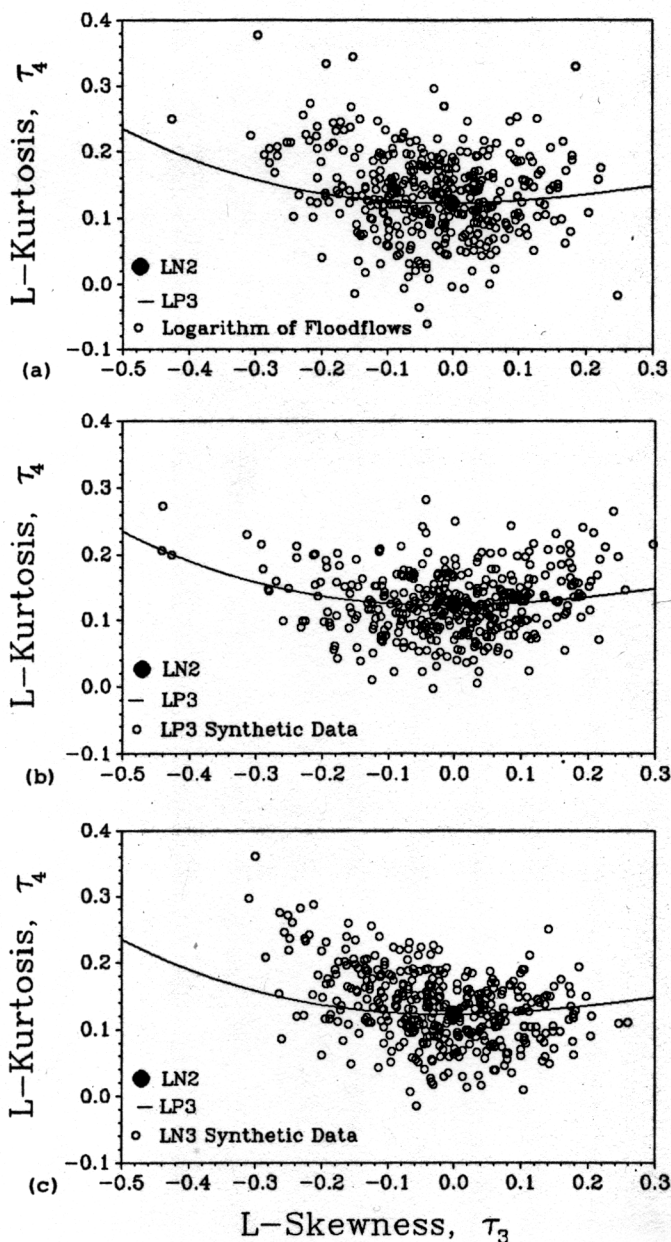


FIG. 4. *L*-Moment Diagrams of: (a) Logarithms of Observed Annual Maximum Floodflows; (b) Log Pearson Type 3 (LP3) Synthetic Data; and (c) Three-Parameter Lognormal (LN3) Synthetic Data

using logarithms of the annual maximum flood flows. For comparison, we plot the theoretical relationship for P3 data, which provides a test on whether the distribution of the logarithms of flood-flow data resembles a P3 distribution, which is equivalent to checking whether the distribution of the flood flows resembles an LP3 distribution. Since the average skewness of the logarithms in this region is approximately zero, it is not surprising that the data cluster around the value $\tau_4 = 0.1226$ and $\tau_3 = 0$, which corresponds to the population values for an LN2 distribution. Here again, in Fig. 4(b and c), we compare synthetic LP3 and synthetic LN3 data, respectively. Similar to Fig. 3, Fig. 4 documents that both synthetic LP3 and synthetic LN3 data behave much like the original flood-flow data.

From all our comparisons in Figs. 3 and 4, we conclude that observed flood-flow samples are well approximated by LP3, LN2, LN3, and GEV probability distributions, yet are poorly approximated by a P3, normal, or Gumbel distribution. Our conclusions are derived from a subjective graphical evaluation of the goodness of fit of alternative distributions. Hosking and Wallis (1993) describe more objective quantitative goodness-of-fit procedures that may be employed in association with *L*-moment diagrams.

