Performance-Based Evaluation of an Improved Robust Optimization Formulation

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Abstract: Much progress has been made in the standardization of uncertainty analysis techniques for simulation modeling but less progress has been made in optimization modeling. Among the various techniques used for optimization modeling under uncertainty, robust optimization (RO) uniquely allows for evaluation and control of the various risks of poor system performance resulting from input parameter uncertainties in water-resources problems. A model formulation was developed that addresses an inadequacy in a previous RO formulation. The importance of evaluating, through postprocessing, RO model results with respect to a range of performance metrics, has been demonstrated rather than a single metric, as has been common in previous studies. An analysis of the tradeoffs between solution robustness (nearness to optimality across all scenarios) and feasibility robustness (nearness to feasibility across all scenarios) illustrates the importance of including these terms in multiobjective water resources decision models. **DOI:** 10.1061/(ASCE)WR.1943-5452.0000389. *© 2014 American Society of Civil Engineers.*

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Introduction

Decision-makers with responsibility over management and expansion of water resources systems are interested in managing a wide range of risks, among them the following: (1) risk of cost overruns (solution robustness), (2) risk of system failure (reliability), (3) magnitude of those failures (vulnerability), and (4) risk of performance deterioration (sustainability). In addition, a decision-maker may want information about the reliability, vulnerability, and sustainability of the system with respect to a variety of decision variables (e.g., withdrawals, storage, transfers, and conservation). The decisions made now depend upon the budget, expectations of future conditions, decision-maker's tolerance for various types of risk, and ability to adapt. Formulating a tool to meet the information needs of a decision-maker and aid in the prescription of a water system design that satisfactorily balances the numerous (and conflicting) objectives, including minimization of risks, can be a challenging task.

The primary contributions of this paper are two-fold. First, we present a revision of a previously developed robust optimization (RO) model is presented (Watkins and McKinney 1997). That formulation's treatment of the risk of water system cost overruns (solution robustness term), modeled as the standard deviation (SD) of possible future water-related costs, is not monotonically increasing and therefore penalizes cost deviations irrationally. This leads to irrational second-stage decisions when a relatively large weight is placed on minimizing cost overruns. The improved formulation remedies the flaw and results in rational second-stage decisions under all cases. Second, we demonstrate the use of postprocessing is demonstrated to provide additional information regarding the anticipated performance of designs recommended by the model. Although it may not be reasonable to include measures of every kind of risk in a multiobjective function (in the example of this paper only two are included at a time, in addition to investment cost), postprocessing can provide decision-makers with information relative to as wide a range of risks as possible and perhaps even lead to adjustments in optimization model formulation. In the example presented in this paper, postprocessing of results informed development of the improved RO model.

Optimization under Uncertainty

According to Sahinidis (2004), there are three general methods for optimization under uncertainty: (1) stochastic programming, (2) fuzzy programming, and (3) stochastic dynamic programming. Stochastic programming includes the following: (1) standard approaches using recourse models [termed two-stage or multistage stochastic linear/nonlinear programs; Sen and Higle (1999) provides an introductory tutorial on stochastic programming], (2) robust optimization as described next, and (3) probabilistic models [chance constraints, attributed to Charnes and Cooper (1959)]. Loucks et al. (1981) and Tung (1986) provide an introduction to and applications of chance constraints to water resources problems. Potentially a fourth class of methods for optimization under uncertainty, evolutionary optimization algorithms (EAs), can be connected to Monte Carlo simulation models of water resources systems e.g., Kasprzyk et al. (2009).

Of all of these approaches, the focus of this paper is on RO and its flexibility in the consideration of performance metrics that might be of interest to a decision-maker. In contrast to chance constraint techniques, for example, which can only limit

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the probability of a violation of a model constraint, RO offers a means of simultaneously controlling the sensitivity of the solution to any uncertain parameters or inputs, and penalizing exponentially larger violations of one or multiple model constraints. There are two forms of RO: (1) those that guarantee satisfaction of hard constraints, and (2) those that apply penalties to violations of soft constraints. The field of manufacturing and engineering science has tended to emphasize formulations that guarantee the satisfaction of hard constraints, thus leading to single optimal robust solutions (Taguchi 1986). In contrast, Mulvey et al. (1995) recommended a mathematical programming approach based on a tradeoff between solution robustness (nearness to optimality across all scenarios) and feasibility robustness (nearness to feasibility across all scenarios). Their RO formulation extends stochastic programming to a multiobjective optimization framework that includes higher moments of the objective value (variance, most commonly) and penalty function(s) on violations of a chosen constraint(s). The scenarios used in RO are discrete points in an empirical probability distribution (or joint probability distribution), generated to represent best current understanding of the relative likelihood of potential future system states.

Robust Optimization Formulations in Water Resource Systems Planning

The RO technique introduced by Mulvey et al. (1995) is relevant to the field of environmental and water resource (EWR) systems planning and management, in which decisions often involve multiple objectives and soft constraints. For example, RO allows evaluation of tradeoffs between expected direct costs (direct costs have market value, such as capital costs and water transfer costs), indirect costs (e.g., conservation programs and water shortages), and performance costs (e.g., reliability), as well as the variability in these metrics (e.g., risk).

A two-stage stochastic programming model may be formulated as follows:

$$\underset{x,y}{\text{Minimize}} \sum_{s\in\Omega} p_s \xi_s \qquad (1)$$

Subject to

$$\mathbf{Ay} = \mathbf{b} \qquad (2a)$$

$$\mathbf{g}(\mathbf{x_s}) + \mathbf{B_s y} = \mathbf{d_s}, \quad s \in \mathbf{\Omega} \qquad (2b)$$

$$\mathbf{x_s}, \mathbf{y} \geq 0, \quad s \in \mathbf{\Omega} \qquad (2c)$$

where $\boldsymbol{\xi} = \mathbf{c}^T\mathbf{y} + f(\mathbf{x})$; ξ_s = value of the realization of the objective function in some future scenario s, which is composed of structural decisions variables $\mathbf{y}$ with associated cost coefficients $\mathbf{c}$ and the operational costs of the system $f(\mathbf{x})$, which are functions of control variables $\mathbf{x}$ that are chosen after the uncertain parameters are observed. Each scenario $s \in \mathbf{\Omega}$ occurs with a probability p_s and $\sum p_s \xi_s$ is the expected value of the objective function. Within the constraint set, $\mathbf{A}$ is a matrix defining the structural constraints of the problem; $\mathbf{b}$ is the corresponding right-hand side (RHS) vector; $\mathbf{g}(\mathbf{x})$ are functions representing the response of the system to the values of the control variables; $\mathbf{B}$ is a matrix of coefficients representing the effects of the structural decisions on system performance; and $\mathbf{d}$ is the corresponding RHS vector.

