

PROBABILITY DISTRIBUTION OF ANNUAL MAXIMUM, MEAN, AND MINIMUM STREAMFLOWS IN THE UNITED STATES

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ABSTRACT: L-moment diagrams are constructed for annual minimum, average, and maximum streamflows at more than 1,455 river basins in the United States. Goodness-of-fit comparisons reveal that the generalized extreme value (GEV), three-parameter lognormal (LN3) and the log Pearson Type III (LP3) distributions provide good approximations to the distribution of annual maximum flood flows. These results are consistent with other L-moment studies. A World Meteorological Organization survey of 54 agencies in 28 countries reveals that the LN3 distribution is not a standard in any country, GEV is a standard in one country, and LP3 is a standard in seven countries. The time is ripe for agencies and countries to reevaluate their standards with respect to the choice of a suitable model for flood frequency analysis. L-moment diagrams also reveal that among numerous alternatives, the Pearson Type III (P3) distribution provides the best fit to both annual minimum and annual average streamflows.

INTRODUCTION

Since the introduction of L-moments (Hosking 1990), numerous investigators have recommended them to assess the goodness of fit of various probability distributions (PDs) to regional samples of streamflow and precipitation [Chowdhury et al. (1991), Hosking and Wallis (1993), Stedinger et al. (1993), and Vogel and Fennessey (1993), and others]. Since the theory of L-moments is covered in the initial studies we assume a familiarity with those studies. We focus on the selection of suitable PDs for sequences of annual maximum, minimum, and average streamflow in all regions of the continental United States. Research documents that regional flood frequency methods are preferred to the use of at-site methods [see Cunneane (1989), Stedinger et al. (1993), and Bobee and Rasmussen (1995) for recent reviews]. Hosking and Wallis (1993) organize regional flood frequency analysis into four stages: (1) Screening of the data; (2) identification of homogeneous regions; (3) choice of a regional PD; and (4) estimation of the regional PD. Normally, if one's final goal is to estimate a regional flood frequency distribution, one should proceed from one stage to the next, without skipping intermediate stages. If a suitable PD can be found for the entire continental United States, the second stage becomes moot in the context of selecting a suitable PD for the nation as a whole. The identification of homogeneous regions (step 2) is normally only important for identifying a regional shape or skew parameter to be used with the regional estimation procedure (step 4); however, this study does not address those issues.

This study uses L-moment diagrams to select a regional PD for sequences of annual maximum flood flows, annual average daily streamflows, and annual minimum low flows. Since their introduction to the water resources literature, L-moment diagrams have been used repeatedly to assess the goodness of fit of flood flows; however, to our knowledge, there is only one study for average annual flows (Vogel et al. 1995), and one study for annual minimum low flows (Pearson 1995).

Numerous goodness-of-fit procedures exist for comparing

the fit of alternative probability distributions to streamflow sequences. Fill and Stedinger (1995) endorse the use of unbiased L-moment estimators over other goodness-of-fit tests for the Gumbel distribution. In spite of the many recent endorsements of the use of unbiased L-moment estimators and L-moment diagrams, Chow and Watt (1994) document that L-moment diagrams alone cannot provide definitive goodness-of-fit assessments, using only a few dozen sites. Chow and Watt (1994) use L-moment diagrams to document that the 25 sites evaluated by Pilon and Adamowski (1992) could have originated from a Gumbel distribution, instead of a generalized extreme value (GEV) distribution. This study constructs L-moment diagrams for more than 1,455 river basins in the United States, in the hope that using a database of this size will allow us to discriminate among alternative PDs more clearly and definitively.

DATABASE OF STREAMFLOW

The data set consists of records of average daily streamflow at 1,570 sites located throughout the continental United States, available on CD-ROM from the U.S. Geological Survey (Slack and Landwehr 1993). Fig. 1 illustrates the location of the sites. This data set, termed the hydro-climate data network (HCDN), includes flow records for sites with record lengths ranging from six to 115 years, with an average of 45.5 years of record per site or a total of 73,231 site years of data. The distribution of record lengths associated with the flow records is illustrated in Fig. 2. All flow records used here are based on water years. Annual maximum and annual minimum flow series are based on average daily flows rather than on instantaneous peak maximum or instantaneous peak minimum flows.

The development of the HCDN was a large undertaking,



FIG. 1. Location of Streamflow Gauges [from Slack and Landwehr (1993)]

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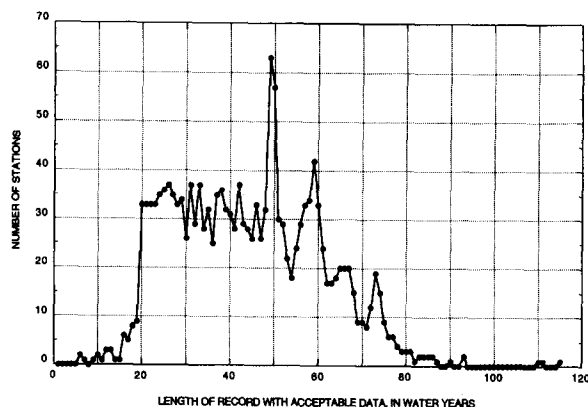


