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Global streamflows – Part 2: Reservoir storage–yield performance

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Summary This is the second of three papers describing hydrologic analyses of monthly and annual streamflow data for a global set of 729 unregulated rivers with at least 25 years of continuous data. Capacity estimates of hypothetical reservoirs are computed for each river using the Sequent Peak Algorithm (SPA), Behaviour analysis and the Gould–Dincer Gamma procedure. Based on SPA and Behaviour procedures, empirical relationships relating reservoir capacity and yield were developed which accounted for 87–96% of the variance in capacity estimates across the global data set of monthly streamflows. The theoretical Gould–Dincer Gamma procedure was also shown to be a suitable technique to estimate reservoir capacity–yield relationships. It is noted that the three procedures are based on different definitions of supply reliability.

Continental variations of the estimated capacities under equivalent conditions are examined. Reservoir performance measures – reliability, resilience and dimensionless vulnerability – are computed and their continental variations described. As a result of these analyses a number of differences are noted about the performance of reservoirs across continental regions. For example, the median continental reservoir capacity as a ratio of the mean annual flow varied by a factor of 9 across the continental regions. Furthermore, based on the reliability metric as an example of reservoir performance, high reliabilities occur in the South Pacific and Europe, slightly less reliable systems in North and South America, lower still in northern Africa, followed by Australia and the lowest value in southern Africa. This distribution follows inversely with the coefficient of variation of annual streamflow between continents.

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Introduction

This is the second in a series of papers utilizing a large and unique data base of global streamflow data for 729 rivers. Over the years the data set has been assessed in detail and we are confident that the data we have used in the analyses reported herein are representative of world-wide unimpacted rivers. As reported elsewhere (Peel, 1999; Peel et al., 2004) considerable effort has been undertaken to ensure that the data are free of errors and are not affected by major water withdrawals nor from upstream reservoir regulation. The first paper in this series examines the streamflow characteristics of unregulated global rivers. The third paper deals with the variations of the hydrologic characteristics as a function of country and climate type of both unregulated and hypothetically regulated rivers. In this paper, reservoir capacity–yield relationships are developed and reservoir performance metrics interpreted for hypothetical reservoirs located on each river.

Hydrologic studies at a continental level are designed to identify those continents that show characteristics which are different from those observed for other continents. This is important in assessing how continents as a whole will behave under climate change (IPCC, 2001) and, as noted elsewhere (McMahon, 1998), in transposing hydrologic models from one continent to another. Furthermore, because hydrology is a major driver of most aquatic fauna ecosystems (Poff et al., 2006), hydrologic differences between continents explain some of the major differences in assemblages of aquatic fauna observed between some continents (Poff et al., 2006). The main purpose of this paper is to establish the size of hypothetical storages necessary to regulate global rivers to specified levels of service, to assess their performance under a range of conditions and to examine variations between continents of three reservoir performance metrics.

Except for the studies by McMahon (1982) and McMahon et al. (1992), we know of no literature that deals with reservoir storage–yield characteristics at the global scale. The important early reports by Kalinin (1971), Korzun et al. (1974), UNESCO (1978), Baumgartner and Reichel (1975) examine *inter alia* the world water balance, parameters of the annual streamflow series and availability of water, but do not consider reservoir storage–yield performance measures, or the impact of seasonal flow characteristics on the storage–yield relation.

Because the capacity of a reservoir is not only a function of the characteristics of the inflow hydrology, but also of the targeted draft (also referred to as yield) and of a performance measure (usually the reliability of being able to meet the demand), it is helpful in our analysis to restrict the range of characteristics that we will examine. As noted in McMahon et al. (2007a) target drafts (expressed as a percentage of mean annual inflow) vary widely across the world. Based on data for Australia, South Africa and the United States we have chosen in this paper to adopt 75% draft for most analyses. For reservoir storage–yield procedures that are based on reliability, a 95% reliability value of meeting demand was adopted. It is also noted in McMahon et al. (2007a) that the median capacity of large reservoirs in Australia and South Africa is equal to 1.28× and 1.22× mean an-

nual flow, respectively. Where it was necessary to specify reservoir capacity as an input to an analysis, we have adopted a capacity equal to the mean annual flow. Although this maybe considered too large for regions with low streamflow variability, it was necessary to restrict the range of analysis proposed, given the breadth of the analyses that we planned to carry out across the three papers.

Following this introduction, section 'Annual and monthly streamflow data' describes briefly the streamflow data used and their general characteristics. The next section discusses the application of four reservoir storage procedures to the global data set of 729 rivers. The procedures considered are the Sequent Peak Algorithm (SPA), the Behaviour analysis, the Gould–Dincer Gamma (G–DG) method and the Extended Deficit Analysis (EDA). In section 'Continental variations of reservoir storage estimates', we discuss the continental variations in the reservoir storage estimates. This is followed in section 'Reservoir storage performance' by an examination of global variations in reservoir performance. Relevant conclusions are drawn in section 'Conclusions'.

Annual and monthly streamflow data

The global streamflow data used in this paper consist of continuous monthly and equivalent annual time series for 729 unregulated rivers with 25 years or more of data. The locations of the rivers are shown in Fig. 1, which suggests the data are reasonably distributed world-wide although there are regional areas poorly represented including Central America (except Panama), equatorial South America, the Middle East, central, south and south-east Asia. These data are a sub-set of a larger cohort of streamflows consisting of 1221 rivers, details for which are discussed in the first of this series of three papers (McMahon et al., 2007c). The general hydrologic characteristics of the rivers adopted herein are summarized in Table 1.

We observe in Table 1 the rivers cover an extremely wide range of hydrologic characteristics and, from the point of view of reservoir capacity analysis, the especially large range of annual Cv should be noted because it is an important variable in reservoir storage–yield analysis as discussed later.

Table 2 displays the hydrologic characteristics in three groups – median values by continent, between Australia–southern Africa (ASA) and the rest of the world (RoW), and the results of all rivers combined. The continental definitions used in this paper follow Peel et al. (2004), which are based on McMahon et al. (1992).