The RO formulation (Mulvey et al. 1995) extends the expected value formulation to capture risk-averse behavior in the objective function and allow soft constraints [Eq. (2*b*)] to be violated at a cost

$$\underset{x,y}{\text{Minimize}} \quad \sigma(\xi_1, \ldots, \xi_s) + \omega\rho(z_1, \ldots, z_s) \qquad (3)$$

subject to Eqs. (2*a*), (2*c*), and an expansion of Eq. (2*b*) including $\mathbf{z}$, a set of infeasibility variables $\{z_1, \ldots, z_s\}$ that measure the amount by which the control constraints are violated, i.e., $\mathbf{g}(\mathbf{x_s}) + \mathbf{B_s y} + \mathbf{z_s} = \mathbf{d_s}$, $s \in \mathbf{\Omega}$, where $\sigma(\xi_1, \ldots, \xi_s)$ is an aggregate objective function containing information pertaining to the performance of the solution under all scenarios; $\rho(z_1, \ldots, z_s)$ is a feasibility penalty function that penalizes constraint violations under all scenarios; and ω = weight indicating the degree of acceptance or rejection of infeasibilities in the solution.

EAs are more flexible than classical mathematical programming formulations with respect to nonlinearities in the objective and constraints and can more easily absorb expansions to the objective function from single-objective to multiobjective to many-objective (Reed and Minsker 2004). However, the focus of this paper is an in-depth exploration of the performance tradeoffs generated using a simple design problem, which does not require the added flexibility of an EA. Thus, the problem is solved using the more classical mathematical programming techniques, which are more computationally efficient and easily allow decisions to be made in two or more stages.

In a review of the literature, Ray (2010) encountered significant inconsistency regarding the use of the term RO in the EWR field. Most examples of this type in EWR involve stochastic programming, as did Mulvey et al. (1995). Some however are built on the framework of fuzzy optimization (Li et al. 2006) or EAs (Cui and Kuczera 2003). Most that have claimed to employ RO in EWR present tradeoffs between immediate capital cost and some other performance metric, although not always (Rosenberg and Lund 2009). Some that closely follow the approach of Mulvey et al. (1995) do not adopt the term RO (Kapelan et al. 2005), whereas others that use the term RO do not (Chung et al. 2009). To place the research reported in this paper in the context of previous applications, *the writers define RO as any optimization technique explicitly incorporating uncertain input data (model parameters) resulting in a tradeoff between multiple objectives, with increasing robustness with respect to one performance metric gained at a cost in some other performance metric*.

Applications of this type of RO in EWR range from water distribution system design (Cunha and Sousa 2010) and wastewater-treatment design (Afonso and Cunha 2007) to the design of large-scale water systems (Escudero 2000), as well as the design of groundwater pump and treatment systems (Ricciardi et al. 2009). Nearly all previous EWR applications involve only feasibility robustness, $\underset{x,y}{\text{minimize}} \sum_{s\in\Omega} p_s\xi_s + \omega\rho(z_1, \ldots, z_s)$, and do not consider solution robustness. Most such examples involve groundwater remediation applications (Bau and Mayer 2006; Ricciardi et al. 2007; Bayer et al. 2008; Alcolea et al. 2009; Ko and Lee 2009). As presented in the literature review by Ray (2010), only a few applications of RO to EWR systems also included solution robustness through minimization of the variance/SD of direct cost in the objective function $\underset{x,y}{\text{minimize}} \sum_{s\in\Omega} p_s\xi_s + \sigma(\xi_1, \ldots, \xi_s) + \omega\rho(z_1, \ldots, z_s)$ (Watkins and McKinney 1997; Suh and Lee 2002; Kawachi and Maeda 2004; Kasprzyk et al. 2009; Ray 2010).

One plausible reason for the failure of these previous studies to incorporate metrics for solution robustness other than computational demand is that the two-stage mean/variance model for incorporation of solution robustness suffers from a fundamental shortcoming (Bai et al. 1997). In particular, the minimization of variance, a symmetrical measure of risk, penalizes outcomes that are better than the expected value just as it penalizes outcomes that are worse. As a result, as documented by King (1993), mean/variance functions do not always increase monotonically, an important requirement of utility functions. Fig. 1 illustrates this,

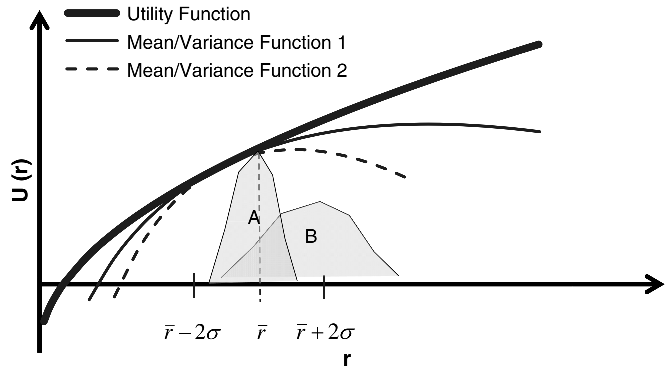

Fig. 1. Mean-variance approximations to a utility function; function 1 appears to be a good approximation, given the SD of return σ; function 2, corresponding to a larger weight on the variance term than in function 1, is a poor approximation and may lead to preference for a stochastically dominated solution (e.g., return distribution A dominated by return distribution B)

showing two mean-variance approximations to a (monotonic) utility function (Watkins 1997). The function incorporating a larger penalty on variance would lead to preference of return distribution A (higher total utility, measured as the aggregate value of mean utility and weighted variance of utility, over a narrow range of r) over B, even though distribution B is stochastically dominant (meaning that in B the system never provides less of what is wanted r than does A). In separate studies, Sen and Higle (1999) and Takriti and Ahmed (2004) demonstrated that the RO model of Mulvey et al. (1995) can be suboptimal when compared to the solution of the more basic two-stage stochastic recourse formulation and that the variability of the RO solution may be underestimated. Takriti and Ahmed (2004) showed that modifying the objective function to minimize positive deviations from a fixed target direct cost remedies this problem, a finding derived and reinforced in the model formulation of this paper.

