OPTIMAL ALLOCATION OF WATER WITHDRAWALS IN A RIVER BASIN

By Jennifer M. Jacobs,[1] Associate Member, ASCE, and Richard M. Vogel,[2] Member, ASCE

ABSTRACT: An increasing number of states use permit programs to coordinate and to control water resource allocations. A general approach is suggested for allocating and permitting water withdrawals in a river basin. A mathematical programming methodology facilitates optimal streamflow allocation while maintaining desired levels of instream flow. The approach uses a graphical tool, the flow duration curve, to illustrate the quantity and frequency of joint streamflow withdrawals in a river basin. The methodology is unique because while it uses mathematical programming methods, it is implemented using a spreadsheet optimization tool, Microsoft Excel Solver, and the solution is illustrated in a graphical form so that nontechnical individuals can easily understand the methodology results. The ability to apply the methodology and clearly explain the results is extremely important since many nontechnical individuals are involved as policymakers in water allocation decisions.

INTRODUCTION

Water allocation problems have challenged water resource engineers for decades. Much of the scientific and engineering literature addressing water allocation problems focuses on the optimal allocation of reservoir releases from single and multiple reservoir systems [see, e.g., Loucks et al. (1981)]. Even when streamflow is not regulated by dams, river basin management is complex. Heightened competition for withdrawals, increasing instream flow regulations, and compelling water quality issues are resulting in water withdrawal permits replacing reasonable use legislation.

Many states have established some form of a permit system to manage water resources. Cox (1994) examined the extensive adoption of permitting programs by eastern states. Permitting allows water management agencies to allocate water in a manner that reflects the state and regional values (Cox 1989). The first task in allocating water withdrawals is determining and explicitly formulating the overall goals of the permit system and establishing permit rules that reflect those goals. National, regional, states, or local goals may include maintenance of instream flow, economic development, protection of established rights, and/or incentives for efficient water use. The permits may allow ongoing restrictions by limiting the duration of the right or restricting water rights during periods of low flow.

Eheart and Lyon (1983) identified and compared alternative designs of marketable water permitting systems. Their work examined the trade-offs between multiple objectives that included economic efficiency, equity, ease of implementation and administration, and maintenance of instream flows. Howe et al. (1986) also examined allocation by means of water markets and suggested administrative approaches for minimizing their shortcomings. Tisdell and Harrison (1992) modeled a water market using game theory. Their goal was to understand how regulatory agencies should initially allocate water to promote its equitable distribution. Lund and Israel (1995) used optimization techniques for planning water transfers in urban water supply systems. Winter (1995) provided a review of recent literature that addressed the optimal and conjunctive allocation of ground- and surface-water resources.

Whether permits are assigned or marketed, regulatory agencies and water users can benefit from understanding the trade-offs that result from competing objectives and constraints. The task of optimally allocating water resources in a river basin may be approached using system analysis. Systems analysis is the applied use of optimization, mathematical programming, operations research, or other mathematical decision-making techniques. Yeh (1985), Loucks et al. (1981), Rogers and Fiering (1986), and others provided a review of systems analysis as applied to water resource problems. Rogers and Fiering (1986) suggested that 10 years ago most applications of systems analysis in water resources were implemented by academic researchers, with few real-world applications. Now, real-world applications of systems analysis methods are much more feasible than in the past; as the optimization methods are more widely understood, desktop computers have the necessary power to perform the analyses, and optimization algorithms are readily available in popular software packages.

Two of the most widely used optimization techniques in the field of water resource management are linear programming (LP) and nonlinear programming (NLP). Historically, the deterministic nature of LP and NLP hindered its application to water resources planning. Using a deterministic approach, hydrologic parameters must be set equal to a constant value, often the mean seasonal inflow, or another value from the historical period of record. Such models cannot account for the natural stochastic variability of streams.

Methodologies such as stochastic LP models (Manne 1962) and chance-constrained LPs (Charnes et al. 1958) can introduce statistical constraints to LPs and NLPs that account for random variables. A chance-constrained LP or NLP is applied to water resource problems by using the cumulative probability distribution of streamflow (Loucks et al. 1981; Loaiciga 1988; Mays and Tung 1992).

A simple hydrologic tool, the flow duration curve (FDC), describes the variable nature of daily streamflow. Vogel and Fennessey (1995) reviewed previous and new application areas for FDCs. Some applications of FDCs include instream flow studies, hydropower, water quality, and water supply studies (Searcy 1959; Warnick 1984; Noss and Gladstone 1987; Mueller and Male 1993; Vogel and Fennessey 1995). For example, Mueller and Male (1993) developed a model for managing ground-water withdrawals. Their model used FDCs to measure the effect of new withdrawals on instream flow.

This paper uses FDCs to quantify the streamflow available for allocation, as did Male and Mueller (1992) and Fennessey (unpublished paper, 1998). The relationship between streamflow reliability and magnitude has also been used in reservoir management. Buras (1985) used flow duration curves to calculate a "reservoir yield function." Male and Mueller (1992) proposed an approach for optimizing the allocations for a river

[1]Asst. Prof., Dept. of Civ. Engrg., Univ. of Florida, Gainesville, FL 32611. E-mail: jjaco@ce.ufl.edu
[2]Assoc. Prof., Dept. of Civ. and Envir. Engrg., Tufts Univ., Medford, MA 02155. E-mail: rvogel@tufts.edu

basin without reservoirs. Their approach reduced the multiple consumptive uses in a river basin to a single "lumped" withdrawal. The methodology introduced here considers the spatial relationship between withdrawals. Fennessey (unpublished paper, 1998) devised a basin water allocation methodology, which has been implemented by the Commonwealth of Massachusetts, that uses FDCs to assess basin-wide and local impact. Alaouze (1989, 1991) used the reservoir yield function to develop a relationship between streamflow magnitude and reliability. Alaouze used the resulting FDC of reservoir releases to allocate releases from a reservoir to different users with varying reliabilities.