Median continental values for the four parameters based on historical annual streamflows – mean annual runoff in mm (MAR), coefficient of variation (Cv), coefficient of skewness (γ) and lag-one serial correlation (ρ) – are listed in Table 2. (It should be noted that the variable MAR (in mm) is used where comparisons between catchments are made, however, we also use the variable mean annual flow (in 10^6 m^3) in reservoir storage related analysis.) We note in the table the ranking of the median values of the MAR (mm) as follows: southern Africa (77), northern Africa (134), Asia (232), Australia (236), South America (327), Europe (435), North America (504) and South Pacific (1244).

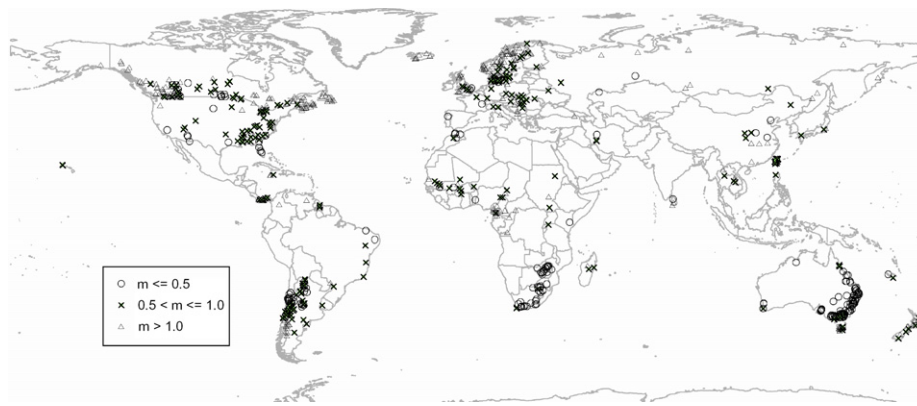


Figure 1 Locations of rivers with 25 or more years of continuous annual and monthly historical streamflow data and values of drift for 75% target draft (drift is defined in section 'Reservoir storage estimates based on global data').

Table 1 The key hydrologic features of the 729 rivers used in this paper

	Area (km ²)	<i>N</i> (years)	MAR (mm)	Cv	γ	ρ
Median	1370	38	354	0.314	0.561	0.119
Maximum	3,475,000	185	5370	2.97	6.14	0.895
90th percentile	55,100	65	1444	0.870	1.724	0.394
10th percentile	132	27	50.9	0.159	−0.069	−0.108
Minimum	0.360	25	0.373	0.0679	−2.22	−0.482

Area is catchment area (km²), *N* is the number of years of continuous annual or monthly data, MAR is the mean annual runoff (mm), Cv is the coefficient of variation of annual flows, γ is the coefficient of skewness of annual flows and ρ is lag-one serial correlation coefficient of annual flows.

The magnitude and rank of these values are different from estimates in previously noted continental water balance reports because the values in Table 2 do not reflect overall continental spatial variations but rather they reflect the median values of the rivers in the global data set.

The statistics MAR and Cv in Table 2 are the two key variables determining the yield of a surface water resources system. Southern Africa exhibits the highest annual Cv being 3.7 times more variable than the least variable region, the rivers from the South Pacific. The Australian continent also exhibits very high Cv values with a median of 0.68. Relative to the global median value of 0.31, the South American rivers also have an above median annual Cv of 0.37. The remaining continental areas – Asia 0.29, northern Africa 0.29, North America 0.25 and Europe 0.24 all have below median Cvs. Based on the 10th and 90th percentile ranges it is noted in Table 2 that the rivers of the Australian continent exhibit the largest range of Cv. Both southern Africa and South America also have a wide range of Cvs with the smallest range being found in Europe and South Pacific. The wide range of Cvs combined with a range of MARs for Australian rivers makes the Australian continent ideal for studying the broad spectrum of surface hydrology. The relatively larger values of Cv for Australia and southern Africa have been attributed to several factors including temperate evergreen vegetation in these regions compared to the rest of the world which, in the main, is characterized by deciduous temperate flora (Peel et al., 2001, 2004).

The rivers of northern Africa, on the whole, exhibit lag-one serial correlations that are approximately 3.5 times larger than the global median. Furthermore, the 10th–90th range of ρ is much larger than elsewhere. Such high autocorrelations have major implications for reservoir storage yield values. To account for such a high value of 0.4, reservoir capacity for carry-over storage requirements needs to be 2.3 times larger than a reservoir with $\rho = 0$ (see discussion in McMahon et al., 2007b).

The ASA–RoW comparisons in Table 2 have been discussed in the first paper in this series (McMahon et al., 2007c). Suffice to note here the much larger median annual Cv and coefficient of skewness for the ASA rivers compared with those in RoW. This observation should not be interpreted to suggest that areas of high Cv will not be found in RoW. On the contrary, there are many regions, for example in south-west of the United States, where large reservoirs are required to counter the high variability in streamflow (see, for example, Vogel et al., 1998). The impact of Cv on reservoir capacity is discussed in the next section.

Reservoir storage estimates based on global data

An appropriate setting to begin this global review is to examine the effect of annual Cv on reservoir storage