For illustration of these RO model limitations, an example is used of a city faced with potential water scarcity and uncertain water demand.

Robust Optimization Formulation: An Example

In this example [from Lund and Israel (1995), as adapted by Watkins and McKinney (1997) and Ray (2010)], a city is subject to water scarcity and must plan accordingly. The objective is to minimize the cost of satisfying next year's water requirements through decisions made now (first stage) and utilization of options once next year's conditions are realized (second stage). The city has only one immediate (first stage) option; it can build desalination capacity or not. Upon making this decision, if local water availability next year is insufficient, the city can take combinations of the following three courses of action: (1) utilize its constructed desalination capacity, (2) make emergency spot-market transfers at market prices, and (3) accept some amount of water conservation (shortage) at a cost.

One possible formulation of this problem is minimization of expected total (direct and indirect) cost, as in Lund and Israel (1995). They represented potential water availability and use in the subsequent year through a set of discrete scenarios with corresponding probabilities. Local water availability and the price of an emergency water transfer were perfectly correlated in accordance with the probabilities assigned to supply scenarios. Water use was also represented as a random input parameter but assigned probabilities independent of those related to supply. Model 1 [Eq. (4)] shows a two-stage stochastic nonlinear program for minimization of the direct cost (in dollars) of decisions made now, plus the expected value of next year's decisions

$$\min Z_1 = c_c Q + \sum_{s\in S}\sum_{r\in R} p_r p_s [c_o Uq_{rs} + c_t Ut_{rs} + \eta(Us_{rs})^\gamma] \quad (4)$$

Subject to

$$\text{maximum use of desalination capacity } Uq_{rs} \le Q \quad \forall_r, \quad \forall_s \quad (5a)$$

$$\text{satisfy water requirement } a_s + Uq_{rs} + Ut_{rs} + Us_{rs} \ge d_r \quad \forall_r, \quad \forall_s \quad (5b)$$

$$\text{nonnegativity } Q, Uq_{rs}, Ut_{rs}, Us_{rs} \ge 0 \quad \forall_r, \quad \forall_s \quad (5c)$$

where Z_1 is the objective function value, total cost (direct and indirect), to be minimized; c_c = unit capital cost of desalination plant [\$/ million cubic meters (MCM)]; Q = capacity of desalination plant (MCM/year); p_r = probability of water requirement event r; p_s = probability of supply event s; c_o = amortized unit operation and maintenance cost of desalination plant (\$/MCM); Uq_{rs} = capacity of desalination plant actually used (MCM/year); c_t = unit cost of water transfer (\$/MCM); Ut_{rs} = quantity of transfer water purchased (MCM/year); Us_{rs} = quantity of water shortage (MCM/year); $\eta Us_{rs}{}^\gamma$ = nonlinear cost of water shortage (\$/MCM); η and γ constants; a_s = local water availability in realization s occurring with probability p_s; and d_r = water requirement in realization r occurring with probability p_r.

Model 1 yields the least-cost solution to the overall problem (both stages). It is an improvement over a deterministic formulation using the mean values of availability and demand because it takes into account uncertainties in next year's water availability and water requirement by minimizing over a range of input scenarios (Table 4, discussed in greater depth in the Results section). It recommends the construction of a certain amount of desalination capacity based upon next year's expected conditions. Less consequentially, it provides values for the second-stage variables under each scenario. Under expected conditions, based upon the prescribed first-stage decision, the city will make use of $E[Uq_{rs}]$ amount of the constructed desalination capacity, transfer $E[Ut_{rs}]$ amount of water, and have a shortage of size $E[Us_{rs}]$. The formulation is an example of expected-value decision-making and does not factor in a decision-maker's likely aversion to risk.

Adding measures of risk to the two-stage stochastic nonlinear program, Model 2 [Eq. (6)] is a multiobjective two-stage robust optimization model (MO-RO; Watkins and McKinney 1997). Model 2 extends Model 1 by additionally considering the SD of second-stage decisions (solution robustness term), plus a penalty on water shortages (feasibility robustness term). These additional terms are weighted by ω_1 and ω_2, respectively, in accordance with the decision-maker's aversion to risk. Each of the weights can be varied to trace out the Pareto-optimal frontier of tradeoffs between expected system performance and robustness

$$\begin{aligned}\min Z_2 = {} & c_c Q + \sum_{s\in S}\sum_{r\in R} p_r p_s \xi_{rs} + \omega_1 \\ & \times \left[\sum\nolimits_{s\in S}\sum\nolimits_{r\in R} p_r p_s \left(\xi_{rs} - \sum_{s'\in S}\sum_{r'\in R} p_{r'} p_{s'} \xi_{r's'}\right)^2\right]^{0.5} \\ & + \omega_2 \sum_{s\in S}\sum_{r\in R} p_r p_s \eta (Us_{rs})^\gamma \end{aligned} \quad (6)$$

Subject to

$$\text{maximum use of desalination capacity } Uq_{rs} \leq Q \quad \forall_r, \quad \forall_s \tag{7a}$$

$$\text{satisfy water requirement } a_s + Uq_{rs} + Ut_{rs} + Us_{rs} \geq d_r \quad \forall_r, \quad \forall_s \tag{7b}$$

$$\text{limit shortage } Us_{rs} \leq 0.10 \cdot d_r \quad \forall_r, \quad \forall_s \tag{7c}$$

$$\text{nonnegativity } Q, Uq_{rs}, Ut_{rs}, Us_{rs} \geq 0 \quad \forall_r, \quad \forall_s \tag{7d}$$

where $\xi_{rs} = c_o Uq_{rs} + c_t Ut_{rs}$. The prime notation indicates that the inner summations over the sets R and S occur separately from the outer summations.