Alaouze's application of the relationship between allocation magnitude and reliability may also be used to optimally allocate streamflow in river basins without reservoirs as demonstrated here. The following application of FDCs to allocating withdrawals in an unregulated basin is in spirit similar to the methodology introduced by Alaouze (1989, 1991) but differs in three respects: (1) The withdrawals are distributed throughout the basin rather than withdrawn from a single point; (2) other constraints are introduced, such as instream flow requirements; and (3) other objectives exist, such as minimizing basin consumptive use and/or prioritizing according to use category.

The purpose of this paper is to present a general method for allocating consumptive water uses that reflects the goals (objectives) established by the permit system and the natural limitations (constraints) of unregulated streamflow availability, yet is easily implemented using common software and readily summarized in graphical terms. Since many permit policies are developed and/or reviewed by nontechnical individuals, the following prescriptive approach requires minimal technical background. Layperson accessibility is coupled with technical comprehensiveness using graphs to describe the impact of proposed allocations on the instream flow and current users. The allocation procedure uses chance-constrained mathematical programming, as developed for reservoir releases, and flow duration curves to optimally allocate spatially distributed withdrawals. A case study for a hypothetical basin using a popular optimization tool, Microsoft Excel Solver, documents the implementation of the proposed methodology.

FDCs

A simple hydrologic tool, the FDC, accounts for the variable nature of streamflow. Fig. 1 illustrates the relationship between streamflow and its associated reliability for an FDC. For a given streamflow q_1, the FDC in Fig. 1 specifies the reliability as r_1. Reliability or the exceedance probability is defined as the probability that the average daily flow will be greater than or equal to the corresponding streamflow. FDCs are defined for specific sites and flow measurement duration; daily streamflows are typically used, though other durations are possible [see Vogel and Fennessey (1994)].

There are a number of methods for constructing an FDC. Traditionally, FDCs have been constructed by simply ranking all streamflows q_i over the period-of-record (Searcy 1959) from the largest to smallest, $q_1, q_2, \ldots, q_S$ where S is the total number of streamflows and $q_i > q_{i+1}$. Each streamflow quantity has a corresponding exceedance probability $r_i = i/(S + 1)$ using the Weibull plotting position. If an FDC is constructed using period-of-record streamflows, then one interprets the exceedance probability or reliability as the reliability of streamflow exceeding some level over the period of record. Alternatively, one can construct an annual-based FDC that represents the exceedance probability or reliability of streamflow exceeding some minimum level in a typical or median year [see Vogel and Fennessey (1994)]. Either approach can be used depending upon the overall goals of the permit program.

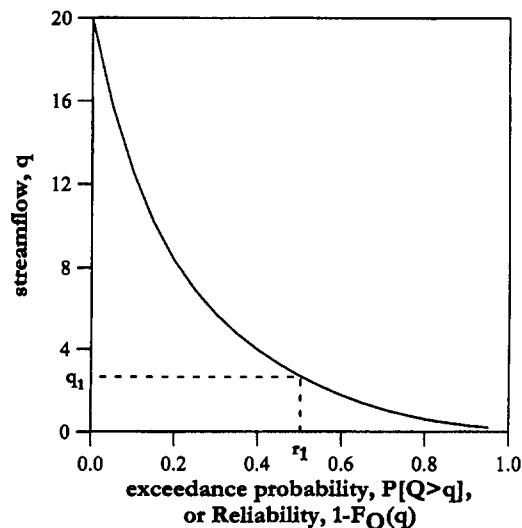Techniques exist for using drainage basin characteristics to

FIG. 1. FDC Exhibiting Relationship between Streamflow Quantity q and Reliability r

create an FDC for ungauged and unregulated sites [see Fennessey and Vogel (1990) for an example and for citations to other studies]. The FDC for the entire river basin provides an excellent overview and summary of the streamflow allocated within the river basin. However, each location in the basin has a unique natural or unregulated FDC. To capture the distributed nature of the individual withdrawals, an FDC for each withdrawal site in the catchment must be developed.