NONPARAMETRIC EVALUATION OF FLOOD-FREQUENCY PROCEDURES

The previous *L*-moment diagrams focused on the ability of alternative probability models to approximate the distribution of flood flows; however, those comparisons did not evaluate the ability of alternative methods to provide estimates of design quantiles. In this section we evaluate the performance of alternative models and parameter-estimation schemes in terms of their ability to predict the 100- and 1,000-year flood flows. We repeat a portion of the experiment performed by Beard (1974) and summarized in Table 1 and in IACWD ("Guidelines" 1982), which led to the original uniform flood-frequency guidelines in the United States. The experiment is conceptually simple and focuses on the important question of how well each model and associated parameter-estimation scheme performs in terms of predicting extreme flood flows with a fixed exceedance probability.

Flood-Flow Frequency Methods Evaluated

The experiment begins by estimating the 100- and 1,000-yr flood flow at each of the 383 sites using the following methods.

Log Pearson Type 3 Distribution (LP3)

The method of moments in log space, as recommended by IACWD ("Guidelines" 1982) is always used for estimating the mean and variance of the logarithms. Four different methods are considered for estimating the skew of the logarithms.

1. LP3- $\gamma = 0$: The skew is always set to zero.
2. LP3- G_1 : The at-site skew estimator described in IACWD ("Guidelines" 1982) is used.
3. LP3- G_r : The at-site probability plot correlation coefficient skew estimator described by Vogel and McMartin (1991) is used.
4. LP3- G_w : The weighted skew estimator described in IACWD ("Guidelines" 1982) is used where the regional skew is assumed to be zero with a mean square error of 0.31. Thomas et al. (1992) found that the mean square error associated with four alternate regional estimators of the skew coef-

ficient for this 10-state region [including the regional map skew from IACWD ("Guidelines" 1982)] were all about equal to the mean square error of the sample of 1,059 gaged records, which was 0.31. Their tests failed to reject the hypothesis that the regional population skew is zero. Hence, Thomas et al. (1992) recommend assuming a fixed regional skew of zero for the entire region and using Tasker's (1978) recommended formula [also see IACWD ("Guidelines" 1982), pages 12–13] for obtaining a weighted estimator of the at-site skew coefficient and the regional skew. Here, the mean square error of the regional skew coefficient is assumed to be 0.31. We term this skewness estimator G_w .

Lognormal Distribution

Two procedures are considered. The two-parameter lognormal procedure (LN2) employs maximum likelihood estimators and the three-parameter lognormal procedure (LN3) employs Stedinger's (1980) estimator of the lower bound ξ along with sample estimators of the mean and variance of $y_i = \ln(x_i - \xi)$, where x_i are the observed flood flows. Stedinger (1980) and Stedinger et al. (1993) summarize these procedures.

Generalized Extreme Value Distribution

The generalized extreme value (GEV) procedure and the Gumbel or extreme value type I procedures (GUM) were also examined. These procedures employ unbiased L -moment estimators of each distributions parameters as described in Stedinger et al. (1993).

Expected Probability Adjustment

Most quantile estimators provide almost unbiased estimates of the percentile of interest. Hence, one expects, on average, the estimated 100- and 1,000-yr events to equal their population values. However, an unbiased estimator of the T -yr event will not, in general, be exceeded with an average probability of $p = 1/T$. Beard (1960), Beard (1978), IACWD ["Guidelines" (1982), Appendix 11], Stedinger (1983), Gunasekara and Cunnane (1991), and Stedinger et al. (1993) discuss this issue in greater detail.

For normal and lognormal samples, *Bulletin 17B* [IACWD ("Guidelines" 1982), Appendix 11] provides formulas for the probabilities that an almost unbiased quantile estimator of the T -yr event will be exceeded. For example, for the $T = 1,000$ -yr event, the expected exceedance probability is $0.001(1.0 + 280/N^{1.55})$, and for the $T = 100$ -yr event the expected exceedance probability is $0.01(1.0 + 26/N^{1.16})$, for normal samples. Note that, for the average sample size employed here ($n = 50$), expected exceedance probabilities for $T = 1,000$ and $T = 100$ -yr events are 0.00165 and 0.0128, respectively, instead of 0.001 and 0.01. Although these corrections were derived for the normal and lognormal distributions, they have been recommended by IACWD ("Guidelines" 1982) for use with the LP3 distribution. We employ these corrections for all of the methods considered. Gunasekara and Cunnane (1991) showed that the expected probability correction for normally distributed samples is approximately valid for other distributions.