Table 2 Characteristics of rivers with 25 years or more of annual streamflows

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Number of rivers	Median length of historical data (years)	Median catchment area (km ²)	Median MAR (mm)	Median annual Cv	Median annual γ	Median annual ρ
Asia	58	34 (26–63) ^a	19,130 (160–1,488,000)	232 (72–692)	0.29 (0.14–0.55)	0.47 (–0.12–1.22)	0.081 (–0.16–0.38)
Australia	114	36 (27–68)	548 (52–7150)	236 (43–1434)	0.68 (0.26–1.22)	1.12 (0.28–2.28)	0.11 (–0.12–0.33)
Europe	158	47 (29–69)	772 (76–17,600)	435 (191–1238)	0.24 (0.15–0.41)	0.41 (–0.045–0.97)	0.10 (–0.052–0.34)
Northern Africa	40	30 (25–66)	65,270 (4020–450,000)	134 (40–483)	0.29 (0.16–0.61)	0.22 (–0.32–1.26)	0.40 (–0.0052–0.75)
North America	195	39 (28–67)	1700 (193–50,300)	504 (101–1523)	0.25 (0.15–0.53)	0.39 (–0.12–1.07)	0.13 (–0.11–0.34)
Southern Africa	48	33 (27–51)	795 (73–18,600)	77 (13–421)	0.82 (0.36–1.09)	1.44 (0.87–2.39)	0.11 (–0.14–0.29)
South America	100	40 (29–52)	2746 (434–46,400)	327 (31–1984)	0.37 (0.21–0.89)	0.72 (–0.049–2.45)	0.13 (–0.12–0.45)
South Pacific	16	29 (26–32)	473 (89–1393)	1244 (713–2217)	0.22 (0.16–0.33)	0.32 (–0.34–0.94)	0.072 (–0.20–0.19)
Australia–southern Africa	162	36 (27–61)	566 (62–9170)	160 (27–1280)	0.71 (0.28–1.12)	1.21 (0.43–2.32)	0.11 (–0.13–0.33)
Rest of world	567	39 (27–66)	1709 (160–74,600)	406 (79–1476)	0.28 (0.15–0.54)	0.42 (–0.12–1.26)	0.12 (–0.10–0.42)
All rivers	729	38 (27–65)	1370 (132–55,100)	354 (51–1434)	0.31 (0.16–0.87)	0.56 (–0.069–1.72)	0.12 (–0.11–0.39)

^a Values in parenthesis are 10th and 90th percentile.

through the drift metric. The term drift (m) was adopted by Pegram (1980) from Troutman (1976) and is also known as the standardised net inflow (Hurst, 1951) and defined by

$$m = \frac{1 - \alpha}{Cv} \quad (1)$$

where α is the draft ratio (or reservoir yield) expressed as a ratio of mean annual inflow and Cv is the coefficient of variation of annual inflows. Hazen (1914) also used this equation in his analysis of reservoir capacities in the northeastern United States. For much of our analysis we adopt $\alpha = 0.75$ and have tabulated median and range of values of drift for each continent in Table 5 (details in columns 4–6 are discussed in section 'Continental variations of reservoir storage estimates'). As noted in Eq. (1) values of drift vary inversely with Cv . As a first approximation, values of drift >1 imply within-year reservoir storage (Vogel and Bolognese, 1995; Vogel et al., 1999) and under this assumption we observe for $\alpha = 0.75$ approximately 50% of the rivers in Europe, North America and the South Pacific region could be assumed to spill every year.

Sequent Peak Algorithm

The capacity of many reservoirs world-wide has been estimated by the graphical mass curve (Rippl, 1883) procedure or, more recently, by its automated form (Thomas and Burden, 1963) known as the traditional Sequent Peak Algorithm (SPA). For detailed descriptions of SPA and its variations, readers are referred to Thomas and Burden (1963), Lele (1987), Adeloje and Montaseri (1998) and McMahon and Adeloje (2005). The SPA computes the minimum required reservoir capacity to meet a target draft for a failure-free operation of an initially full storage reservoir, over the historical streamflow record.

Fig. 2 illustrates the results of applying SPA to the rivers in the global data set for 50% and 90% target drafts based on monthly data and relating the storage estimates to the standard deviation of annual inflows. The relationships are very strong for both drafts with more than 92% of the variance being accounted for by the standard deviation of annual flows. Fig. 2 shows that, in general, the required reservoir capacity for 90% target draft is approximately $4.0/0.8 = 5$ times that for 50% draft.

We explored the best relationship for the global rivers between SPA storage capacity (based on monthly flows) and draft, record length, mean annual flow, standard deviation of annual flows, annual coefficient of skewness and annual auto-correlation. At the outset, it should be noted that the relationships will be for hypothetical reservoirs ignoring reservoir net evaporation. Readers wishing to pursue this and similar practical issues are referred to relevant texts, for example McMahon and Adeloje (2005). Table 3 sets out the cross-correlation matrix between the key potential variables, and it is noted that all eight variables (MAF, annual σ , annual Cv , annual γ , annual ρ , m , N , and catchment area) were found to correlate with SPA (75% draft) at the 5% level of significance. However, the correlations among five of the variables (Cv , γ , ρ , m and N) are very weak.

To establish a generalized relationship between SPA storage estimates and the key streamflow variables

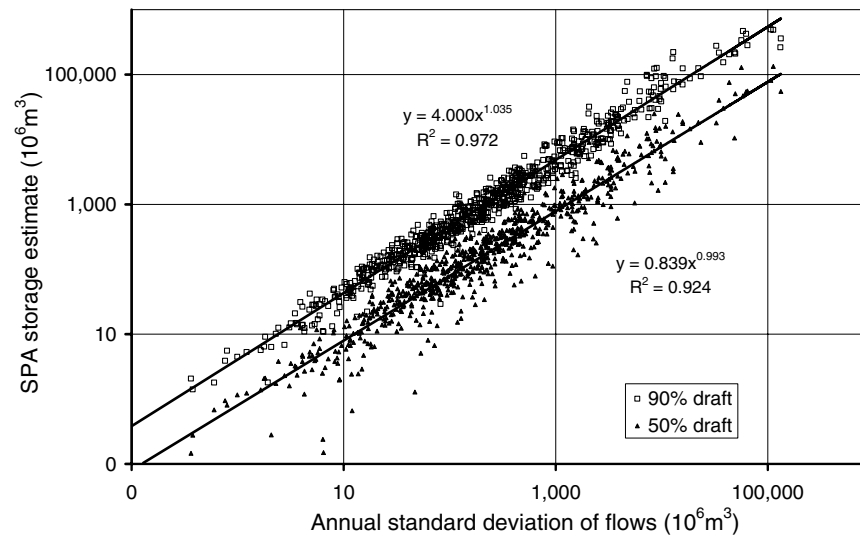


Figure 2 Sequent Peak storage estimates (based on monthly streamflow data) versus standard deviation of annual flows. The equation is based on weighted least squares.