FIG. 2. Distribution of Record Lengths Associated with Flow Records

TABLE 1. Summary of Number of Sites Used in L-Moment Diagrams

Type of streamflow series (1)	Number of discordant sites (2)	Number of sites dropped due to zero flows (3)	Total number of sites used (4)
Average annual flows	89	0	1,481
Annual minimum low flows	78	37 sites with all zeros	1,455
Annual maximum flood flows	80	0	1,490
Logarithms of average annual flows	44	19	1,507
Logarithms of annual minimum low flows	47	398	1,125
Logarithms of annual maximum flood flows	90	18	1,462

which included screening the data in a variety of ways. Data specialists at each U.S. Geological Survey (USGS) district office along with data specialists at the national headquarters in Reston, Virginia, reviewed records based on the following criteria: (1) Availability of data in electronic form; (2) record lengths in excess of 20 years unless site location is under-represented; (3) accuracy rating of records had to be at least "good" as defined by USGS standards; (4) no overt adjustment of "natural" monthly streamflows by flow diversion, augmentation, ground-water pumping, or other forms of regulation; (5) only measured discharge values are tabulated, no reconstructed or estimated records are used.

We follow the steps outlined in Hosking and Wallis (1993) for identifying a regional parent PD for the continental United States. The first step is to screen the data so that gross errors and inconsistencies can be eliminated. For this purpose Hosking and Wallis (1993) introduced a discordancy measure D_i , which allows for an objective determination of which sites, i , in a region, are grossly discordant with the group as a whole. For each flow series, sites were dropped with discordancy measures $D_i > 3$. In addition, construction of L-moment diagrams for the natural logarithm of flows required dropping sites with any zero observations. Table 1 summarizes the number of sites dropped due to discordancy criteria and zero observations. There were 37 sites in arid portions of the United States for which all observations of annual minimum streamflow were zero. With the exception of the series of logarithms of annual minimum low flows, there are at least 1,455 of the 1,570 original sites (or 93%) remaining for all flow series after discordant sites and sites with zeros are dropped.

Evaluation of Potential Trends in Annual Streamflow

The HCDN was specifically designed for studies of unimpaired streamflows so that basic hydroclimatic relationships could be explored without complications due to anthropogenic influences. The USGS HCDN only contains streamflow data for basins in which there has been no obvious adjustment of natural streamflow through hydraulic diversion and/or regulation. Although the hydrology of these basins is probably subject to some degree of anthropogenic influence such as land-use modifications, we hypothesize that the effects are not discernible and that the flow series are approximately stationary. To test this hypothesis, the 1,481 time series of streamflow were evaluated to determine whether any temporal trends were evident using a simple linear regression test. The linear regression test uses a t -test at each site to determine whether or not there is a significant linear relationship between average annual streamflow and time. The model $Q_t = a + bt + \epsilon$ is fit at each site, where Q_t is annual flow in year t , a and b are estimated parameters, t is year, and ϵ are model errors. Under the null hypothesis of no linear trend we assume $Q_t = \alpha + \beta t + \epsilon$ with $\beta = 0$. In this case the significance levels associated with the estimates b should be uniformly distributed over the interval $[0, 1]$. We found that computed significance levels were below 0.05 at 13.7% (or 203) of the sites, whereas under the null hypothesis, one expects only 5% (or 74) of the significance levels associated with b to be below 0.05. Therefore, we suspect that roughly $13.7 - 5\% = 8.7\%$ of the sites exhibit linear temporal trends. Theoretically, one expects the computed significance levels to follow a uniform distribution. The 1,481 computed significance levels had a mean of 0.405 and standard deviation of 0.299. Under the null hypothesis of uniform significance levels, one expects a mean of 0.5 and a standard deviation of 0.289. Vogel and Kroll (1989) introduced a uniform probability plot correlation coefficient test to evaluate whether significance levels obtained from the repeated application of n independent hypothesis tests follow a uniform distribution. A uniform probability plot of the computed significance levels produces an almost perfect linear plot with a probability plot correlation coefficient (PPCC) equal to 0.99136. This PPCC value is slightly lower than the critical test statistic (0.9994) using an overall 5% significance level; hence, we must reject the null hypothesis. Nevertheless, temporal trends are only apparent at a small fraction of the sites; therefore, for practical purposes, we still assume no trends are evident. Such trend tests should not be taken too seriously, since annual streamflows are known to be serially correlated and trend tests are known to lack power and yield more Type I errors than one expects when applied to serially correlated time series (von Storch and Navarra 1995). Yevjevich (1977) used variance spectral analysis of 748 annual streamflow records in the United States to show that they are at least temporarily stationary processes for the periods of time and record lengths considered.