The FDC has interesting graphical properties and features. The area underneath the FDC represents the average daily streamflow (Vogel and Fennessey 1994). For any withdrawal location, there are three categories of allocated streamflow: (1) Instream flow q_s; (2) upstream allocation q_u (streamflow not available for use due to present and/or future consumption by an upstream user); and (3) point of withdrawal streamflow allocation q_w. These three categories can be represented graphically using an FDC as shown in Fig. 2. The actual water allocated, on average, to each use is denoted by regions. For example, the water withdrawal rate that is allocated to streamflow location w, q_w, is denoted by the Region ABCD in Fig. 2. The three allocation categories are stacked according to priority of allocation. The highest priority allocation appears on the bottom of the FDC. The reliabilities associated with the instream flow q_s and the point of withdrawal allocation q_w are given as r_s and r_w, respectively. The reliability for the upstream allocation q_u is not shown in this graph since it must be ob-
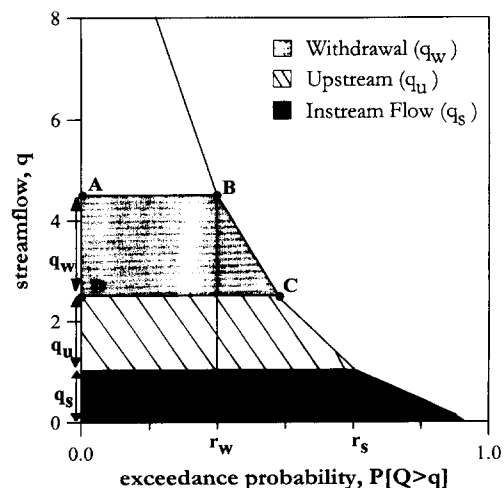

FIG. 2. Streamflow Allocation for Instream Flow Requirement, Upstream Users, and User at Site

tained using the upstream site's FDC and the instream flow at that site.

STREAMFLOW ALLOCATION MODELS

Different withdrawal permit programs manage streamflow allocations with different objectives. One possible goal is to maximize the overall allocation of water for productive use in the basin. Using this model, programs may be formulated to reflect user prioritization or the economic value of water allocations by weighting schemes [see Male and Mueller (1992)]. The problem is finding the allocation that maximizes the objective function, subject to restrictions based on streamflow availability, individual withdrawal requirements, instream flow requirements, and reliability limitations. The nonlinear streamflow availability constraints due to nonlinear FDCs are transformed to linear constraints using a piecewise linearization technique. The general chance-constrained model for maximizing water allocation for productive use is given as

$$\text{Max } Z = \sum_{i=1}^{N} w_i q_i \tag{1}$$

subject to

$$q_i \leq a_i, \quad \forall \, i = 1, 2, \ldots, N \tag{2}$$

$$q_i \geq p_i, \quad \forall \, i = 1, 2, \ldots, N \tag{3}$$

$$r_i \geq r_{\min,i}, \quad \forall \, i = 1, 2, \ldots, N \tag{4}$$

$$\Pr[q_{i,\text{TOT}} \leq Q_{r_i}] = r_i, \quad \forall \, i = 1, 2, \ldots, N \tag{5}$$

$$q_i \geq 0, \quad \forall \, i = 1, 2, \ldots, N \tag{6}$$

$$r_i \geq 0, \quad \forall \, i = 1, 2, \ldots, N \tag{7}$$

where w_i = weight for site i; q_i = permitted withdrawal quantity for site i; N = number of withdrawals; a_i = withdrawal amount requested for site i; p_i = existing permitted withdrawal at site i; r_i = streamflow reliability for withdrawal i; $r_{\min,i}$ = minimum acceptable streamflow reliability for withdrawal i; and Q_{r_i} = streamflow available with reliability r_i at site i. The total streamflow allocated to site i, $q_{i,\text{TOT}}$, including instream flow, consumptive upstream use, and withdrawal at the site is

$$q_{i,\text{TOT}} = q_{s_i} + q_i + \sum_{j=1}^{N} c_j u_{ij} q_j \tag{8}$$

where q_{s_i} = instream flow requirement at site i; c_j = consumptive loss coefficient for withdrawal j; u_{ij} = upstream coefficient, where $u_{ij} = 1$ if withdrawal j is upstream of site i and 0 otherwise; and q_j = withdrawal quantity at site j. A consumptive loss is the amount of water lost to the river basin at the withdrawal location. Evaporation, out of basin transfer, or other reasons may cause this loss. If the unconsumed portion of a withdrawal is returned to the stream below the withdrawal location, then $c_j = 1$ when $i = j$. Hence, the last term on the right-hand side of (8) is total streamflow consumed upstream of site i.

Constraint set (2) limits the quantity allocated to each user to that requested by the user. Constraint set (3) protects existing permit quantities. Constraint set (4) establishes a minimum value for the reliability of each permitted withdrawal where $r_{\min}$ may be a basin-wide constant or may reflect individual withdrawal reliability requirements. Constraint sets (6) and (7) ensure the decision variables q_i and r_i are nonnegative.

The remaining streamflow constraint set (5) uses a chance-constraint to establish the probability that the amount of streamflow allocated will not exceed the amount of streamflow available. At each withdrawal location i, the probability that the total allocated streamflow (the sum of the instream flow

requirement and all consumptive withdrawals at and upstream of point i) is less than the available streamflow is r_i.