Table 3 Cross-correlation matrix (as R^2) of pairs of variables: SPA, mean annual flow, standard deviation of annual flows, coefficient of skewness of annual flows, auto-correlation of annual flows, historical record length and catchment area^a

Variable	MAF	Annual σ	Annual Cv	Annual γ	Annual ρ	Annual m	N	Catchment area
Monthly SPA (75% draft)	0.593	0.714	0.011	0.015	0.037	0.049	0.024	0.532
MAF	1	0.932	0.017	0.006	0.009	0.088	0.014	0.700
Annual σ		1		0.008	0.017	0.068	0.016	0.701
Annual Cv			1	0.642	0.0005	0.528	0.007	0.013
Annual γ				1	0.002	0.346	0.002	0.004
Annual ρ					1	0.0008	0.031	0.029
Annual m						1	0.004	0.070
N							1	0.013
Area								1

^a Values given in italics are not statistically significant at the 5% level.

(following Vogel and Stedinger (1987) we adopted the mean, standard deviation and auto-correlation of annual flows, and the length of the historical record, and added the coefficient of skewness to account for the non-normality of flows), we used a weighted least squares regression (WLS) analysis but restricted the analysis to the range 30–80% drafts in 10% increments and for SPA values greater than zero (a total of 4293 storage estimates are available and were used to develop WLS regression equations). The weights in the WLS were based on the historical record available for each river. The final equation was of the form

$$\text{MonSPA} = a\mu^b\sigma^c\gamma_{\text{mod}}^dD^eN^f \quad (2)$$

where MonSPA is the storage estimate (10^6 m^3) based on monthly flows, D is the targeted draft (10^6 m^3), μ and σ are, respectively, the mean and the standard deviation of annual inflows (10^6 m^3), and γ_{mod} is the coefficient of skewness of annual flows in which zero and negative skewness is set to 0.001 and N is the length of historical data (years). The latter variable which was statistically significant at 5%

level was added to the regression as SPA capacity estimates are a function of record length. Values of the coefficients a , b , c , d , e and f are listed in Table 4, columns 4, 5, 6, 7, 8 and 12, respectively, along with the regression results including the standard error of estimate (%) (SEE), the R^2 adjusted value (RSq_{adj}) and R^2 predicted value (RSq_{pred}). The results of two statistical tests are also presented in the table. The first examines the collinearity of the independent variables based on the variance inflation factor (Montgomery and Peck, 1982) and the second checks whether the residuals are normally distributed using the Ryan–Joiner probability plot correlation statistic (Ryan and Joiner, 1976); these appear in columns 16 and 17.

The results of calibrating Eq. (2) are listed as model 1 in Table 4. The following points are noted about the analysis and results. Firstly, ρ , the lag-one serial correlation coefficient, is not included in the model as it is not statistically significant at the 5% level. Although auto-correlation is important for an at-site analysis and its range across the global data set is large (ρ from -0.49 to 0.90), it has little effect on the regression when compared

Table 4 Regression coefficients and statistics for models to estimate reservoir capacities using Eqs. (2) and (4)–(6)

Model	Data based on drift	N	Constant (10^6 m^3)	$\ln \mu$	$\ln \sigma$	$\ln \gamma_{\text{mod}}$	$\ln D$	$\ln Z_q$	$\ln m$	$\ln Cv$	$\ln N$	SEE (%)	RSq _{adj} (%)	RSq _{pred} (%)	Collinear ^a	Normal residual ^b
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
<i>Monthly sequent peak algorithm</i>																
1	All	4293	3.894 [*]	−3.073	1.581	−0.071	2.503				0.170	73.8	92.7	92.7	Yes	Reject
2	<1	1632	0.161		1.032				−1.592	−1.087	0.208	44.3	96.5	96.5	No	Reject
3	≥1	2661	0.0987		1.004				−2.016	−1.876	0.132	94.8	90.1	90.0	No	Reject
<i>Monthly Behaviour analysis</i>																
4	All	12,413	1.932	−3.254	1.599	−0.074	2.670	1.445				94.9	89.8	89.9	Yes	Reject
5	<1	4890	0.0719		1.028			1.616	−1.829	−1.067		56.5	94.7	94.7	No	Reject
6	≥1	7523	0.0269		1.003			1.307	−1.987	−2.156		116.2	87.1	87.0	No	Reject

Values in column 4 are in volume units (10^6 m^3) whereas those in columns 5–12 are coefficients of natural logarithm units, since they are exponents in all the equations.

^a Collinearity is checked by the variance inflation factor (VIF). For $VIF > 4$, the regression coefficients are poorly estimated according to Montgomery and Peck (1982).

^b Normality of residuals is tested using the Ryan–Joiner statistic which is a correlation based statistic (Ryan and Joiner, 1976).

* All constants and coefficients for models 1, 2, 4, 5, and 6 are statistically significant at <0.0005 level of significance. Those for model 3 are statistically significant at <0.001 level of significance.

with the range of the other key variables that are included in the regression, e.g., the mean annual flow varies across the 729 rivers from 0.134 to $1,301,000 \times 10^6 \text{ m}^3$. Secondly, the signs of the coefficients are stable and consistent with theory, i.e., capacity is negatively related to the mean and the skewness but positively related to standard deviation, draft and historical record length. Thirdly, the standard error of estimate is relatively large but not excessively so (see discussion in McMahon et al., 2007a) keeping in mind the equation would be used only for screening purposes during preliminary surface water supply assessments. Fourthly, both R^2 values are high indicating high explanatory power and that a single observation does not exert an unusual influence on the estimated model coefficients. Fifthly, the regression shows strong multicollinearity (detected using the variance inflation factor) especially among the variables μ , σ and D . As a result, although the predicted values still have high numerical accuracy, the standard error of estimate (SEE) value will be larger than without collinearity. Sixthly, the Ryan–Joiner probability plot test suggests the residuals are not well approximated by a normal distribution. However, the probability plot (not reproduced here) shows the residuals do closely follow the normal curve and deviate only for the lower 3% of values. Given the large sample size ($n = 4293$), it is reasonable to assume the residuals are approximately normally distributed. Model 1 (Table 4) should not be applied outside the ranges of the variables used in its development, i.e., for 30–80% draft and for $N \geq 25$ years of historical streamflow data. It should be noted that the model is based on 4374 (729 rivers \times 6 drafts) storage values excluding 81 estimates that are zero.