L-MOMENT DIAGRAMS

An L-moment diagram compares sample estimates of the L-moment ratios L-cv, L-skew, and L-kurtosis with their population counterparts for a range of assumed distributions. An advantage of L-moment diagrams over other goodness-of-fit procedures is that one can compare the fit of several distributions to many samples of data using a single graphical instrument. Another advantage of L-moment diagrams over ordinary product moment diagrams is that L-moment ratios are approximately unbiased for all probability distributions, unlike ordinary product moment ratios, which are significantly biased (Vogel and Fennessey 1993). Vogel and Fennessey show that L-moment diagrams are always preferred to ordinary product

TABLE 2. Coefficients of Polynomial Approximations of L-cv, τ_2 , as Function of L-skewness, τ_3 ($\tau_2 = \sum_{k=0}^7 A_k \tau_3^k$)

Coefficient (1)	LN2 (2)	GAM (3)	W2 (4)	GP (5)
A_0	—	—	0.17864	0.33299
A_1	1.16008	1.74139	1.02381	0.44559
A_2	-0.05325	—	-0.17878	0.16641
A_3	—	-2.59736	—	—
A_4	-0.10501	2.09911	-0.00894	—
A_5	—	—	—	0.09111
A_6	-0.00103	-0.35948	-0.01443	—
A_7	—	—	—	-0.03625

Note: The approximations are good for $-0.1 < \tau_3 < 1$, except for GAM, in which case they are only good for $0 < \tau_3 < 1$.

moment diagrams, regardless of the sample sizes, probability distributions, or skews involved.

In this study, L-moment diagrams are constructed using the unbiased L-moment estimators introduced by Hosking (1990) and recommended by Stedinger et al. (1993), Vogel and Fennessey (1993), and Hosking and Wallis (1995). The theoretical relationships among L-skewness and L-kurtosis for the Pearson Type III (P3), GEV, three-parameter lognormal (LN3), three-parameter Weibull (W3), and the lower bound of a Wakeby distribution (WLB) shown as curves in the figures here, are obtained from the polynomial approximations given in Hosking (1991) and Stedinger et al. (1993). The curve labeled WLB represents the lower bound of the L-kurtosis–L-skew space, which can be captured by the five-parameter Wakeby distribution. The theoretical relation between L-skew τ_3 , and L-kurtosis τ_4 , for the three parameter Weibull (W3) PD is obtained by noting that the third and fourth L-moment ratios for the W3 distribution equal $-\tau_3$ and τ_4 for the GEV distribution.

Theoretical relationships among L-cv and L-skewness for two-parameter distributional alternatives are only available in exact form from Hosking (1990) and Stedinger et al. (1993). To facilitate the construction of L-moment diagrams in future studies, we developed polynomial approximations to the theoretical relationships between L-cv (τ_2) and L-skew (τ_3) for the two-parameter lognormal (LN2), Weibull (W2), Gamma (GAM), and generalized Pareto distribution (GP). The approximations are summarized in Table 2. The approximations are not intended for detailed analytical calculations—use the exact relations given in Hosking (1990) and Stedinger et al. (1993) for that—but are sufficiently accurate for plotting theoretical L-moment relationships for comparison with sample L-moments, as is performed later on.

In each of the following experiments, L-moment diagrams compare sample estimates of L-cv, L-skewness and L-kurtosis, using small points, with theoretical curves for various PDs. Since the L-moment ratio estimators L-cv, L-skewness and L-kurtosis are approximately unbiased, regardless of the underlying PD, one expects approximately half the points to lie above the theoretical curve and half to lie below the curves. To evaluate this, we plot locally weighted scatterplot smoothes (LOWESS) on some of the L-moment diagrams. The LOWESS smooth (Cleveland 1979) provides a nonparametric regression estimate of the relationship between L-kurtosis and L-skewness for comparison with the theoretical curves.

FLOOD FLOW FREQUENCY MODELS: GLOBAL SURVEY

Cunnane [(1989), Appendix 6] summarizes the results of a worldwide survey of flood frequency methods prepared for the World Meteorological Organization in 1984, which is partially reproduced in Table 3. The survey involved 55 agencies from 28 countries. Some countries reported use of more than one distribution as a standard. Of the six distributions reported in

TABLE 3. Summary of Frequency of Use of Various Probability Distributions [from Cunnane (1989), Appendix 6]

Probability distribution (1)	Number of agencies in which it is used as a standard (2)	Number of countries in which it is used as a standard in one or more agencies (3)
EV1 gumbel	18	10
EV2 extreme value type 2	3	3
GEV generalized extreme value	5	1
LN2 lognormal	16	8
P3 Pearson Type III	12	10
LP3 log-Pearson Type III	17	7

Table 3, the most common distributions appear to be the Gumbel (EV1), two-parameter lognormal (LN2), Pearson Type III (P3), and log-Pearson Type III (LP3) distributions. Only one country uses the GEV distribution in spite of its recent popularity, documented later on.

Farquharson et al. (1987) fit a GEV distribution to annual flood flow data at 1,121 gauging stations in 70 different countries using probability weighted moments. Although they did not assess the goodness of fit of a GEV distribution, they did provide regional growth curves to allow for comparisons of regional flood frequency curves in different regions.

McMahon et al. (1992) and Finlayson and McMahon (1992) used ordinary product moment diagrams to explore the probability distribution of annual maximum flood flows at 974 stations around the world. Their evaluations reveal that the LP3 distribution provides a good fit to observed flood flow data, whereas the LN2, GUM, and GAM distributions do not. Those comparisons are not definitive because ordinary product moment diagrams produce misleading results because estimates of ordinary product moment ratios such as the coefficient of variation and skewness contain significant bias (Vogel and Fennessey 1993), especially for small and highly skewed samples.

Onoz and Bayazit (1995) explored the goodness of fit of seven different probability distributions to 1,819 site years (compared with the 73,231 site years of data in this study) of flood flow data available at 19 gauging stations around the world with record lengths ranging from 60 to 165. Using numerous goodness-of-fit procedures, including L-moment diagrams, they conclude that the GEV distribution provides the “best fit” at a majority of the sites, using all tests.