In addition to the introduction of risk factors into the objective function, there are two significant differences between Models 1 and 2. The shortage cost function is no longer minimized along with the expected cost of desalination and water transfers. It has been removed from the expected direct cost summation and placed on its own, as a weighted function of the risk of shortage (indirect cost). To better evaluate the tradeoff between the expected value and the SD of direct cost, the amount of water conservation in each scenario is limited to 10% of that scenario's water requirement, thereby hedging against the unlikely scenario in which there is high water requirement and low water availability, resulting in an extreme water shortage. In this example, the top design priority then becomes to hedge against catastrophe.

Performance Metrics

The performance metrics explicitly incorporated into Model 2 are the expected direct cost and two risk metrics: (1) solution robustness (i.e., the minimization of variance of total direct cost), and (2) feasibility robustness (i.e., the added penalty on indirect shortage costs). In addition, three other common performance metrics are evaluated in a postprocessing analysis: (1) reliability, (2) vulnerability, and (3) sustainability.

Reliability, as defined by Hashimoto et al. (1982), is the probability α that a system is in a satisfactory state. Eq. (8) presents reliability as the probability that the system's output state, denoted by the random variable X, is not a member of F, the set of all unsatisfactory (failure) outputs. The definition of what constitutes a failure is somewhat subjective and in practice stakeholders would arrive at such a definition through consensus. In the writers' case, a failure is defined as the occurrence of a nonpreferred outcome of any magnitude (e.g., a shortage of any size Us_{rs} or a water transfer of any size Ut_{rs})

$$\alpha = 1 - \sum_{s \in S} \sum_{r \in R} P[X_{rs} \in \mathbf{F}] \tag{8}$$

Relative vulnerability v quantifies the relative magnitude of failure, given that one occurs. Relative vulnerability ranges from 0–1, with the scenario of maximum system vulnerability producing the failure of greatest magnitude. Although Loucks (1997) presented vulnerability as the expected magnitude of shortage relative to the maximum shortage, in this study vulnerability is presented as the expected magnitude of a nonpreferred outcome (e.g., a shortage Us_{rs} or a water transfer Ut_{rs}) relative to the magnitude of the water requirement

$$v = \frac{E[X_{rs} | X_{rs} \in \mathbf{F}]}{E[d_r]} \tag{9}$$

Sustainability can be defined in a number of ways. From Loucks (1997), a sustainable alternative is one in which there are no long term decreases in the level of welfare produced by the system. If the statistical measures for reliability α, resilience r, and in-vulnerability $(1 - v)$ range from 0–1 with higher values preferred over lower values, as is true in this paper, then Loucks (1997) suggested defining sustainability ϕ as the product

$$\phi = \alpha \cdot (1 - v) \cdot r \tag{10}$$

Unfortunately, a single-year model provides no insight into the resilience of the system and hence the writers assume $r = 1$. The sustainability of a water system could be evaluated using a multistage (contiguous stages) optimization model, including resilience in the definition of sustainability (Cai et al. 2002). However, Kjeldsen and Rosbjerg (2004) observed that the high correlation between vulnerability and resilience guarantees that solutions with low vulnerability also are marked by high resilience. They therefore recommended against the use of both vulnerability and resilience in the Loucks (1997) equation for sustainability, and thus the assumption that $r = 1$ in Eq. (10) may be reasonable.

Tables 1–3 summarize the input data for this problem. Next year's available water supply, spot market transfer price, and quantity required are modeled as random normal variables. Local water

Table 1. Water Supply Scenarios for Input to the Robust Optimization Model

Supply scenario	Probability of supply event p_s	Local water availability event a_s (MCM/year)	Spot market price event c_t (\$/MCM)
1	0.000078	0.0	300,000
2	0.000489	20.0	281,250
3	0.002403	40.0	262,500
4	0.009245	60.0	243,750
5	0.027835	80.0	225,000
6	0.065591	100.0	206,250
7	0.120978	120.0	187,500
8	0.174666	140.0	168,750
9	0.197413	160.0	150,000
10	0.174666	180.0	131,250
11	0.120978	200.0	112,500
12	0.065591	220.0	93,750
13	0.027835	240.0	75,000
14	0.009245	260.0	56,250
15	0.002403	280.0	37,500
16	0.000489	300.0	18,750
17	0.000078	320.0	0

Table 2. Water Requirement Scenarios for Input to the Robust Optimization Model

Water requirement scenario	Probability of water requirement event p_r	Water requirement d_r (MCM/year)
1	0.00088	140.0
2	0.02951	160.0
3	0.23559	180.0
4	0.46803	200.0
5	0.23559	220.0
6	0.02951	240.0
7	0.00088	260.0

Table 3. Cost Coefficients

Coefficient	Value
Desalination capital cost c_c	\$30,000/MCM
Desalination operations and management cost c_o	\$80,000/MCM
Shortage function cost coefficient η	6,000
Shortage function cost coefficient γ	2

availability is generated from a normal distribution with $\mu =$ 160 MCM/year and coefficient of variation $C_v = 0.25$, the spot market transfer price with $\mu = \$150{,}000$/MCM and $C_v = 0.25$, and the water requirement with $\mu = 200$ MCM/year and $C_v = 0.08$. The supply function and water requirement functions are then discretized to formulate the scenario-based optimization model in *GAMS* (Brooke et al. 1992). The local water availability and spot market water transfer price are assumed to be perfectly correlated. For example, if local/regional water is scarce next year, the price of a transfer will be higher than average. The water use target is assumed to be independent of water availability.

Model Results

Solving this problem deterministically, using mean values for water requirement (200 MCM/year), water availability (160 MCM/year), and cost of transfer ($150,000/MCM), the total (first and second stage) expected direct cost would be $3.4 million and the expected penalty on shortage would be $500,000, for a total objective function value of $3.9 million. The modeler would recommend the construction of a desalination plant of size 30.83 MCM/year and expect the operator to use it all ($Uq = 30.83$ MCM). The modeler would expect no transfer ($Ut = 0$ MCM) and a shortage of just less than 10 MCM ($Us = 9.17$ MCM). The cost of this deterministic solution under uncertainty is much higher (Table 4). If the deterministically prescribed first-stage decision ($Q = 30.83$) is subjected to the range of possible future scenarios (not just their expected values), with optimal decisions made in the second stage, the total direct cost would be $5.4 million, with a $C_v \approx 1$. This is because the quality of the deterministic solution is entirely contingent upon the (arbitrary) realization of the singular expected scenario.