Reservoir behaviour is dominated by either over-year storage where part of the stored water is carried over to the following years or within-year storage (in which spills usually occur annually) and models 2 and 3 in Table 4 reflect this difference. However, the equations should not be used to estimate specifically over-year or within-year storage estimates. Readers seeking such analysis are referred to McMahon et al. (2007a) and Adeloje et al. (2003). The reservoir capacity data (4293 estimates) were divided into 1632 estimates based on the drift <1 and 2661 estimates for drift ≥ 1 . Furthermore, noting the collinearity of model 1, we take the opportunity to develop a model in which the predictor variables are relatively independent, leading to higher stability of model parameters and the model has high predictive power. The model is based on the following structure of the simplest form of the Gould–Dincer procedure (Vogel and McMahon, 1996):

$$S = kz_p^2 m^{-1} \sigma \quad (3)$$

where S is the estimated reservoir capacity, z_p is the standardised normal variate for a given system failure reliability p , m is drift given in (1) and σ is the standard deviation of the annual inflows. Following some preliminary analysis, models 2 and 3 are presented, representing over-year and within-year reservoir systems based on Eq. (3). The models both took the following form:

$$\text{MonSPA} = a\sigma^g m^h Cv^i N^j \quad (4)$$

where g , h , i and j are the regression coefficients as defined in Table 4, columns 6, 10, 11 and 12, respectively, for the two models and the constant term a appears in column 4.

As observed in Table 4, model 2 has signs of the coefficients that are consistent with theory, there is no significant multicollinearity (although C_v appears in two of the variables), the model accounts for more variance than Eq. (2) and SEE is considerably reduced to $\pm 44\%$. As in model 1, the normality test of the residuals was rejected but again failure occurred because the lowest 2% of residuals were underestimated, although the remainder fitted the normal curve satisfactorily.

In contrast to model 2 and model 1, the model 3 performance in terms of R^2 and SEE is less satisfactory, due to the effect of within-year variations in inflow and, hence, reservoir contents are not adequately modelled by annual parameters alone. This model is included here for completeness. It should be noted, however, that for values of $m \sim 1$, models 2 and 3 yield storage estimates that differ, on average, by about 18%.

Behaviour storage estimates

Like SPA, Behaviour or simulation estimates of reservoir capacity are based on applying the continuity equation of storage with relevant inputs to, and outputs from, the storage and determining the required capacity to meet a target draft for given reliability. In this paper, reliability is defined as the ratio of the number of months or years that the reservoir is able to meet the target draft to the total number of months or years of historical inflows used in the simulation. We adopted the so-called Standard Operating Policy which assumes the demand will be satisfied if there is sufficient water, otherwise whatever is available is supplied until the reservoir is empty (McMahon and Adeleye, 2005).

Using the global monthly data set, we computed for each river the required storage capacities using a Behaviour analysis to meet drafts in the range of 30–80% of mean historical flow in 10% increments and for three monthly time reliabilities of 90%, 95% and 98%. Excluding reservoir capacity estimates that were zero, a total of 12,413 estimates were available (4890 for drift < 1 and 7523 for drift ≥ 1) and were used to develop WLS regression equations. Following the approach used in the SPA analysis, the structure of the three Behaviour models were the same as SPA except z_δ replaced N as follows:

$$\text{MonBev} = a\mu^b\sigma^c\gamma_{\text{mod}}^d D^e z_\delta^f \quad (\text{for all data}) \quad (5)$$

$$\text{MonBev} = a\sigma^g m^h C_v^i z_\delta^j \quad (\text{separately for models with drift} < 1 \text{ and } \geq 1) \quad (6)$$

where MonBev is the storage estimate (10^6 m^3) based on monthly flows and z_δ is the standardised normal variate and $100(1 - \delta)\%$ is the probability of the reservoir running dry (failing) in any month.

The coefficients for the three Behaviour based models, designated as models 4, 5, and 6, are listed in Table 4. As expected, there is considerable consistency between these coefficients and those based on SPA. It should also be noted

that for values of $m \sim 1$, storage estimates differ between the Behaviour models 5 and 6 by about 26%.

Summary comment regarding SPA and Behaviour models

Overall, the proportion of variance accounted for by the six models is high (at least 87%) given the wide variability in flows, record lengths and probable errors in the global data set. However, the standard errors are also large for the two overall models (1 and 4) and for models 3 and 6, so one needs to take care in using the equations in more than reconnaissance analyses.

Gould–Dincer Gamma

The Dincer procedure, developed by T. Dincer and reported in McMahon and Mein (1978), is a theoretical approach based on normally distributed annual flows to estimate the mean first passage time to emptiness for a given reservoir capacity and target draft. McMahon et al. (2007b) have designated the procedure as Gould–Dincer Normal to distinguish it from Gamma and Lognormal distributed inflows termed, respectively, Gould–Dincer Gamma and Gould–Dincer Lognormal. The latter two methods were proposed by Gould (1964) and G. Annandale. A detailed description of these three complementary approaches is given in McMahon et al. (2007b). However, as noted in a complementary paper (McMahon et al., 2007c), global annual streamflows are, on the whole, best approximated by a Gamma distribution. In the Gould–Dincer Gamma (G–DG) equation that follows, the basic equation for normally distributed inflows has been transformed to Gamma by the Wilson and Hilferty (1931) transformation. The method is not applicable to within-year storage estimates. The complete equation to account for the first three moments and auto-correlation of annual flows is given as follows (McMahon et al., 2007b):