Experimental Procedure

Using each of the foregoing methods, we counted the number of times an observed annual maximum floodflow exceeded the estimated $T = 100$

and $T = 1,000$ -yr flood flow with and without the use of an expected probability adjustment. The results are reported in Table 2. If one assumes that the 383 sites are independent, and that floods occur independently from one year to the next at each site, then the number of exceedances, X , follows a binomial distribution with mean $E[X] = mp$ and variance $\text{Var}[X] = mp(1 - p)$, where m is the number of independent trials and p is the exceedance probability associated with each event ($p = 1/T$). There are $m = 19,196$ site-years of data (or independent trials) across the 383 sites. Hence, on average, over many such experiments, one expects to observe approximately $E[X] = 19$ and $E[X] = 192$ exceedances of the 1,000- and 100-yr events, respectively. Furthermore, since X follows a binomial distribution, we can estimate the 95% likely interval as the range of values one might expect (95% of the time) over repeated experiments of this kind. The 95% intervals ($x_{0.025}$, $x_{0.975}$) are reported in Table 2 and are found from

$$0.95 = \sum_{x=x_{0.025}}^{x_{0.975}} \binom{m}{x} p^x (1-p)^{m-x} \dots\dots\dots (2)$$

where $m = 19,196$ and $p = 0.01$ and 0.001 for the 100- and 1,000-yr events, respectively.

Results

Table 2 reports the number of observed flood flows, out of the entire sample of $m = 19,196$ site-yr of flood flows that exceeded the 100- and 1,000-yr events. We document the results with and without the expected probability adjustment corresponding to eight different methods. The only procedure that leads to observed exceedances that always fall within the 95% likely interval, both with and without the expected probability adjustment, is the LP3- G_w method, the method recommended by IACWD

TABLE 2. Number of Observed Flood Flows that Exceed 100- and 1,000-Yr Flood Flow Computed from Each Station's Complete Record Using Eight Different Methods^a

Method (1)	NUMBER OF EXCEEDANCES			
	$T = 100$		$T = 1,000$	
	No expected probability adjustment (2)	Expected probability adjustment (3)	No expected probability adjustment (4)	Expected probability adjustment (5)
LP3- $\gamma = 0$	176 ^a	217 ^a	25 ^a	34
LP3- G_i	244	292	75	88
LP3- G_r	164	210 ^a	45	52
LP3- G_w	172 ^a	216 ^a	18 ^a	26 ^a
LN2	165 ^a	207 ^a	21 ^a	33
LN3	172 ^a	229	8	16 ^a
GEV	152	215 ^a	11 ^a	13 ^a
GUM	377	443	132	161
Theoretical expectation	192	192	19	19
95% likely interval	(165, 219)	(165, 219)	(11, 28)	(11, 28)

^aResults fall inside 95% likely interval.

("Guidelines" 1982). The only methods leading to observed exceedances that always fall within the 95% likely interval with the expected probability adjustment are the LP3- G_w and the GEV procedures. Since an expected probability adjustment is required to reproduce the theoretical expectation and to produce results that fall within the 95% likely interval of the number of exceedances we place greater weight on the results with the expected probability adjustment. Unfortunately, an exact expected probability adjustment is unavailable for the GEV and LP3 methods, hence our results for those cases are approximate.

For the $T = 1,000$ -yr event, the LP3- G_w , LN3, and GEV procedures are the only methods that produce exceedances that fall within the 95% likely interval when the expected probability adjustment is made. This result agrees with our previous conclusions based on L -moment diagrams, which showed that these distributions seemed to behave most like the observed flood-flow samples. For the $T = 100$ -yr event, similar results are obtained since now the LP3- $\gamma = 0$, LP3- G_r , LP3- G_w , LN2, and GEV procedures are the only methods that produce exceedances that fall within the 95% likely interval when the expected probability adjustment is made.

Vogel and McMartin (1991) showed that the at-site PPCC skew estimator G_r should always perform as well or better than the at-site skew estimator G_1 . This result is verified in Table 2, which reveals that the LP3- G_r procedure was always an improvement over the LP3- G_1 procedure.

Unfortunately, experiments such as the one reported in Table 2 can never be definitive, because actual flood-flow samples are cross correlated in space. Cross correlation reduces the amount of regional experience, implying that one needs more basins to obtain convergence between the theoretical number of exceedances and the observed number of exceedances. Cross correlation of the flood-flow samples implies that there are effectively fewer than 19,196 independent samples leading to a wider 95% likely interval than is reported in Table 2.