Flood Flow Frequency and L-Moments: Literature Review

Numerous investigators have applied L-moment diagrams to assess the goodness of fit of various PDs to regional samples of flood flows: Wallis (1988), Nathan and Weinmann (1991), Gringas and Adamowski (1992), Pilon and Adamowski (1992), Pearson (1991), Karim and Chowdhury (1993), Vogel et al. (1993), Rao and Hamed (1994), and Onoz and Bayazit (1995). These initial studies are summarized in Table 4. Even though these studies involve flood flow samples throughout the world (Australia, New Zealand, Canada, the United States, and Bangladesh), all studies recommend the use of the GEV distribution. Table 4 reveals that with the exception of the studies by Pearson (1991) and Vogel et al. (1993b), the recommendations from the remaining six studies are based on relatively small regional samples. Nevertheless, together, the nine studies exploit 944 individual samples of annual maximum flood flows across the globe, including some of the longest records available in the world (Onoz and Bayazit 1995). Apparently L-moment diagrams reveal an emerging consensus regarding the choice of a regional parent PD.

TABLE 4. Previous Assessments of Flood Frequency Using L-Moment Diagrams

Reference (1)	Location (2)	Recommended probability distribution (3)	Number of sites (4)
Onoz and Bayazit (1995)	The World	GEV	19
Karim and Chowdhury (1993)	Bangladesh	GEV	31
Gringas and Adamowski (1992)	New Brunswick, Canada	GEV	53
Pilon and Adamowski (1992)	Nova Scotia, Canada	GEV	25
Pearson (1991)	South Island, New Zealand	EV1, EV2, GEV	275
Nathan and Weinmann (1991)	Central Victoria, Australia	GEV	53
Vogel et al. (1993a)	Australia	GEV, ^a GP, ^b LP3, LN3	61
Wallis (1988)	Eastern United States	GEV	55 ^c
Vogel et al. (1993b)	Southwestern United States	LN3, LN2, GEV, and LP3	383
This study	Continental United States	LN3, GEV, and LP3	1,490

Note: GEV = generalized extreme value distribution, GP = generalized Pareto distribution, LP3 = log-Pearson Type III distribution, LN3 = three-parameter lognormal distribution, and LN2 = two-parameter lognormal distribution.

^aThe GEV distribution provides a good fit in the regions dominated by rainfall during the winter months.

^bThe GPA distribution provides a good fit in the regions dominated by rainfall during the summer months.

^cWallis (1988) used 44 of the sites used by Jain and Singh (1987).

This study exploits a much larger regional database in the continental United States (1,490 basins) to evaluate the consistency of this emerging consensus on a regional PD. Among the nine studies summarized in Table 4, only the two studies by Vogel et al. (1993) and Onoz and Bayazit (1995) evaluated the goodness of fit of a LP3 distribution. Together, the two studies by Vogel et al. (1993) exploit 444 basins in the United States and Australia, and indicate that the GEV, LP3, and a three-parameter lognormal (LN3) PD provide equally acceptable models for the distribution of flood flows.

Probability Distribution of Annual Maximum Flood Flows in the United States

Fig. 3 compares the observed and theoretical relations between L-cv and L-skew for the flood flows. Fig. 3 documents that none of the two-parameter PDs: GAM, GUM, or LN2 approximate the observed relationship between L-cv and L-skew.

Fig. 4 compares the observed relationship between L-kurtosis and L-skew of annual maximum flood flows with the theoretical PDs: P3, GEV, LN3, W3, normal, and GUM. Also shown using a very thick curve, is the LOWESS smooth. Fig. 5 is identical to Fig. 4 with the sample L-moments removed. Fig. 4 documents that, overall, the GEV and LN3 distributions are the only three-parameter distributions evaluated which approximate the observed relationship between L-kurtosis and L-skewness. Of those two distributions, Fig. 5 documents that the LN3 distribution provides a slightly better fit to the LOWESS smooth than does the GEV distribution. None of the other distributions considered in Figs. 4 and 5 provide an adequate model of flood flows for the entire nation. Fig. 6 is an L-moment diagram for the logarithms of the annual maximum flood flows. Fig. 6 illustrates that the theoretical curve for a LP3 distribution closely approximates the relationship described by the observed L-moments and summarized using the

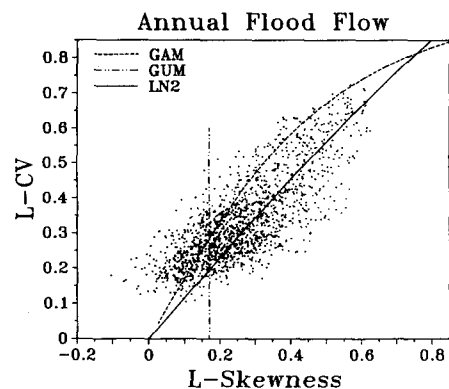


FIG. 3. L-Moment Diagram Illustrating Relationship between L-cv and L-skewness for Annual Maximum Flood Flows