The previously discussed results motivate use of the stochastic model, Model 1, which results in a direct cost of $E[\text{direct cost}] =$ 5.4 million, and indirect costs of $E[\text{cost } Us] = \$540{,}000$, for a total objective function value of $E[Z_1] = \$5.9$ million, all of which are superior to the deterministic solution applied under uncertainty. The recommendation would be for the construction of a larger desalination plant of size 52.4 MCM/year, with the expectation that just more than half of it ($E[Uq] = 29.7$ MCM) would be used under average conditions. The stochastic model indicates that the expected transfer and shortage would be 6.9 and 7.5 MCM, respectively. The stochastic model also reduces the SD of the direct to approximately $4.5 million. Other performance metrics are computed through postprocessing of the optimization model results.

The stochastic solution (Model 1) is both more feasible (performance metrics in terms of shortages) and more cost-effective (SD of direct cost) across a wider range of scenarios than the deterministic solution. Robustness can be improved further by introducing the basic MO-RO model (Model 2). To demonstrate this, for each solution of Model 2, the Q must be fixed in Model 1 and the second-stage operational variables are optimized. These postprocessed second-stage solutions (Uq, Ut, and Us) are compared to those produced by Model 2. The value of Model 2 is seen in its ability to control the SD of the direct cost. By varying the weight ω_1 in the objective function of Model 2 (and fixing corresponding Q values in Model 1 for comparison), a tradeoff curve is created between the expected direct cost and SD of direct cost (Fig. 2). The greatest reductions in the SD of direct cost are made during the initial increases in ω_1, with diminishing returns thereafter. Likely, a decision-maker would prefer to choose a desalination plant capacity with the $E[\text{direct cost}]$ approximately the same for both Model 2 and Model 1 but with the SD of cost SD(cost) significantly reduced; however, decision-makers would almost certainly not want to choose a point at the far right-hand of the plot in which the decrease in SD(cost) has been won at a considerable increase in expected cost relative to the stochastic solution.

Unfortunately, Model 2 can overestimate the reduction in variance due to generation of irrational second-stage decisions. Both Models 1 and 2 begin with $Q = 52.4$ MCM/year, in which the solutions are the same (high SD and low expected direct cost; Fig. 2). However, as Q increases, the expected direct cost of the Model 1 result rises more slowly than the expected direct cost of Model 2 and the SD of the Model 1 result falls more slowly. This occurs because Model 2 provides a wider range of operational choices than does Model 1. When other performance measures are considered, the two methods yield strikingly different results (Figs. 3–5). Figs. 3–5 demonstrate the performance of Model 2 relative to the postprocessed second-stage indications of failure (Ut, Us) of Model 1.

Fig. 3 shows that, in terms of shortages, by using more of the desalination design capacity, Model 2 appears to make great gains in reliability. Model 1, choosing to use nearly the same amount of desalinated water regardless of the design capacity, does not realize gains in reliability in accordance with increases in Q. Neither does it result in vulnerability increases as does Model 2. The end result is that Model 2 leads to more reliable and sustainable results in terms of shortages for design capacities between approximately 100 and 170 Mm^3/year, yet counter-intuitively Model 2 leads to more vulnerability to shortage events in that range.

Table 4. Comparison of Deterministic and Stochastic Model Results

Variable definition	Model result	Deterministic solution with no uncertainty	Deterministic solution under uncertainty	Model 1
Desal plant capacity	Q, MCM/year	30.8	30.8	52.4
E requirement	$E[d]$	—	200.0	200.0
Desal capacity used	$E[Uq]$, MCM	30.8	20.4	29.7
Transfer water used	$E[Ut]$, MCM	0	14.7	6.9
Shortage	$E[Us]$, MCM	9.17	9.0	7.5
Total objective value	$E[Z]$, \$1 million	3.896	6.141	5.908
Expected direct cost	$E[\text{cost}]$, \$1 million	3.392	5.427	5.370
SD of direct cost	SD cost, \$1 million	—	5.459	4.472
Expected cost of shortage	$E[\text{cost } Us]$, \$1 million	0.504	0.714	0.538
Expected size of shortage if one occurs	$E[Us]\|$shortage event	—	11.9	10.0
Reliability	α	—	0.245	0.245
Vulnerability	v	—	0.06	0.05
Sustainability	ϕ	—	0.230	0.233

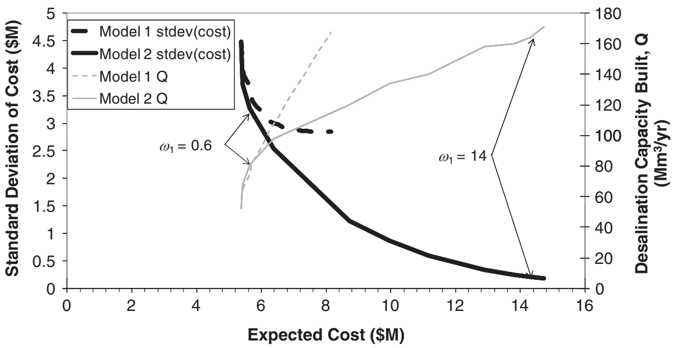

Fig. 2. Tradeoff between expected direct cost and solution robustness, Model 2 versus Model 1; to perform this analysis, desalination capacity Q in Model 1 is treated as a parameter (not a variable) and fixed to a value corresponding to the given weight ω_1 in Model 2