Assuming average daily streamflow Q is treated as a random variable with cumulative probability distribution function

$$F_Q(q) = P[Q \leq q] \tag{9}$$

The cumulative distribution function F_Q describes the cumulative probability distribution or the nonexceedance probability associated with the random variable Q. The complement of the cumulative distribution function, $1 - F_Q(q)$, is the FDC because it describes the exceedance probability or the reliability that flow will exceed Q. $1 - F_Q(q)$ may also be written as $P[Q \geq q]$. For a given value of r, Q_r denotes the inverse of the FDC or that value of streamflow that is exceeded with probability or reliability r. Thus, the chance-constraint formulation in (5) has a deterministic equivalent of

$$q_{i,\text{TOT}} \leq Q_{r_i} = F_i^{-1}(1 - r_i), \quad \forall \, i = 1, 2, \ldots, N \tag{10}$$

where $q_{i,\text{TOT}}$ is related to the r_i by the site i FDC. The FDC represented by the function Q_{r_i} may be made piecewise linear with mixed integer linearization (Loucks et al. 1981). Linearization is achieved by defining K segments, each having slope $s_{i,k}$, horizontal length $r_{i,k}$, beginning at reliability $R_{i,k}$. The reliability r_i is then defined as the sum of the K values of $r_{i,k}$

$$r_i = \sum_{i=1}^{K} [R_{i,k} + r_{i,k}], \quad \forall \, i = 1, 2, \ldots, N \tag{11a}$$

where

$$\sum_{k=1}^{K} z_{i,k} = 1, \quad \forall \, i = 1, 2, \ldots, N \tag{11b}$$

and $z_{i,k}$ is an integer. An additional constraint must be added to limit the length of each $r_{i,k}$ to its maximum length

$$r_{i,k} < (R_{i,k+1} - R_{i,k}) z_{i,k}, \quad \forall \, i = 1, 2, \ldots, N, \quad \forall \, k = 1, 2, \ldots, K \tag{11c}$$

The result of the formulation is that, at most, one of the $r_{i,k}$ and one of the $z_{i,k}$ are greater than zero so that if $z_{i,k} = 1$, then $r_i = R_{i,k} + r_{i,k}$. The linearized form of constraint (5) is then

$$q_{i,\text{TOT}} \leq \sum_{k=1}^{K} [s_{i,k} r_{i,k} + z_{i,k} Q_{i,k}], \quad \forall \, i = 1, 2, \ldots, N \tag{12}$$

where each linearized FDC has K segments with slope $s_{i,k}$; and $Q_{i,k}$ = streamflow available with reliability $R_{i,k}$. Usually, it is not reasonable to allocate streamflow with reliabilities smaller than 50 or 60% due to the limited use for streamflow with low availability. Therefore, the linearization begins at a reliability level larger than 0. The FDC is assumed to be concave over the region of interest. A convex FDC could also be linearized using a linear mixed-integer optimization procedure (see Loucks et al. 1981, pp. 57–76).

The FDC linearization is only necessary when the problem is solved with an LP algorithm. The advantage of an LP algorithm is that the globally optimal solution is more readily found for the LP than for the NLP. The LP has, at most, one feasible region, and the optimal solution is always found on the surface where the constraints intersect. The NLP could have several feasible regions, each with several locally optimal solutions. The final optimization model is solved for objective function (1) constrained by (2)–(4), (6), (7), (11), and (12), where (11) and (12) replace (5).

SOLVING ALLOCATION PROBLEM

A secondary objective of this research project was to implement the methodology by means of a computer tool that is

easy to employ and readily accessible to the water permitting community. The Microsoft Excel Solver tool can optimize this model using mathematical programming. Excel Solver analyzes a variety of mathematical programming problems with straightforward optimization algorithms: LP problems are solved by the Simplex method; NLP problems are solved by the Generalized Reduced Gradient method (Lasdon and Smith 1992); and mixed-integer programming problems use the "branch and bound" method. The tool provides a solution as well as sensitivity analysis. In addition, the ability to easily sort data and perform repetitive calculations makes a spreadsheet an excellent tool for using historical streamflow data to construct and linearize an FDC.

The Excel Solver tool is not as powerful as other optimization algorithms. Problems are limited to 200 decision variables. The number of constraints is unlimited for LP problems, but limited to 100 for NLP problems. The memory required by Excel Solver increases with the number of variables times the number of constraints. Other tools, such as the optimization solver MINOS (Murtagh and Saunders 1983), can handle thousands of variables and constraints and use improved algorithms or more sophisticated implementations of the Excel Solver's algorithms to save time and memory.

APPLICATION OF DEVELOPED METHODOLOGY

Model Implementation

The methodology was applied to a hypothetical unregulated river basin. The river basin is composed of two separate streams, S1 and S2, that converge downstream to form a single stream S3. There are three possible withdrawal locations. Site 1 on S1, Site 2 on S2, and Site 3 on S3. Table 1 shows the reliabilities and corresponding streamflow for the linearized FDCs. The FDCs for Sites 1 and 2 are the same. It is assumed that no withdrawal permits currently exist.

TABLE 1. Linearized FDCs for Potential Withdrawal Sites

Reliability r (1)	Streamflow, Q_r		
	Site 1 (2)	Site 2 (3)	Site 3 (4)
0.1	80.10	80.10	200.25
0.5	3.36	3.36	8.40
0.6	1.80	1.80	4.50
0.7	0.92	0.92	2.30
0.8	0.42	0.42	1.05
0.95	0.06	0.06	0.15

The proposed methodology was solved for five different allocation request scenarios. Table 2 lists the model inputs for each scenario. The model inputs for each scenario are the instream flow requirement at each site and the withdrawal request characteristics including requested withdrawal rate, minimum reliability, consumptive loss coefficient, and prioritization weight.

Scenarios 1 and 2 consider withdrawal requests from Sites 1 and 2. For Scenario 1, Sites 1 and 2 have the same instream flow requirements, reliability requirements, and consumptive loss coefficients. Scenario 2 examines the role of reliability on allocation by modifying the reliability requirements in Scenario 1.