Interestingly, Gunasekara and Cunnane (1992) reached almost identical conclusions to ours by repeating Beard's experiment with synthetic flood-flow data. Gunasekara and Cunnane (1992) generated 40-yr synthetic flood-flow traces at 300 artificial sites from seven different parent probability distributions. They employed 13 different model/parameter estimation procedures to estimate the 10-, 100-, and 1,000-yr flood flow at each site. They found that for both the 100- and 1,000-yr events, the regional LP3- G_w and the at-site GEV procedures employed here reproduced the expected number of exceedances better than any of the procedures they evaluated. Those results are identical to ours except that we also found that the LN2 and LN3 procedures performed well. Gunasekara and Cunnane (1992) did not consider the LN3 procedure, and the reason they rejected the LN2 procedure was probably due to the fact that the logarithms of their artificial samples exhibited nonzero skew, unlike the logarithms of observed flood flows in this study.

CONCLUSIONS

The primary objective of this study was to evaluate the ability of several distributional alternatives for modeling annual maximum flood flows in a 10-state region of the southwestern United States. L -moment diagrams revealed that the log Pearson type 3 (LP3), lognormal (LN2), three-parameter lognormal (LN3), and generalized extreme value (GEV) distributions all

provide equally acceptable approximations to the observed distribution of flood flows at 383 sites in the region depicted in Fig. 1.

To assess the ability of alternative flood-frequency models and parameter-estimation schemes to estimate design quantiles, we also repeated a portion of the original nonparametric experiment performed by Beard (1974), which led to the IACWD ("Guidelines" 1982) mandate to recommend the LP3- G_w procedure in the United States. Those experiments also document that the LP3- G_w , LN2, LN3, and GEV procedures are all equally acceptable procedures for modeling flood flows in this region. These conclusions are surprisingly similar to the results of a recent study by Gunasekara and Cunnane (1992) that employed synthetic flood-flow data instead of flood-flow observations.

It is satisfying to report that all our evaluations consistently reach similar conclusions. Yet these results do not imply that the current IACWD ("Guidelines" 1982) guidelines are satisfactory. Recent studies by Potter and Lettenmaier (1990) and others demonstrate that regional index-flood procedures for the GEV distribution with L -moment estimators should be more accurate and more robust than the type of at-site procedures described here and recommended by IACWD ("Guidelines" 1982). Stedinger et al. (1993) summarize index-flood procedures. Nevertheless, this study reveals that index-flood procedures need not be restricted to the GEV distribution because the LN2, LN3, and LP3 distributions are suitable alternatives, at least in the southwestern United States.

Finally, we emphasize, as Potter (1987) and others did that improvements in regional flood-frequency analysis are usually derived from at-site procedures because the single-site (at-site) model lies at the heart of all regional procedures. Since this study focuses on the adequacy of various distributional hypotheses, treating each site independently, our analyses could not reveal any information regarding the adequacy of regional procedures that exploit the GEV, LN2, LN3, or LP3 distributions. We suggest, however, that future studies that seek to develop improved regional flood frequency procedures consider using the GEV, LN2, LN3, LP3, and possibly other procedures as the basis for such methods.

ACKNOWLEDGMENTS

We are grateful to Michael Hynes for his assistance with the computer programming. We are also grateful to J. R. Wallis and L. R. Beard for their comments on an earlier version of this manuscript. This research was supported by the University of Melbourne and Tufts University, and it was completed while the first writer was on sabbatical leave in Melbourne, Australia.

APPENDIX. REFERENCES

- Beard, L. R. (1960). "Probability estimates based on small normal-distribution samples." *J. Geophys. Res.*, 65(7), 2143–2148.
- Beard, L. R. (1974). "Flood flow frequency techniques." Center for Research in Water Resources, Univ. of Texas at Austin, Austin, Tex.
- Beard, L. R. (1978). "Impact of hydrologic uncertainties on flood insurance." *J. Hydr. Div.*, ASCE, 104(11), 1473–1483.
- Benson, M. A. (1968). "Uniform flood-frequency estimating methods for federal agencies." *Water Resour. Res.*, 4(5), 891–908.
- Chowdhury, J. U., Stedinger, J. R., and Lu, L. (1991). "Goodness-of-fit tests for regional GEV flood distributions." *Water Resour. Res.*, 27(7), 1765–1776.