$$S_{G-DG} = \frac{\sigma^2}{(\mu - D)\gamma^2} \left(\frac{1 - \rho^3}{(1 - \rho^2)^{1.5}} \right)^{-2} \left[\left\{ 1 + \frac{\gamma z_p}{6} \left(\frac{1 - \rho^3}{(1 - \rho^2)^{1.5}} \right) - \frac{\gamma^2}{36} \left(\frac{1 - \rho^3}{(1 - \rho^2)^{1.5}} \right)^2 \right\}^3 - 1 \right]^2 \left(\frac{1 + \rho}{1 - \rho} \right) \quad (7)$$

where S_{G-DG} is the Gould–Dincer Gamma storage estimate, μ and σ are, respectively, the mean and standard deviation of annual flows, γ is the coefficient of skewness of annual flows and ρ is the lag-one serial correlation coefficient, D is draft and z_p is the standardized variate at 100p% probability of non-exceedance of annual flows. The mean, standard deviation, draft and storage size all have the same volume units. In the next section, reservoir capacities estimated using G–DG are compared with estimates based on the Extended Deficit Analysis.

Extended Deficit Analysis

An interesting measure of accumulated streamflow deficit is the Extended Deficit Analysis (EDA) (Pegram, 2000). In this project, we have slightly modified the basic method which

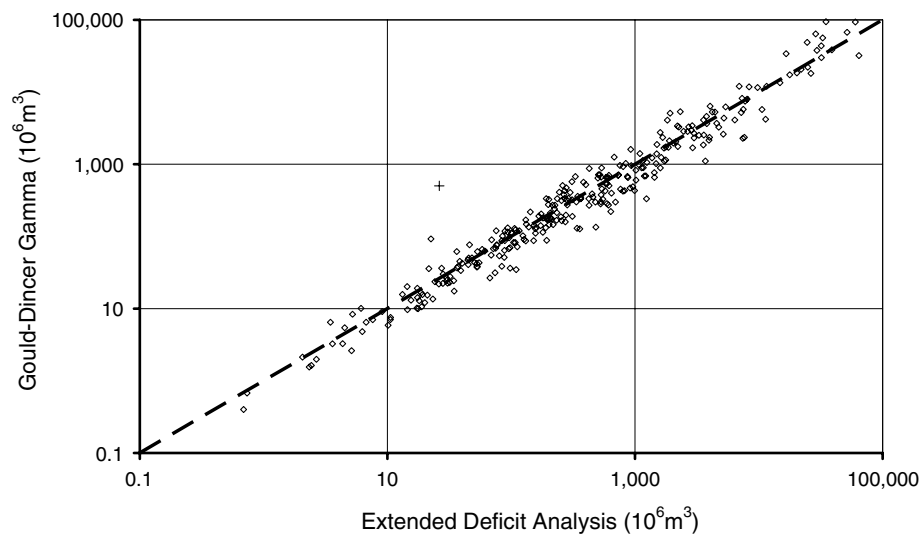


Figure 3 Storage estimates for 75% target draft by Gould–Dincer Gamma procedure (99% annual reliability) compared with estimates based on an Extended Deficit Analysis (1/100 year recurrence) restricting the analysis to $0.4 < m < 1.0$ and $n_{crit} > 1$ year and for rivers with $\gamma < 4.0$. The 1:1 line is dashed.

is described in McMahon and Adeleye (2005). The modification is outlined in McMahon et al. (2007a). The procedure allows one to compute for a given recurrence interval, the deficit from a hypothetical full storage based on annual streamflows. For comparison we have plotted in Fig. 3 reservoir capacity estimates based on the G–DG equation (for 75% targeted draft and 99% annual reliability) versus 1/100 year deficits for 75% draft using EDA. Although the definitions of failure are different for the two methods – G–DG estimates the mean first passage time to emptiness from a full reservoir, whereas EDA estimates the recurrence interval of reservoir deficits – it has been shown in McMahon et al. (2007b) that for the range $0.4 < m < 1.0$ the two failure definitions are approximately equal. Thus for this range and ensuring that the critical period to failure is > 1 year (an assumption in the G–DG method), values are plotted in Fig. 3 and show a satisfactory relationship between the two procedures.

In order to assess the similarity of the capacity estimates by the two procedures, a weighted least squares regression was applied excluding the outlier indicated in Fig. 3 with a cross. The overestimate by G–DG relative to the EDA value for this river is mainly due to an annual auto-correlation value of 0.74. The slope of the WLS regression, without an intercept term, is 0.991 which is significantly different from one at the 95% level of significance but not at the 99% level. Removal of the outlier ensured the normality of the regression residuals but slightly decreased the slope from 0.992 to 0.991. From this analysis we can conclude that within the range adopted in Fig. 3, the G–DG and EDA methods provide similar estimates of reservoir capacity for all practical purposes.

Continental variations of reservoir storage estimates

In Table 5 large variations between continents in values of drift (holding draft constant) are evident. These variations

in drift reflect the variations in the annual C_v between continents. We would expect, therefore, that this effect would be carried through to reservoir storage estimates and is revealed by the high negative correlation (0.85) observed between the median continental SPA estimates (Table 5, column 5) and the median continental drift values (Table 5, column 3). The range and spatial distribution of drift values are shown in Fig. 1. From the figure, we note the need for large storages (small values of drift) in eastern Australia, southern Africa and in South America about the 30°S line of latitude.

Consider the SPA values, which are ratios of the mean annual flow, in Table 5. The median storage for southern Africa is $2.89 \times \text{MAF}$, which is more than 4 times larger than the world median values of 0.67. The median value of storage required for Australian rivers is also relatively speaking very large – $1.95 \times \text{MAF}$. Northern Africa (1.10) and South America (0.79) also require reservoirs of capacity larger than the world median value. North America and Asia have similar needs ($0.56 \times \text{MAF}$) with Europe being smaller (0.47) and the smallest storages being required for the South Pacific (0.32).

The median of Behaviour reservoir capacity estimates are summarized by continent in Table 5, column 6. As expected there is little change between the Behaviour ranking and those for SPA. G–DG was not included in this comparison because it is not applicable to within-year storage estimates, i.e., when drift > 1 . The square of the correlations (R^2) among the median continental storage values for the three estimation techniques are very high: SPA vs. EDA = 0.97, SPA vs. Behave = 0.98, and Behave vs. EDA = 0.93.