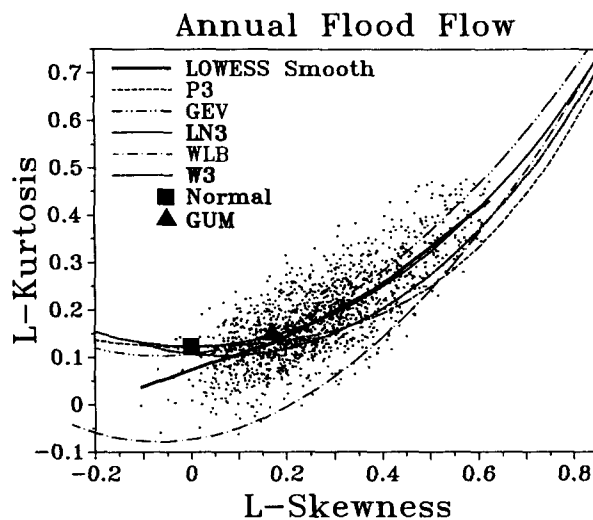


FIG. 4. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for Annual Maximum Flood Flows

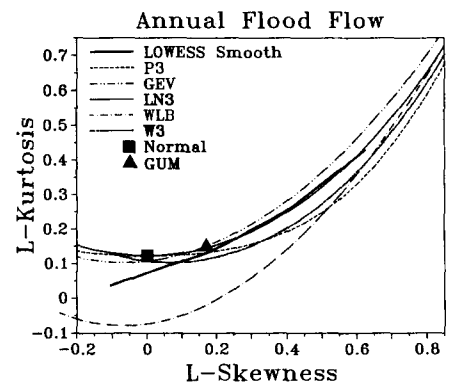


FIG. 5. Comparison of LOWESS Smooth with Theoretical Relationships between L-kurtosis and L-skewness for Annual Maximum Flood Flows

LOWESS smooth. Fig. 6 also documents that a LN2 model is not adequate to describe flood flow populations, nationwide, as was shown in Fig. 3. In summary, Figs. 3–6 document that the GEV, LN3, and LP3 distributions are all adequate PDs to model the frequency of annual maximum flood flows in the continental United States. This conclusion is identical to that reached by Vogel et al. (1993b) using 383 sites in southwestern United States. It is likely that the Pareto-based distribution introduced by Durrans (1994) would also be acceptable for modeling flood flows throughout the continental United States.

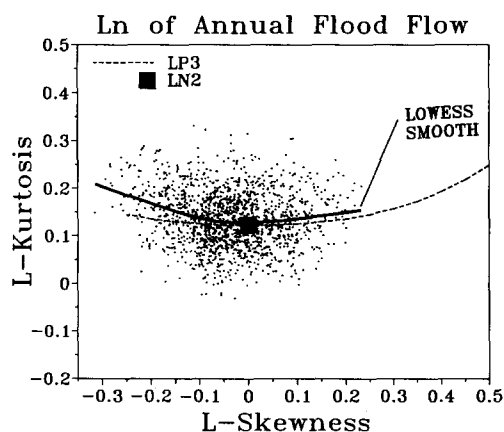


FIG. 6. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for Logarithms of Annual Maximum Flood Flows

LOW FLOW FREQUENCY ANALYSIS: LITERATURE REVIEW

There is much less literature on low flow frequency analysis than on flood flow frequency analysis. To our knowledge, the only study that employed L-moment diagrams to evaluate the probability distribution of annual minimum streamflow is by Pearson (1995). Pearson constructed L-moment diagrams for nearly 500 positive annual minimum flow series in New Zealand. He found that no single two-parameter or three-parameter model was adequate to capture observed L-moment relationships throughout New Zealand. Delleur et al. (1988) and Vogel and Kroll (1989) review numerous studies that have compared the fit of alternative probability distributions and parameter estimation procedure to sequences of annual minimum one- and seven-day low flows.

Most previous studies have used a relatively small number of sites. Some of the studies performed in the United States include a study of 23 rivers in Massachusetts by Vogel and Kroll (1989), 34 rivers scattered across the continental United States by Matalas (1963), 20 rivers in Virginia by Tasker (1987), 37 stations in the Missouri river basin by Joseph (1970), and 195 stations in Indiana by Delleur et al. (1988). Those studies recommend both probability distributions and parameter estimation procedures: we do not consider estimation procedures here. Matalas (1963) recommended the use of either the P3 or W3 distributions for sequences of annual minimum one-day and seven-day streamflows across the United States. Tasker (1987) recommended the use of either the LP3 or W3 distributions for annual minimum seven-day flows for Virginia. Vogel and Kroll (1989) recommended the use of the LN2, LN3, LP3, or W3 distributions for annual minimum seven-day flows in Massachusetts. Joseph (1970) recommended the use of the GAM distribution for annual minimum 14-day flows in the Missouri river basin. Delleur et al. (1988) recommended the use of the LN2, LN3, and W3 distributions for most Indiana streams. Others have recommended derived distributions of low flow (Fiering 1964; Perzyna and Gottschalk 1994) or other distributions not considered here, such as log Boughton (Loganathan et al. 1985) PD or use of a SME-MAX transformation to normalize flow series (Prakash 1981).