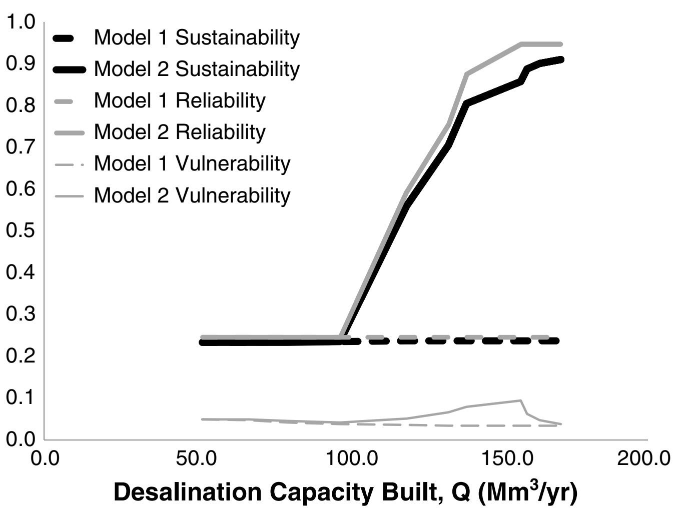

Fig. 3. Comparison of model performance in terms of shortages, Model 2 versus Model 1, controlled as described in the caption of Fig. 2

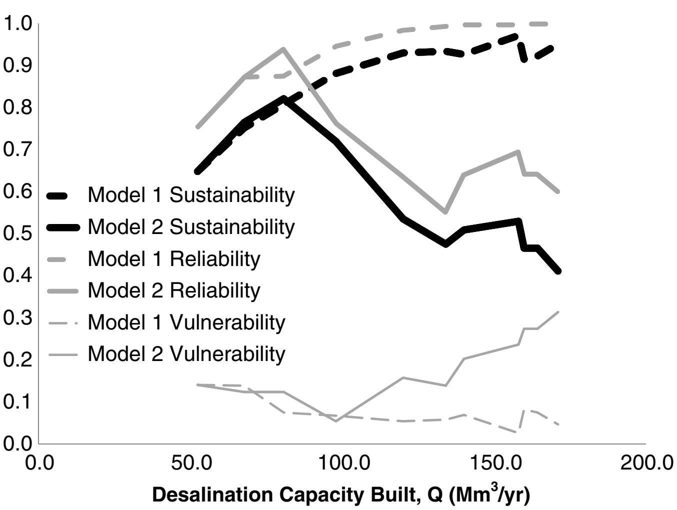

Fig. 4. Comparison of model performance in terms of transfers, Model 2 versus Model 1, controlled as described in the caption of Fig. 2

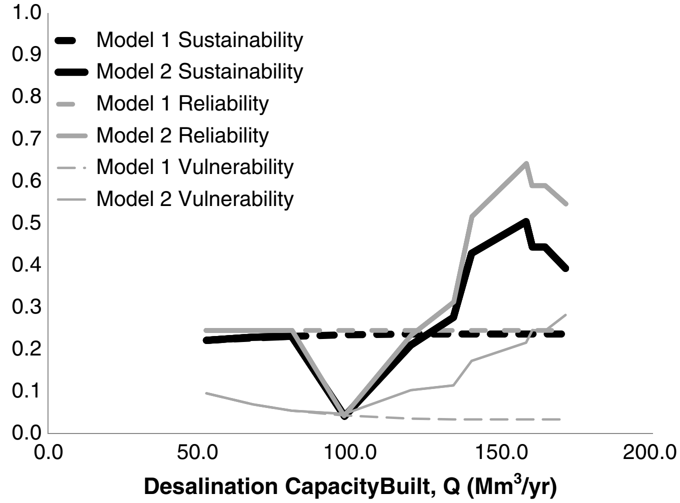

Fig. 5. Comparison of model performance in terms of all recourse actions, Model 2 versus Model 1, controlled as described in the caption of Fig. 2

A comparison of the results with respect to transfers (Fig. 4) indicates poorer performance measures for Model 2. Fig. 4 shows that the difference in the performance of Models 1 and 2 stems in part from the total quantity of water supplied (or oversupplied) by transfers. Taking a closer look at second-stage decisions generally, Fig. 5 shows the system reliability, vulnerability, and sustainability in terms of all recourse actions (shortage and/or transfer, e.g., reliability is the probability that neither a transfer nor a shortage is needed). The combination of the two metrics removes some of the variability in Model 2 and almost all the variability from the Model 1 results. Using Model 1, there are slim gains to be made in system performance metrics through an increase in desalination plant capacity.

At each fixed Q in Model 1 (Table 5), the total water supplied ($E[\text{avail}] + E[Uq] + E[Ut] + E[Us]$) exactly equals the water use requirement of 200 MCM/year. To reduce the SD of direct cost, Model 2 sometimes supplies more water (through surplus transfers and desalination usage) than required. If the decision-maker has reason to be risk averse with respect to water transfers (from a tenuously allied neighbor, for example), then this finding has particular significance. Even under scenarios of abundant local water availability, Model 2 still recommends large usage of desalinated water in excess of water-use requirements just to reduce the variance in direct costs. This is an example of the model behavior (Fig. 1), in which the tradeoff between expected cost and SD of cost seems to have value only to a point, after which it would be irrational to choose a solution with a smaller SD. Watkins (1997) noted this breakdown of the expected cost/SD formulation: "In reality, once [the system capacity] is chosen and a given scenario s is observed, the response which maximizes second-stage return would be preferred to one which accepts a smaller return in order to minimize the variance between the observed return and the returns under all other scenarios considered in the model (i.e., all those which were not observed)."

Revised MO-RO Formulation

Ray (2010) developed three alternative MO-RO formulations to Model 2, with the preferred formulation, Model 3, given in Eq. (11). Model 3 penalizes the square of positive deviations from a fixed target cost (Takriti and Ahmed 2004). By considering only

Table 5. Snapshot of Models 1, 2, and 3 Solution Robustness Results

Variable definition	Model result	Model 1	Model 2 with $\omega_1 = 8$, $\omega_2 = 0$	Model 3 with $\omega_1 = 15$, $\omega_2 = 0$
Desal plant capacity	Q, MCM/year	52.4	158	158
E[requirement]	$E[d]$	200.0	200.0	200.0
Desal capacity used	$E[Uq]$, MCM	29.7	80.8	36.6
Transfer water used	$E[Ut]$, MCM	6.9	14.4	0.0
Shortage	$E[Us]$, MCM	7.5	1.0	7.5
Total nonlocal supply	Sum $E[U]$s	44.1	96.3	44.1
Expected supply and local water availability[a]	sum $E[U]s + E$[avail]	200.0	252.2	200.0
Excess supply	$a + U -$ demand	0.0	52.2	0.0
Expected direct cost	E[cost], \$1,000	5.4	12.9	7.7
SD of direct cost	SD cost, \$1,000	4.5	0.3	2.5
Expected cost of shortage	E[cost Us], \$1 M	0.5	0.1	0.6
Expected size of shortage if one occurs	E[Us]|shortage event	6.7	18.8	9.9
Reliability	α	0.245	0.946	0.245
Vulnerability	v	0.050	0.094	0.049
Sustainability	ϕ	0.233	0.858	0.233