Another significant feature in Table 5 is the difference in the median reservoir capacities between ASA and RoW. For the three storage–yield techniques the ratio of ASA to RoW is 4.9 (EDA), 3.8 (SPA) and 4.6 (Behaviour). These values are consistent with the ratio of ASA to RoW median C_v values of 2.5 (Table 2, column 6).

Table 5 Reservoir storage estimates (as ratios of mean annual flow) for 75% targeted draft

	Number of rivers	Median drift	Median EDA (1/100 year recurrence)	Median SPA	Median Behaviour (95% monthly reliability)
(1)	(2)	(3)	(4)	(5)	(6)
Asia	58	0.86 (0.46–1.76)	0.40 (0.15–2.16) ^a	0.56 (0.30–1.91)	0.28 (0.17–1.00)
Australia	114	0.36 (0.20–0.96)	1.88 (0.35–5.07)	1.95 (0.46–4.92)	1.19 (0.20–3.34)
Europe	158	1.05 (0.61–1.67)	0.30 (0.11–0.79)	0.47 (0.20–0.94)	0.25 (0.078–0.48)
Northern Africa	40	0.86 (0.41–1.52)	1.48 (0.21–4.27)	1.10 (0.26–3.35)	0.43 (0.13–2.29)
North America	195	1.01 (1.70–0.47)	0.42 (0.11–1.63)	0.56 (0.28–1.72)	0.30 (0.16–0.95)
Southern Africa	48	0.30 (0.23–0.69)	3.32 (0.73–6.59)	2.89 (0.60–4.84)	1.93 (0.33–3.45)
South America	100	0.68 (0.28–1.21)	0.72 (0.25–3.02)	0.79 (0.34–2.98)	0.35 (0.13–2.24)
South Pacific	16	1.11 (0.76–1.54)	0.31 (0.11–1.67)	0.32 (0.16–0.42)	0.16 (0.091–0.23)
Australia–southern Africa	162	0.35 (0.22–0.88)	2.12 (0.44–5.10)	2.16 (0.51–4.92)	1.30 (0.20–3.43)
Rest of world	567	0.91 (0.46–1.63)	0.43 (0.12–1.88)	0.57 (0.26–1.91)	0.28 (0.13–1.00)
All rivers	729	0.80 (0.29–1.57)	0.56 (0.13–3.49)	0.67 (0.28–3.23)	0.32 (0.14–2.10)

^a Values in parenthesis are 10th and 90th percentile.

Reservoir storage performance

Metrics of storage performance

Three reservoir storage performance metrics are examined in this paper. The first metric is the monthly time reliability and is the proportion of months during a simulation that a reservoir can meet the target draft. It is defined as

$$R_m = \frac{N_s}{N_m}; \quad 0 < R_m < 1 \quad (8)$$

where R_m is the monthly time-based reliability, N_s is the number of months that the target draft can be met and N_m is the number of months in the simulation. As noted earlier, this measure is usually adopted in the practical application of the Behaviour diagram and can be equated to a probability of failure as $(N_m - N_s)/N_m$. It is different from the definition of the mean time to failure from a full reservoir, which is used in the Gould–Dincer procedure.

The second metric, we have adopted herein, is resilience. This indicates how quickly a reservoir will recover after a failure or emptiness and is defined according to Hashimoto et al. (1982) as

$$\varphi = \frac{f_s}{f_d}; \quad f_d \neq 0 \quad (9)$$

where φ is the Hashimoto resilience estimate, f_s is the number of individual failures in a simulation and f_d is the total period of all failures.

Hashimoto's dimensionless vulnerability parameter (Hashimoto et al., 1982) is the third metric we examined. This metric estimates the average volumetric severity of failures during periods when the reservoir is unable to meet the targeted draft. It is defined as

$$\eta = \frac{\sum_{j=1}^{f_s} \max(s_j)}{D_f \times f_s} \quad (10)$$

where η is Hashimoto's dimensionless vulnerability metric, f_s is the number of individual failures in a simulation, s_j is the volumetric shortfall in draft during the j th continuous

failure and D_f is the targeted draft during failures (more details of these and other metrics are discussed in McMahon and Adedoye, 2005).

Continental variation of reservoir storage performance

In this section, we discuss the three key reservoir performance measures – time reliability, resilience and vulnerability – computed for the global rivers and summarized at the continental scale in Table 6. The table is based on applying a monthly Behaviour analysis to each river in the world data set for 75% targeted draft and for a storage equal to the mean annual flow. In Table 6, column 3, which shows the median continental values of monthly time-reliability, we observe a picture highly correlated, as expected, with that observed for the reservoir capacity requirements listed in Table 5. For the hypothetical storages, at a continental scale high reliabilities occur in the South Pacific and Europe, slightly less reliable systems in North and South America, lower still in northern Africa, followed by Australia and the lowest value in southern Africa.

The resilience index (Table 6, column 4) shows a similar pattern to column 3. The low estimates of resilience suggest that empty reservoirs recover relatively slowly. Again southern Africa and Australia have relatively lower resilience than reservoirs in other continents. However, except for South Pacific with a median φ of 0.42, the remaining continents have resilience values in the narrow range of 0.22–0.27.

The third metric, dimensionless vulnerability, measures the severity of the shortfall when a reservoir fails to meet the targeted draft. High values, greater than 0.8, occur in southern Africa and Australia (Table 6, column 5). Of interest, northern Africa also exhibits a relatively high value of 0.73. This is consistent with the relatively high median reservoir capacity required for this region (northern Africa ranks 3 in SPA estimates). As expected the three reservoir performance metrics have consistent rankings across the three measures, except for South America which has more variable rankings than the other continents.