Probability Distribution of Annual Minimum Low Flows in the United States

Fig. 7 compares the observed and theoretical relations between L-cv and L-skew for the annual minimum low flows in the continental United States. None of the two-parameter distributions (LN2, W2, or GAM) considered in Fig. 7 adequately represent the observed relationship between L-cv and L-skew

for the entire nation. Fig. 7 reveals that sequences of annual minimum low flow exhibit a more dramatic range of variability (L-cv) and skewness (L-skew) than the sequences of annual maximum flood flows documented in Fig. 3. This is, in part, due to the large number of zero observations associated with sequences of annual minimum low flows. Pearson (1995) found that the annual minimum flow series exhibit greater variability than annual maximum flow series in New Zealand, even with the zero observations removed.

Figs. 8 and 9 compare observed and theoretical relationships between L-kurtosis and L-skewness for the annual minimum low flows. Also shown using a very thick curve is the LOWESS smooth. Fig. 9 is identical to Fig. 8 with the sample L-moments removed. Figs. 8 and 9 reveal that among the dis-

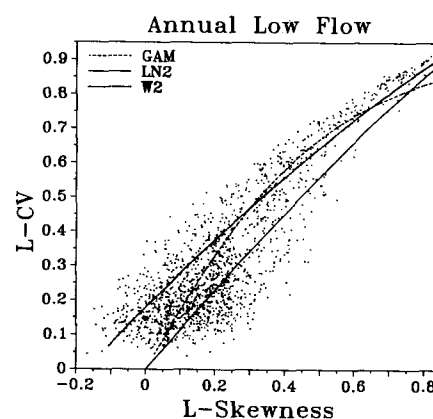


FIG. 7. L-Moment Diagram Illustrating Relationship between L-cv and L-skewness for Annual Minimum Low Flows

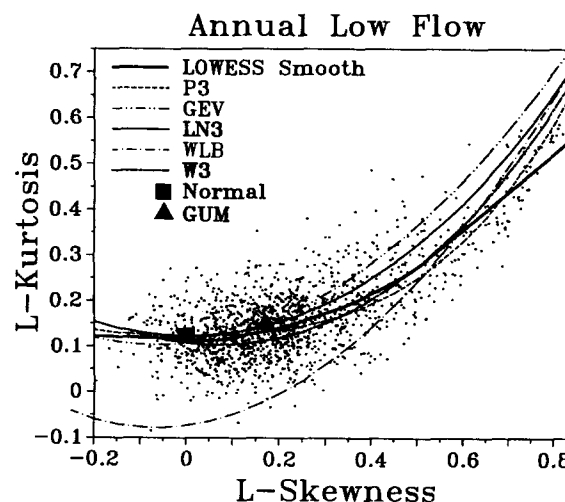


FIG. 8. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for Annual Minimum Low Flows

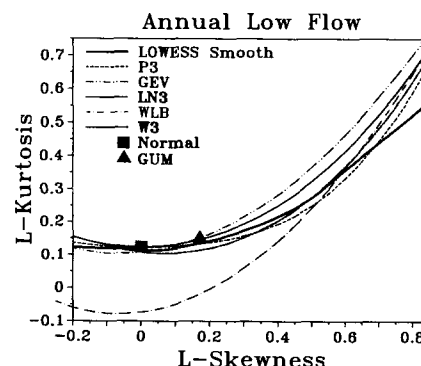


FIG. 9. Comparison of LOWESS Smooth with Theoretical Relationships between L-kurtosis and L-skewness for Annual Minimum Low Flows

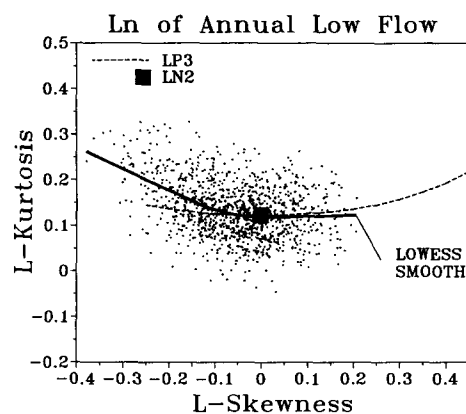


FIG. 10. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for Logarithms of Annual Minimum Low Flows

tributions considered, the P3 and the W3 provide the best fit to the LOWESS smooth and, of those two choices, the P3 distribution provides the best fit overall to the observed relation between L-kurtosis and L-skew. Other three-parameter alternatives such as LN3, GEV, and even the two-parameter GUM and GAM models perform well for smaller values of L-skew; however, none of them capture the tail behavior of annual minimum flows for arid regions, which exhibit frequent zeros and hence large L-skews.

Fig. 10 compares the observed and theoretical relations between L-kurtosis and L-skew for the natural logarithms of the annual minimum low flows. Again, as in Fig. 6 for annual maximum flood flows, the LP3 distribution approximates the LOWESS smooth, documenting the overall flexibility of the LP3 distribution for modeling low flows. Although the LOWESS smooth deviates slightly from the theoretical LP3 curve for negative L-skews in Fig. 10, the LP3 distribution approximates the overall relationship between observed L-kurtosis and L-skewness for annual minimum flows relatively well.

On the basis of Figs. 7–10 we conclude that among all the distributions considered for modeling the frequency of annual minimum low flows, the P3, W3, and LP3 distribution are adequate nationwide models, though the P3 distribution is preferred overall. This result is consistent with Matalas (1963), which was the only previous study that examined sequences of annual minimum low flows across broad regions of the United States. Most previous studies, cited earlier, never even considered the P3 distribution for modeling annual minimum low flows, in spite of Matalas's recommendations.