Scenarios 3–5 have withdrawal requests from all three sites. Scenario 3 explores the relationship between upstream and downstream withdrawals. For this scenario, the withdrawal requests from Sites 1 and 2 are the same as the requests in Scenario 1. All three sites have the same withdrawal requests, reliability requirements, and consumptive loss coefficients. Scenario 4 examines the impact of a reduced upstream consumptive loss on the downstream streamflow availability by decreasing Site 1's consumption rate. Scenario 5 studies how a prioritization weighting scheme influences streamflow allocation by assigning a higher weight to Site 3 than to Sites 1 and 2.

Analysis of Results

Table 2 shows the streamflow allocated q_i and the corresponding reliability r_i by site for each scenario. Using each site's FDC, the streamflow allocations for each scenario are displayed in Figs. 3–7. The FDCs are shown with reliabilities from 0.4 to 1.0.

Scenario 1 has the identical allocation for both Sites 1 and 2. The allocation for Site 1 is shown in Fig. 3. Instream flow was allocated first and given the highest priority. The remaining streamflow available with reliability ≥ 0.60 is 1.3. As this amount is less than the 2.0 that was requested, the entire 1.3 was allocated to the withdrawal request.

The degree to which a site can handle uncertainty of water availability can have a large impact on the quantity of water available at the site. The effect of changing the minimum reliability, as examined by Scenario 2, is shown in Fig. 4. The different allocation results for Sites 1 and 2 are shown in Figs. 4(a and b), respectively. Increasing the minimum reliability to $r_{min} = 0.7$ for Site 1 decreased the available streamflow to 0.42.

TABLE 2. Model Input and Output for Selected Withdrawal Scenarios

| Scenario (1)	Site i (2)	Model Input					Model Output	
		Instream flow requirement, $q_{s,i}$ (3)	Requested withdrawal, a_i (4)	Minimum reliability, $r_{min,i}$ (5)	Consumptive loss coefficient, c_i (6)	Weight, w_i (7)	Permitted withdrawal, q_i (8)	Reliability of q_i, r_i (9)
1 (Fig. 3)	1	0.5	2.0	0.6	0.75	1	1.3	0.6
	2	0.5	2.0	0.6	0.75	1	1.3	0.6
	3	1.0	—ᵃ	—ᵃ	—ᵃ	—ᵃ	—ᵃ	—ᵃ
2 (Fig. 4)	1	0.5	2.0	0.7	0.75	1	0.42	0.7
	2	0.5	2.0	0.5	0.75	1	2.00	0.56
	3	1.0	—ᵃ	—ᵃ	—ᵃ	—ᵃ	—ᵃ	—ᵃ
3 (Fig. 5)	1	0.5	2.0	0.6	0.75	1	1.3	0.6
	2	0.5	2.0	0.6	0.75	1	1.3	0.6
	3	1.0	2.0	0.6	0.75	1	1.5	0.6
4 (Fig. 6)	1	0.5	2.0	0.6	0.60	1	1.3	0.6
	2	0.5	2.0	0.6	0.75	1	1.3	0.6
	3	1.0	2.0	0.6	0.75	1	1.75	0.6
5 (Fig. 7)	1	0.5	2.0	0.6	0.75	1	1.3	0.6
	2	0.5	2.0	0.6	0.75	1	0.7	0.67
	3	1.0	2.0	0.6	0.75	1.5	2.0	0.6