Table 6 Reservoir performance metrics for 75% targeted draft from a reservoir of capacity equal to mean annual flow

	Number of rivers	Median monthly reliability	Median resilience	Median dimensionless vulnerability
(1)	(2)	(3)	(4)	(5)
Asia	58	0.66 (0.48–0.79) ^a	0.24 (0.16–0.35)	0.68 (0.42–0.89)
Australia	114	0.51 (0.33–0.77)	0.18 (0.12–0.33)	0.82 (0.59–0.95)
Europe	158	0.74 (0.60–0.94)	0.27 (0.20–0.46)	0.58 (0.30–0.84)
Northern Africa	40	0.62 (0.45–0.86)	0.22 (0.15–0.40)	0.73 (0.31–0.98)
North America	195	0.69 (0.53–0.85)	0.26 (0.17–0.40)	0.66 (0.45–0.82)
Southern Africa	48	0.46 (0.34–0.76)	0.16 (0.11–0.28)	0.86 (0.52–0.95)
South America	100	0.69 (0.51–0.89)	0.23 (0.097–0.39)	0.55 (0.33–0.80)
South Pacific	16	0.86 (0.78–0.93)	0.42 (0.25–0.50)	0.47 (0.24–0.56)
Australia–southern Africa	162	0.50 (0.33–0.77)	0.17 (0.13–0.32)	0.83 (0.53–0.95)
Rest of world	567	0.70 (0.53–0.88)	0.25 (0.17–0.43)	0.61 (0.36–0.86)
All rivers	729	0.67 (0.44–0.88)	0.24 (0.14–0.40)	0.69 (0.38–0.89)

^a Values in parenthesis are 10th and 90th percentile.

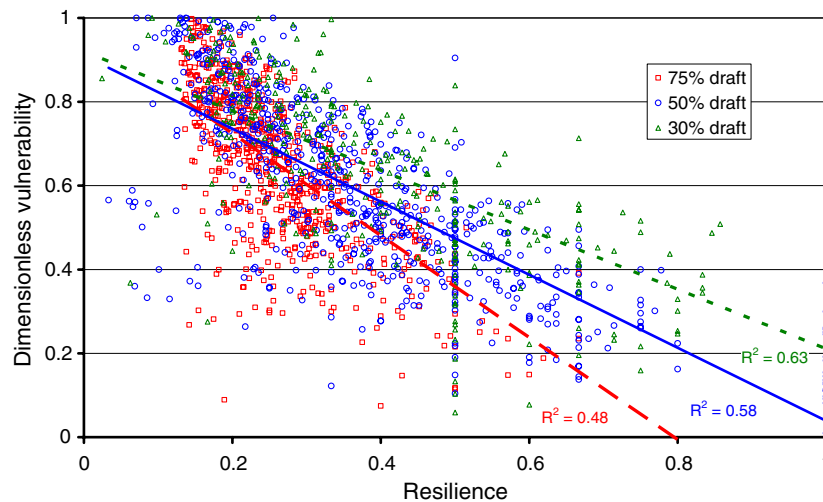


Figure 4 Comparison of dimensionless Hashimoto vulnerability versus Hashimoto resilience estimates (for storages equal to mean annual flow and for 30%, 50% and 75% targeted draft) (analysis is based on monthly flows).

In Fig. 4 dimensionless vulnerability is plotted against resilience for the monthly time series based on the rivers from the global data set assuming a hypothetical reservoir equal in capacity to the mean annual flow and target drafts of 30%, 50% and 75%. As noted elsewhere (McMahon et al., 2006) the global rivers show that the two metrics, dimensionless vulnerability and resilience, are approximately complementary. Based on this much larger data set than that used in McMahon et al. (2006) the line of best fit through all the data in the figure has a slope of -0.76 , suggesting that the complementary relationship (a slope of -1) is not particularly strong. Furthermore, the relationship is not very linear.

Conclusions

This is the second of three papers in a series dealing with the hydrologic characteristics of unregulated and hypothet-

ically regulated rivers on a global basis. Based on the analyses described herein we have identified the following conclusions:

1. The literature that deals with the characteristics of hypothetically regulated global rivers is sparse.
2. We have examined the variations of unregulated and regulated flow characteristics among continents and between Australia–southern Africa and the rest of the world.
3. In terms of the median annual coefficient of variation C_v of streamflows, the continents are ranked as follows: southern Africa 0.82, Australia 0.68, South America 0.37, northern Africa 0.29, Asia 0.29, North America 0.25, Europe 0.24 and South Pacific 0.22.
4. Excluding northern Africa with a median value of annual auto-correlation of unregulated streamflows of 0.40, the median value for the other continents is approximately 0.11.

5. Approximately 50% of hypothetical reservoirs with 75% draft located on rivers in Europe, North America and the South Pacific exhibit drift values $> \sim 1$ which implies within-year reservoir behaviour (the reservoir would spill every year).
6. Typically, the required reservoir capacity computed using the Sequent Peak Algorithm (SPA) for 90% target draft is approximately 5 times that for 50% draft.
7. We developed six empirical equations to calculate reservoir capacity based on SPA and Behaviour reservoir capacity estimates. Using data for all the rivers, reservoir capacities were found to be satisfactorily related to the annual standard deviation, the drift (based on annual Cv and draft ratio), the annual coefficient of variation and, for SPA, the historical record length and, for the Behaviour equation, the standardised variate representing reliability. For both sets of analyses, equations for storage estimates for drift < 1 had greater predictive power than those for drift ≥ 1 .
8. The Gould–Dincer Gamma equation produced reservoir capacity estimates that are consistent with the deficits produced using Extended Deficit Analysis for equivalent failure conditions.
9. The median continental reservoir capacity estimates as a ratio of the mean annual flow varied by a factor of 9 across the continental regions.
10. Three reservoir performance metrics – monthly time reliability, resilience and dimensionless vulnerability – were computed from the output of a Behaviour analysis applied to monthly flows of the global rivers. The three metrics have very consistent rankings across the three measures, except for South America which appears to have a lower median dimensionless vulnerability value than the other continents.
11. Comparing dimensionless vulnerability with resilience for a range of drafts across the 729 global rivers, the complementary relationship between the two variables is not particularly strong.

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