FREQUENCY ANALYSIS FOR ANNUAL AVERAGE STREAMFLOWS—LITERATURE REVIEW

The literature on the frequency analysis of annual average streamflows, like the literature on low flows, is sparse when compared to the literature on flood flows. Using a database of annual average flows at 974 sites around the world, McMahon et al. (1992) and Finlayson and McMahon (1992) document that approximately 60% of world streams have distributions that are not significantly different from the normal distribution based on a 5% significance-level skewness test. A closer examination by McMahon et al. (1992) revealed that only about 31% of the sites in Australia and South Africa are approximately normal, compared with 71% of the sites across the rest of their global database. Similarly, smaller and earlier studies using 137 and 140 basins around the world, by Kalinin (1971) and Yevjevich (1963), respectively, found that annual flows at about 70% of the sites were (approximately) normally distributed.

Markovic (1965) used the chi-square goodness-of-fit statis-

tic to compare the fit of N, LN2, LN3, GAM, and P3 distributions to sequences of annual average streamflow at 446 sites in the western United States. Markovic recommended the use of the Gamma distribution; however, his results indicate that the P3, LN2, and LN3 distribution provide suitable alternatives. Similar to the previously cited studies, the normal distribution provided a suitable alternative at 72% of the sites considered by Markovic. Löff and Hardison (1966) used probability plots to assess the goodness of fit of the normal, lognormal, and Weibull distributions to annual average flows across 22 water-resource regions of the United States. They chose the normal distribution for the New England, Ohio, Tennessee, Mississippi, Missouri, Colorado, and Pacific Northwest basins; the lognormal distribution for the Delaware, Hudson, Southeastern, Upper Rio Grande, Pecos, and Central Pacific basins; and the Weibull distribution for the remaining basins. Vogel et al. (1995) used L-moment diagrams, Hosking's (1990) L-moment normality test, and probability plot correlation coefficient hypothesis tests to show that annual average streamflows at 166 sites in the northeastern United States are well-approximated by a normal distribution.

Derived Distribution of Annual Average Streamflow

Eagleson (1978) derives the probability distribution of annual streamflow, assuming a simple precipitation-yield function with storm arrivals following a Poisson process and storm depths following a gamma distribution. A simpler approach is presented here. Fiering (1967) introduced a simple water-balance rainfall-runoff model consisting of two equations: one for streamflow and one for ground-water storage. Salas and Smith (1981) showed that those two equations could be combined to yield the single rainfall-runoff equation

$$Q_t = (1 - c)Q_{t-1} + (1 - a - b)P_t - [(1 - a - b)(1 - c) - ac]P_{t-1} \quad (1)$$

where Q_t and P_t = streamflow and precipitation in year t , respectively; and coefficients a , b , and c = model parameters. Salas and Smith (1981) point out that this conceptual rainfall-runoff model may be interpreted as an autoregressive moving average (ARMA) (1, 1) stochastic streamflow model, with the noise terms based on the precipitation series. In a recent evaluation of times series of annual average precipitation for the entire United States, using L-moment diagrams, Guttman et al. (1993) recommend modeling the P_t with a P3 distribution. If the P_t in (1) follow a normal distribution, then the Q_t will follow a normal distribution. On the other hand, if P_t follows either a GAM or P3 distribution, then Q_t can be approximated by a GAM or P3 distribution, respectively. Therefore, it is not surprising that Markovic (1965) recommended the gamma distribution for annual flows, nor is it surprising that most of the annual flow series evaluated by Yevjevich (1963), Kalinin (1971), McMahon et al. (1992), and Vogel et al. (1995) are well-approximated by a normal distribution.

Probability Distribution of Annual Streamflow in the U.S.

Fig. 11 illustrates the relationship between the observed and theoretical L-cv and L-skew for the annual average streamflow series. The annual average streamflows are not nearly as skewed as either the annual maximum floods or the annual minimum low flows. Fig. 11 documents that of the two-parameter distributions considered—GAM, LN2, W2, N, and GP—the GAM and W2 distributions are the only ones that generally capture the observed L-cv–L-skew relationship. Even though the W2 distribution provides a reasonably good fit to the observed relation between L-cv and L-skew, we drop

it from further consideration because it is an extreme-value distribution and annual average streamflow is not an extreme-value random variable. The GAM distribution is clearly favored over the N and even LN2 distributions.

Fig. 12 compares the theoretical and observed relations between L-kurtosis and L-skew for the annual average flows. Also shown is the LOWESS smooth. Fig. 12 illustrates that both the P3 and LN3 distributions provide a good fit to the observed L-moments and the LOWESS smooth; yet, of those two PDs, the P3 is closer to the LOWESS smooth. Fig. 13 compares the theoretical and observed relations between L-kurtosis and L-skew for the natural logarithms of the annual average flows along with a LOWESS smooth. Fig. 13 shows that an LP3 PD also captures the observed tail behavior, though the simpler LN2 PD, again, does not. In summary, Figs. 11–13 document that annual average streamflows are