Note: Reliability, vulnerability, and sustainability are with respect to shortages only, not total recourse actions.
[a]The expected local water availability is always 160 MCM. However, in certain supply scenarios the local water availability is greater than the requirement (wet years). In those scenarios neither shortage, transfer, nor use of desalination capacity occurs, but the local water availability exceeds the water requirement. The average of all of those excesses is 4.1. That 4.1 MCM is not supplied, but is available for supply, and results from a direct summation of the sum(E[U]values) and local water availability E[avail]. Table 5 shows the true expected water supply and the actual expected excess supply.

positive cost deviations, the objective function remains monotonic, as required for a utility function (Fig. 1)

$$\min Z = c_c Q + \sum_{s\in S}\sum_{r\in R} p_r p_s \xi_{rs} + \omega_1 \left\{ \sum_{s\in S}\sum_{r\in R} p_r p_s \cdot \max\,[0, (\xi_{rs} - \xi^T)]^2 \right\}^{0.5} + \omega_2 \sum_{s\in S}\sum_{r\in R} p_r p_s \eta U s_{rs}^{\gamma} \tag{11}$$

The subsequent continuous approximation is used for the discontinuous max(•) term

$$\max[0, (\xi_{rs} - 5370158)] = \frac{\sqrt{(\xi_{rs} - \xi^T)^2 + \varepsilon^2} + (\xi_{rs} - \xi^T)}{2} \tag{12}$$

where ξ^T is the fixed target cost; and ε is a very small constant, in this case 0.0001. The fixed target cost is chosen to be the expected direct cost of Model 2 with a zero weight on cost deviation $\omega_1 = 0$, E[cost] = \$5.370158 million.

Fig. 6 presents the tradeoff between expected direct cost and solution robustness for Model 3, a result substantially different from Model 2. Fig. 6 shows that Model 3 results very nearly

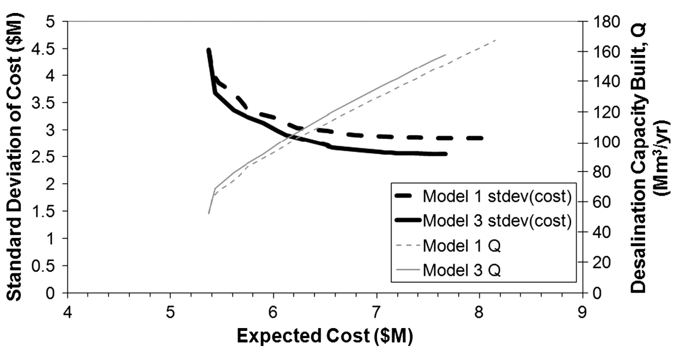

Fig. 6. Tradeoff between expected direct cost and solution robustness, Model 3 versus Model 1; to perform this analysis, desalination capacity Q in Model 1 is treated as a parameter (not a variable) and fixed to a value corresponding to the given weight ω_1 in Model 3

are in accordance with the results obtained from the Model 1 approach. At high values of ω_1, Model 3 achieves slightly better (smaller) SD at slightly better (smaller) direct cost than Model 1. All other models explored in the research reported in this paper and in Ray (2010) won smaller SD values than Model 1 at higher direct cost. In the results of Model 3, there is the possibility for outperforming Model 1 in terms of both shortages and transfers simultaneously.

Model 3 recommends operation of the system in essentially the same fashion as does Model 1 (Table 5). Unlike Model 2, Model 3 does not achieve its decrease (almost 45%) in SD of direct cost through irrational oversupply of water. It does not spuriously inflate reliability and sustainability with respect to shortage. Rather, it reduces reliance on highly cost-variable water transfers during dry years through increased (though not radically increased) use of its excess desalination plant capacity and accepting some amount of expected water shortage at a (fairly stable) direct cost. However, the improved optimality robustness of Model 3 comes at an elevated direct cost (up 43%) and a nearly 50% increase in $E[Us]$|shortage. This is a rational tradeoff for the decision-maker to consider and Model 3 provides a tool for its evaluation.

Holding ω_1 constant and varying the weight on the water shortage term ω_2 in the objective function of Model 3, the resulting tradeoffs between expected direct cost and expected cost of water shortage in accordance with increasing desalination capacity can be evaluated (Fig. 7). Model 1 results (Fig. 7) were created by holding constant $\omega_2 = 1$ and varying the design capacity Q to match that resulting from the corresponding Model 3 run. The point farthest to the left in Fig. 7 (high expected cost of shortage and low expected direct cost) is the result when $\omega_2 = 0$. As ω_2 is increased ($\Delta\omega_2 = 0.2$), the expected cost of shortage decreases and the expected direct cost increases. As in the case for solution robustness, the greatest gains in feasibility robustness are made during the initial increases in ω_2, with diminishing returns thereafter.