ᵃNot applicable.
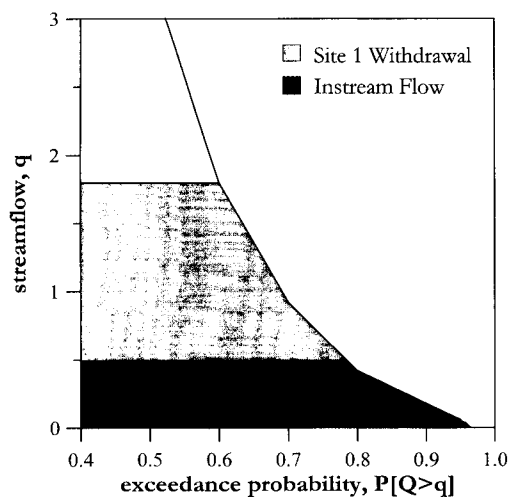
FIG. 3. Optimal Withdrawal Allocations for Scenario 1 at Site 1 as Illustrated by Site's FDC Including Instream Flow Requirement and Withdrawal at Site 1
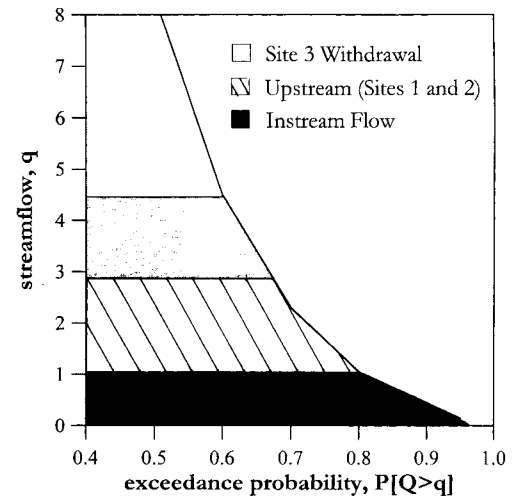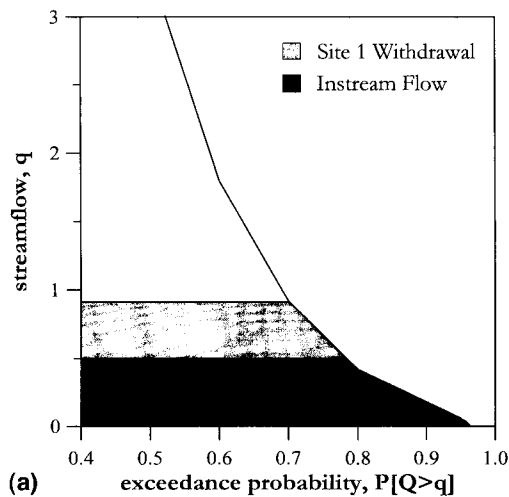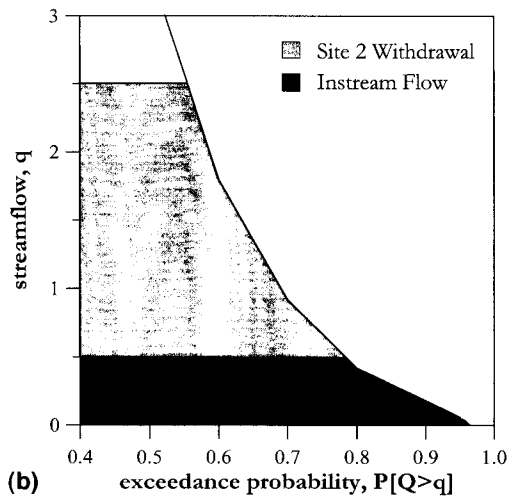

(a)

(b)

FIG. 4. Optimal Withdrawal Allocations for Scenario 2 at: (a) Site 1 with Site's FDC Including Instream Flow Requirement and Withdrawal at Site 1; (b) Site 2 with Site's FDC Including Instream Flow Requirement and Withdrawal at Site 2

Decreasing the minimum reliability to r_{min} = 0.5 allowed the full requested withdrawal to be available at Site 2.

For Scenario 3, the streamflow allocation to Sites 1 and 2 were identical to Scenario 1 (see Fig. 3). The streamflow available for allocation at Site 3 was reduced by the withdrawals at upstream sites. Fig. 5 displays the Site 3 FDC and the al-
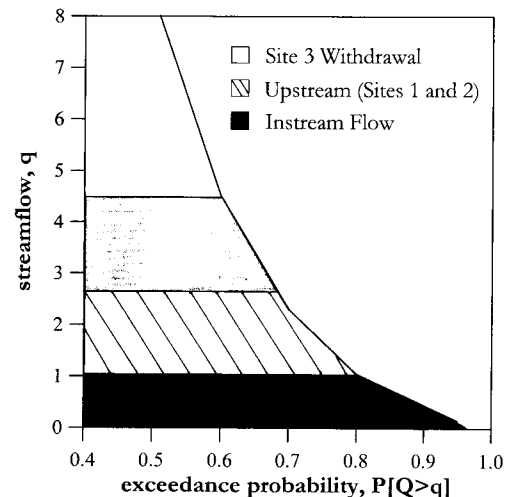

FIG. 5. Optimal Withdrawal Allocations for Scenario 3 at Site 3 as Illustrated by Site's FDC Including Instream Flow Requirements, Consumptive Withdrawals at Upstream Sites, and Withdrawal at Site 3

FIG. 6. Same as Fig. 5 but for Scenario 4

locations for the instream flow requirement, the upstream consumptive withdrawals, and the Site 3 withdrawal. Fig. 5 shows how the water used by Sites 1 and 2 reduced the streamflow available at Site 3 for the model input in Scenario 3.

In Scenario 4, the consumption rate at Site 2 was reduced to 60%. When a site changes the rates at which it returns water to the stream it effectively increases or decreases the streamflow available downstream. Fig. 6 shows that the decreased consumption rate upstream increased the streamflow available at Site 3.

The prioritization-weighting scheme in Scenario 5 gave Site 3 the highest priority for streamflow allocation. Fig. 7(a) shows that this change reduced the streamflow allocated at Site 2 as compared to the streamflow that was allocated in Scenarios 3 and 4. Fig. 7(b) illustrates that the Site 3 withdrawal allocation for this scenario was larger than the Site 3 allocation in Scenario 3. In addition, the weighting scheme reduced the total allocated streamflow from 4.1 in Scenario 3 to 4.0 in Scenario 5.