- Cunnane, C. (1989). "Statistical distributions for flood frequency analysis." *Operational Hydrol. Rep. No. 33*, WMO-No. 718, World Meteorological Organization, Geneva, Switzerland.
- "Guidelines for determining flood flow frequency." (1967). *Bull. 15, Hydrology Commission*, U.S. Water Resources Council, Washington, D.C.
- "Guidelines for determining flood flow frequency." (1982). *Bull. 17B, Hydrology Subcommittee*, OWDC, U.S. Geological Survey, Reston, Va.
- Gunasekara, T. A. G., and Cunnane, C. (1991). "Expected probabilities of exceedance of normal flood distributions." *J. Hydro.*, 128, 101–113.
- Gunasekara, T. A. G., and Cunnane, C. (1992). "Split sampling technique for selecting a flood frequency analysis procedure." *J. Hydro.*, 130, 189–200.
- Hosking, J. R. M. (1990). "L-moments: Analysis and estimation of distributions using linear combinations of order statistics." *J. Royal Statistical Soc., B*, 52(2), 105–124.
- Hosking, J. R. M. (1991a). "Approximations for use in constructing L-moment ratio diagrams." *Res. Rep. RC-16635*, IBM Research Division, T. J. Watson Research Center, Yorktown Heights, N.Y.
- Hosking, J. R. M. (1991b). "Fortran routines for use with the method of L-moments, version 2." *Res. Rep. RC-17097*, IBM Research Division, T. J. Watson Research Center, Yorktown Heights, N.Y.
- Hosking, J. R. M., and Wallis, J. R. (1987). "An index-flood procedure for regional rainfall frequency analysis." *EOS Trans., American Geophysical Union*, 68, 312.
- Hosking, J. R. M., and Wallis, J. R. (1993). "Some statistics useful in regional frequency analysis." *Water Resour. Res.*, 29(2), 271–281.
- Jain, D., and Singh, V. P. (1987). "Estimating parameters of EVI distribution for flood frequency analysis." *Water Resour. Bull.*, 23(1), 59–71.
- Potter, K. W. (1987). "Research on flood frequency analysis: 1983–1986." *Rev. Geophys.*, 25(25), 113–118.
- Potter, K. W., and Lettenmaier, D. P. (1990). "A comparison of regional flood frequency estimation methods using a resampling method." *Water Resour. Res.*, 26(3), 415–424.
- Stedinger, J. R. (1980). "Fitting lognormal distributions to hydrologic data." *Water Resour. Res.*, 16(3), 481–490.
- Stedinger, J. R. (1983). "Design events with specified flood risk." *Water Resour. Res.*, 19(2), 511–522.
- Stedinger, J. R., Vogel, R. M., and Foufoula-Georgiou, E. (1993). "Frequency analysis of extreme events—Chapter 18," *Handbook of hydrology*, D. Maidment, ed., McGraw-Hill Book Co., New York, N.Y.
- Tasker, G. C. (1978). "Flood frequency analysis with a generalized skew coefficient." *Water Resour. Res.*, 14(2), 373–376.
- Thomas, B. E., Hjalmanson, H. W., and Waltemeyer, S. D. (1992). "Methods for estimating magnitude and frequency of floods in the southwestern U.S." *Open File Report*, U.S. Geological Survey, Tucson, Ariz.
- Vogel, R. M. (1986). "The probability plot correlation coefficient test for the normal, lognormal, and Gumbel distributional hypotheses." *Water Resour. Res.*, 22(4), 587–590.
- Vogel, R. M., and Fennessey, N. M. (1993). "L-moment diagrams should replace product moment diagrams." *Water Resour. Res.*, 29.
- Vogel, R. M., and Kroll, C. N. (1989). "Low-flow frequency analysis using probability-plot correlation coefficients." *J. Water Resour. Plng. and Mgmt.*, ASCE, 115(3), 338–357.
- Vogel, R. M., McMahon, T. A., and Chiew, F. H. S. (1993). "Floodflow frequency model selection in Australia." *J. Hydro.*, (Apr.).
- Vogel, R. M., and McMartin, D. E. (1991). "Probability plot goodness-of-fit and skewness estimation procedures for the Pearson type 3 distribution." *Water Resour. Res.*, 27(12), 3149–3158.
- Wallis, J. R. (1988). "Catastrophes, computing and containment: Living in our restless habitat." *Speculation in Sci. Technol.*, 11(4), 294–315.