Fig. 7 shows that simply increasing the system desalination capacity Q will not result in a decrease in the expected cost of shortages. Models 1 and 3 each have available the same Q but over the range tested in this feasibility-robustness experiment produce very different results with respect to the expected cost of water shortages. Over the range of Q considered, there is not a great variation

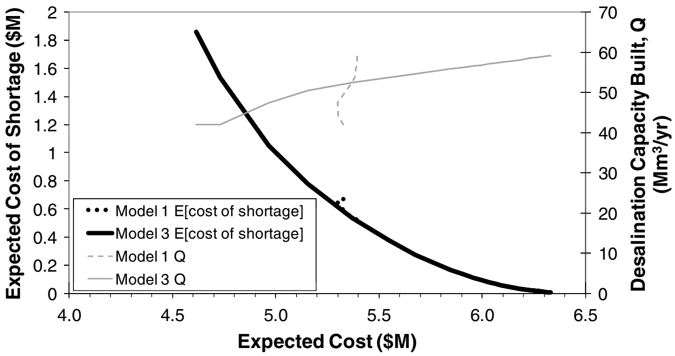

Fig. 7. Tradeoff between expected direct cost and feasibility robustness, Model 3 versus Model 1; to perform this analysis, desalination capacity Q in Model 1 is treated as a parameter (not a variable) and fixed to a value corresponding to the given weight ω_2 in Model 3

in system performance (reliability, vulnerability, and sustainability) in accordance with variations in ω_2. Most notable is the increased reliance on transfers in accordance with a decreased tolerance for shortages. This is demonstrated in the higher vulnerability of Model 2 to large transfers and lower sustainability in terms of transfers at larger values of ω_2 (larger values of Q).

The Model 3 formulation is able to reduce the cost of shortages dramatically by increasing ω_2 (Fig. 7). However, increasing ω_2 improves neither the reliability, vulnerability, nor sustainability of the design with respect to shortages, relative to the simple least-cost solution. In Model 1, $[Us_{rs}]|$shortage remains the same as Q increases. In Model 3, $E[Us_{rs}]|$shortage falls from 20.2 to 1.1 MCM/year. Due to the exponential cost function (as discussed previously) it is cost-effective to allow some small shortage in almost every scenario, decreasing the system reliability but not adding appreciably to total cost. Reliability remains low even at ω_2 values as high as 100.

Conclusion

This paper aimed at a deeper understanding of the design choices recommended by MO-RO models. Compared to a single-objective stochastic programming model (Model 1), the traditional MO-RO model [Model 2, the original development of which is credited to Mulvey et al. (1995)] effectively reduced the SD of the direct cost and cost of shortage over a range of design decisions, up to a certain point. Previous studies that optimize capital cost plus a single performance metric have neither demonstrated the operational tradeoffs inherent in the model's achievement of a low SD of direct cost nor a high reliability with respect to shortages. The case study illustrated, for example, that a low SD of direct cost might be achieved at the cost of a high vulnerability with respect to shortages and a significant dependency on water transfers. Without adequate consideration of the second-stage decisions (through postprocessing), the full ramifications of the model recommendations might be lost and the decision-maker ill-informed.

It should be stressed that the Model 1 formulation as it has been employed for the sake of comparison to other model versions in this work (holding Q constant and optimizing only second stage decisions) here is not an actual two-stage optimization model. If all of the first-stage decisions are already enumerated and applied to Model 1, then it produces logical results with which the decision maker should be informed. However, when the first-stage is high-dimensional, an MO-RO approach is needed.

The improved MO-RO formulation (Model 3), the solution robustness element of which minimized squared positive deviations from a fixed target cost, addressed known deficiencies in Model 2 and demonstrated them in a water resources context. Based upon comparative assessments using multiple performance metrics, Model 3 demonstrated the best ability to simultaneously control solution robustness and performance with respect to shortages, transfers, and total recourse actions.

The linkages between solution robustness and feasibility robustness are important. Improved optimality robustness of Model 3 comes at a cost to feasibility robustness (reduced shortages). This is a rational tradeoff for the decision-maker to consider and Model 3 provides a tool for its evaluation. As in the case for increasing weights on the solution robustness element in the objective function, the greatest gains in feasibility robustness are made during the initial increases in the weight on the shortage penalty function ω_2 with diminishing returns thereafter. Over the range of Q considered, a great variation in system performance was not observed in accordance with variations in ω_2. Most notable is the increased reliance on transfers with a decreased tolerance for shortages.

The three primary objectives minimized were (1) expected direct cost, (2) direct cost deviations, and (3) expected cost of water shortage, with three additional metrics: (1) reliability, (2) vulnerability, and (3) sustainability (evaluated by postprocessing). Future studies might experiment with different primary and secondary objectives to identify the ideal configuration of objectives for each category of decision-maker (budget-constrained, intolerant of water transfers, and so on). An EA formulation of the type developed by Kasprzyk et al. (2009) may be useful in this regard. However, given that EAs allow much greater complexity in their tradeoffs compared with traditional mathematical programming techniques (limited to simpler objective functions), thorough evaluation of system performance as presented in this paper may become even more important as a check on the reasonableness of those solutions.

Comparisons with alternative formulations of the MO-RO model results might be insightful: (1) reduction in the variance of recourse actions (Vladimirou and Zenios 1997), or (2) a more direct application of the ε-constraint method (Kawachi and Maeda 2004) and/or the robust chance constraint method described by Xu et al. (2009). In many cases, simulating second-stage decisions without the need to optimize them, focusing the optimization only on the choice of first-stage decisions (Kapelan et al. 2005), may be preferable.

The shortage of every scenario was limited to 10% of that scenario's water requirement, thereby hedging against the unlikely catastrophic scenario in which there is high water requirement and low water availability. This limitation on water shortage had a powerful effect on the model's results. The expected shortage could have been limited to 10% of the expected requirement across all scenarios, i.e., $E(\text{shortage}) \leq 0.10 \cdot E(\text{water requirement})$). When this was tested, the formulation yielded numerous scenarios with water shortages far in excess of 10% of the given scenario's water requirement. The result seemed to be overly permissive of shortages, so much so that the solutions to worst-case (high requirement and low water availability) scenarios allowed essentially a complete lack of water in the city. In light of this, the more conservative (and more common) approach of hedging against catastrophe was chosen, as unlikely as that catastrophe might be. Given the ability of worst-case scenarios with low probabilities to influence the entire set of results, it seems that this design choice should be reexamined and improved in the next version of the MO-RO model. The design choice described in this paper (hedging against catastrophe) has been advocated by those who value water infrastructure designs that function reasonably well no matter the climate scenario (Stakhiv 2011) and improvement in the controls imposed upon those solution-dominating, catastrophic (but unlikely) scenarios might have far-reaching ramifications. Adaptive designs that are able to adjust incrementally to increasing probabilities on system-straining scenarios are important in this respect.

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