EXTENSIONS

There are many similar allocation problems that could be structured similarly to the preceding model; some possible extensions are provided as follows:
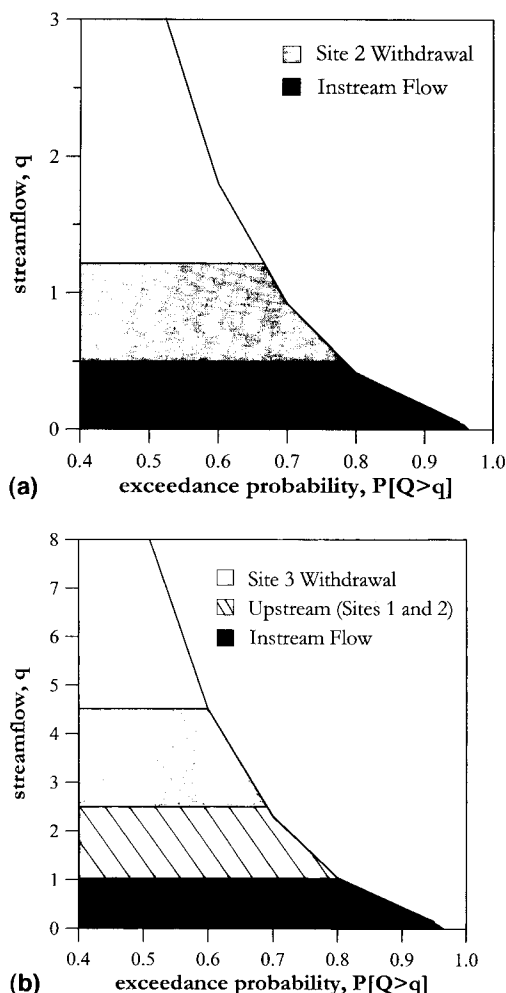
FIG. 7. Optimal Withdrawal Allocations for Scenario 5 at: (a) Site 2 with FDC Including Instream Flow Requirement and Withdrawal at Site 2; (b) Site 3 as Illustrated by Site's FDC Including Instream Flow Requirements, Consumptive Withdrawals at Upstream Sites, and Withdrawal at Site 3

1. A single applicant could make withdrawals from multiple wells within a river basin. The inclusion of multiple wells would allow applicants to obtain one permit per basin that reflects their overall allocation requirement while considering each well's impact on streamflow separately.

2. A constraint could be introduced that schedules withdrawal reductions during low flow periods. Low flow constraints could provide real-time allocation of water during low flow periods as described by Fennessey (unpublished paper, 1998).

3. The period-of-record FDC or annual FDC could be replaced by a seasonal-based FDC. A seasonal FDC could be used to formulate a withdrawal permit that would allow a withdrawal to change quantity or reliability seasonally.

4. The constraint set (3) that protects existing permitted values could be relaxed. This would allow a more flexible analysis during permit renewals.

5. This model assumes the value of water is fixed. As surface water is a scarce, but renewable resource, a more efficient allocation might result from extending the model to include the value of water. One approach would be to develop marginal net benefit curves for each withdrawal (Tietenberg 1992) and to incorporate discretized versions of the net benefit curves into the analysis.

CONCLUSIONS

An objective methodology was developed using flow duration curves and a chance constrained mathematical program to determine the allocation of streamflow to competing multiple users in a river basin. The methodology provides the optimal allocation expressed as withdrawal rates and corresponding reliabilities. A flow duration curve for each site is used to illustrate the site's withdrawal and any upstream withdrawals. Allocation problems are solved using an interactive optimization algorithm, Microsoft Excel Solver. This methodology is flexible and may be modified to include specific goals associated with permit programs that were not considered here.

The methodology was used to determine the withdrawal rates and reliabilities for a hypothetical river basin. Competing streamflow uses included instream flow requirements and multiple users with different withdrawal rates, minimum reliabilities, consumption rates, and basin locations. The optimal allocation was found for several objective functions. The basin's streamflow allocation was graphically displayed using FDCs. It was found that FDCs logically presented the optimized allocation and facilitated comparison between allocations resulting from different competing goals and requirements.

The allocation method presented in this paper may be used during the initial development of a permitting system to study the impact of proposed rules and during implementation of a permitting system for illustrating the existing withdrawals and the streamflow available for future allocations. Well-formulated goals and constraints are the backbone of this methodology. This methodology is not a substitute for water permitting programs, but it can be used during the planning phase of a permit program, to help policymakers and other interested parties understand the impact proposed goals and constraints have on streamflow allocation.

LIMITATIONS

Implementation of a permit program is a complex process that balances streamflow availability with user requirements, program staff, and budget constraints. The methodology currently includes many basic allocation concerns and offers suggestions for model extensions. Special withdrawal requests, such as specifically timed withdrawals requirements, may hinder the methodology's effectiveness. In addition, the result of the methodology is an allocation with quantity and reliability values for withdrawals. The methodology does not indicate how withdrawal reductions should be implemented or when the reductions are likely to occur. This methodology should be implemented with input from individuals who are knowledgeable of the specific requirements of the basin and its users.

ACKNOWLEDGMENTS

APPENDIX I. REFERENCES

Alaouze, C. M. (1989). "Reservoir releases to uses with different reliability requirements." *Water Resour. Bull.*, 25(6), 1163–1168.

Alaouze, C. M. (1991). "Transferable water entitlements which satisfy heterogeneous risk preferences." *Australian J. of Agric. Economics*, Australia, 35(2), 197–208.

Buras, N. (1985). "An application of mathematical programming in planning surface water storage." *Water Resour. Bull.*, 21(6), 1013–1020.

Charnes, A., Cooper, W. W., and Symonds, G. H. (1958). "Cost horizons and certainty equivalents: An approach to stochastic programming of heating oil." *Mgmt. Sci.*, 4(3), 225–263.