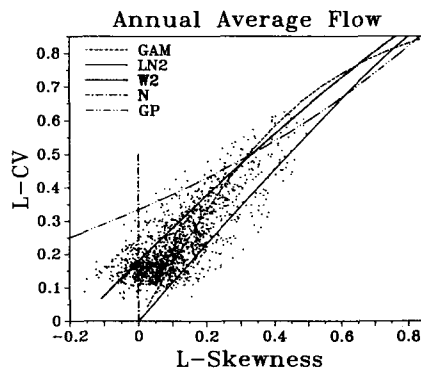


FIG. 11. L-Moment Diagram Illustrating Relationship between L-cv and L-skewness for Annual Average Streamflows

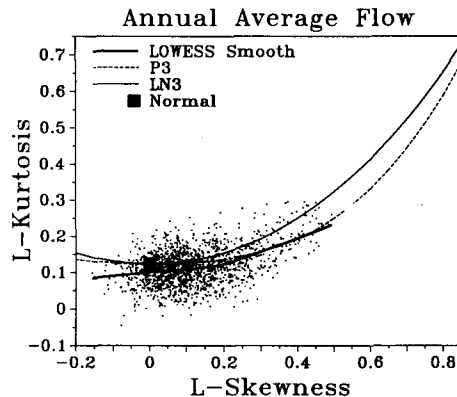


FIG. 12. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for annual average streamflows

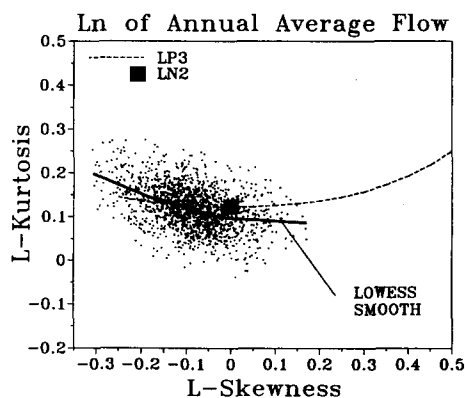


FIG. 13. L-Moment Diagram Illustrating Relationship between L-kurtosis and L-skewness for Logarithms of Annual Average Streamflow

well-approximated by either a P3, LP3, or LN3 distribution, with the P3 distributions performing best overall. However, if a two-parameter distribution is required, the GAM distribution performs best among the alternatives, and a theoretical justification is also provided.

CONCLUSIONS

The goal of this study was to evaluate the goodness of fit of alternate PDs to sequences of annual minimum, average, and maximum streamflows in the continental United States. We will never know, with certainty, the true population from which observed streamflows arise, yet studies such as this can provide some guidance on which PDs provide a reasonable approximation. A review of recent flood frequency studies, which use L-moment diagrams, reveals that a consensus is beginning to emerge regarding suitable PD approximations. This study reveals that the LN3, LP3, and GEV models are all acceptable models in the continental United States, whereas other three-parameter alternatives such as the P3 and W3 and two-parameter alternatives such as N, GAM, GUM, and LN2 are not acceptable for the entire continent. These results are consistent with previous L-moment studies in the southwestern United States and Australia (Vogel et al. 1993). Other recent studies, which used L-moment diagrams in Bangladesh (Karim and Chowdhury 1993), Canada (Gringas and Adamowski 1992; Pilon and Adamowski 1992), New Zealand (Pearson 1991), Australia (Nathan and Weinmann 1991), eastern United States (Wallis 1988), and the globe (Onoz and Bayazit 1995) have all recommended the use of the GEV distribution for modeling annual maximum flood flows. Apparently a consensus is emerging and the time is ripe for agencies and countries, throughout the world, to reevaluate their standards with respect to the choice of a suitable model for flood frequency analysis.

There is much less literature on low flow and average flow frequency analysis than on flood flows. To our knowledge, there are no studies that apply L-moment diagrams to annual minimum streamflows. This study reveals that annual minimum flows in the United States are best approximated by a P3 distribution, yet the LP3 or W3 distributions will suffice. Matalas (1963) recommended the P3 distribution, yet most studies since the Matalas study have not even considered that distribution. This study also documents that none of the two-parameter distributions considered (N, LN2, GAM, or W2) are adequate for modeling low flows everywhere in the United States.

Of all the two-parameter distributions considered, Fig. 11 documents that the GAM distribution is the only PD which captures the observed relationship between L-cv and L-skew of average annual flows in the United States. Although the GAM distribution will probably suffice, Figs. 12 and 13 illustrate that the P3, LN3, and LP3 distributions provide a better approximation to the observed L-moment ratios of average annual flows than any of the two-parameter PDs considered. Given the theoretical justification provided for the GAM and P3 distributions in (1), we recommend the use of either of those PDs for modeling average annual flows in the United States, unless L-skews are near zero, in which case an N model will also suffice. For L-skew near zero, the P3, GAM, and N models are nearly equivalent.

L-moment diagrams have revealed that the LP3 distribution is a remarkably flexible distribution, able to accurately fit more than 1,455 series of annual maximum, average, and minimum streamflows in the United States. It is well-known (Wallis 1988; Vogel and McMartin 1991) that even if flows originate from populations other than LP3, standard goodness-of-fit procedures lack the power to discriminate against those alternatives. It is unlikely that all the different types of flow series examined here arise from the same LP3 population (or any

other particular population considered in this or other studies), yet we are unable to exclude that model as a viable alternative.

APPENDIX. REFERENCES

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