Cox, W. E. (1989). "State water allocation law: Its traditional and emerg-

ing roles." *Conference on Water, Laws, and Management.* Frederick E. Davis, ed., Am. Water Resour. Assn., Bethesda, Md.

Cox, W. E. (1994). "Reallocation impacts of eastern water-law changes." *J. Water Resour. Plng. and Mgmt.*, ASCE, 120(4), 476–484.

Eheart, J. W., and Lyon, R. M. (1983). "Alternative structures for water rights markets." *Water Resour. Res.*, 19(4), 887–894.

Fennessey, N., and Vogel, R. M. (1990). "Regional flow duration curves for ungaged sites in Massachusetts." *J. Water Resour. Plng. and Mgmt.*, ASCE, 116(4), 530–549.

Howe, C. W., Schurmeier, D. R., and Shaw, W. D., Jr. (1986). "Innovative approaches to water allocation: The potential for water markets." *Water Resour. Res.*, 22(4), 439–445.

Lasdon, L. S., and Smith, S. (1992). "Solving sparse nonlinear programs using GRG." *J. Computing*, 4(1), 2–15.

Loaiciga, H. A. (1988). "On the use of chance-constraints in reservoir design and operations modeling." *Water Resour. Res.*, 24(11), 1969–1975.

Loucks, D. P., Stedinger, J. R., and Haith, D. A. (1981). *Water resource systems planning and analysis.* Prentice-Hall, Inc., Englewood Cliffs, N.J.

Lund, J. R., and Israel, M. (1995). "Optimization of transfers in urban water supply planning." *J. Water Resour. Plng. and Mgmt.*, ASCE, 121(1), 41–48.

Male, J. W., and Mueller, F. A. (1992). "Model for prescribing groundwater use permits." *J. Water Resour. Plng. and Mgmt.*, ASCE, 110(4), 543–561.

Manne, S. D. (1962). "Product mix alternatives: Flood control, electric power and irrigation." *Int. Econ. Rev.*, 8(1), 30–54.

Mays, L. W., and Tung, Y. K. (1992). *Hydrosystems engineering and management.* McGraw-Hill Inc., New York.

Mueller, F. A., and Male, J. W. (1993). "A management model for specification of groundwater withdrawal permits." *Water Resour. Res.*, 29(5), 1359–1368.

Murtagh, B. A., and Saunders, M. A. (1983). "MINOS 5.4 User's Guide." *Rep. SOL 83-20R*, Sys. Optimization Lab., Stanford Univ., Stanford, Calif.

Noss, R. R., and Gladstone, I. (1987). "Flow variable discharge permits." *Water Resour. Bull.*, 23(5), 761–766.

Rogers, P. P., and Fiering, M. B. (1986). "Use of systems analysis in water management." *Water Resour. Res.*, 22(9), 146S–158S.

Searcy, J. K. (1959). "Flow-duration curves." *Paper 1542-A*, U.S. Geological Survey Water-Supply, U.S. Govt. Printing Ofc., Washington, D.C.

Tietenberg, T. H. (1992). *Environmental and natural resources economics.* HarperCollins, New York.

Tisdell, J. G., and Harrison, S. R. (1992). "Estimating an optimal distribution of water entitlements." *Water Resour. Res.*, 28(12), 3111–3117.

Vogel, R. M., and Fennessey, N. (1994). "Flow duration curves. I: New interpretation and confidence intervals." *J. Water Resour. Plng. and Mgmt.*, ASCE, 120(4), 485–504.

Vogel, R. M., and Fennessey, N. (1995). "Flow duration curves. II: A review of applications in water resources planning." *Water Resour. Bull.*, 31(6), 1029–1039.

Warnick, C. C. (1984). *Hydropower engineering.* Prentice-Hall, Inc., Englewood Cliffs, N.J., 59–73.

Winter, T. C. (1995). "Management of groundwater and surface water." *Rev. of Geophys.*, 33, Supplement.

Yeh, W. W.-G. (1985). "Reservoir management and operations models: A state-of-the-art review." *Water Resour. Bull.*, 21(12), 1797–1818.

APPENDIX II.   NOTATION

The following symbols are used in this paper:

a_i = amount of water requested for withdrawal i;
c_j = consumption coefficient for withdrawal j;
K = number of piecewise-linear segments of flow duration curve;
mgd = million gallons per day;
N = number of withdrawals;
p_i = existing permitted allocation for withdrawal i (mgd);
Q_r = rate of streamflow for given reliability r (mgd);
Q_{r_i} = rate of streamflow for given reliability r for withdrawal i (mgd);
q = streamflow allocation (mgd);
q_s = instream flow allocation (mgd);
q_{s_i} = instream flow allocation at withdrawal site i (mgd);
q_u = upstream flow allocation (mgd);
q_w = point of withdrawal allocation (mgd);
r = streamflow reliability;
r_i = streamflow reliability for withdrawal i;
r_s = instream flow reliability;
r_w = point of withdrawal reliability;
$s_{i,k}$ = slope of kth segment of piecewise-linear flow duration curve for withdrawal i;
u_{ij} = withdrawal coefficient where $u_{ij} = 1$ if withdrawal j is at or upstream from withdrawal i and 0 otherwise;
w_i = weight for user i; and
Z = value of objective function (mgd).

Subscripts

i = index of withdrawal;
j = index of upstream withdrawals; and
k = index for piecewise-linear section of flow duration curve.