THE RELIABILITY, RESILIENCE, AND VULNERABILITY OF OVER-YEAR WATER SUPPLY SYSTEMS

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The practice of developing storage-reliability-yield (S-R-Y) relationships for water supply systems is divided into two schools of thought. In the United States, S-R-Y relationships are often based on the probability of no-failure reservoir operations over an N-year planning period. Elsewhere, S-R-Y relationships are based on the steady-state probability of failure corresponding to a specific reservoir capacity-yield combination. These two independent views of system reliability are merged using a two-state Markov model, along with existing analytical S-R-Y relationships which describe the behavior of over-year water supply systems fed by autoregressive normal and lognormal inflows. Monte Carlo simulation experiments confirm that a two-state Markov model can represent the relationships between reservoir system reliability and resilience for a wide class of over-year water supply systems. The two-state Markov model combined with existing analytic S-R-Y relationships provide a very general theoretical foundation for understanding the tradeoffs among reservoir system storage, yield, reliability, resilience, and vulnerability.

INTRODUCTION

Two general classes of reservoir systems exist: over-year and within-year systems. Within-year systems are characterized by reservoirs which typically refill at the end of each year. Such systems are particularly sensitive to seasonal, monthly, and even daily variations in both the hydrologic inflows and the system yield. Over-year systems do not usually refill at the end of each year; such systems are particularly prone to water supply failures (empty reservoirs) during periods of drought that extend over several years. Here we define a failure as the inability of a reservoir system to provide the contracted demand in a given year. Water supply failures for within-year systems tend to be short-lived, in comparison with over-year systems, since within-year systems tend to refill on an annual basis. Naturally, all reservoir systems exhibit some combination of over-year and within-year behavior. However,

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for the moment, consider two reservoir systems having an equal steady-state probability of a failure, q, in a given year, one being a system dominated by exclusively over-year behavior and the other dominated exclusively by within-year behavior. During an N-year period, one would expect Nq failures. However, for the within-year system those failure sequences will typically last only a few days or months, whereas for the over-year system, a typical failure may last years (if no new water is imported and demand curtailment programs are not implemented).

A prerequisite to the proper operation, management, and design of over-year reservoir systems is a thorough understanding of the likelihood, duration, and magnitude of potential reservoir system failure sequences. For this purpose, the storage-reliability-yield (S-R-Y) relationship is one important ingredient. However, reliability statements alone do not convey information regarding the consequences of failure (system vulnerability) or the ability of a system to recover from failure (system resilience). This study formulates an approximate, yet general approach for understanding the overall behavior of over-year reservoir systems focusing attention on both the S-R-Y relationship and the frequency, magnitude, and duration of reservoir system failures.

Storage reservoirs tend to be large and complex systems requiring equally complex mathematical models to simulate their behavior. Historically, one modelling approach has been replaced by, or appended to, another more complex one to deal with such issues as the Hurst phenomenon, model parameter uncertainty, optimal operations, spatial and temporal disaggregation schemes, etc. What is lacking are simple, reasonably accurate "back-of-the-envelope type" methods which give insight into a wide range of reservoir storage system characteristics and reliability indices before one embarks on a complex modelling expedition. Such "back-of-the-envelope" methods would also be useful for the education of water supply analysts.

Most current textbooks in the U.S. recommend the simulation of water supply system behavior using either the historical record or synthetic streamflow traces, in conjunction with the sequent peak algorithm (see for example Loucks et al., 1981). Yet such exercises do not always impart much knowledge of overall reservoir system behavior other than the desired S-R-Y relationship. What is needed are simple, yet accurate expressions which can be easily exploited to describe the resiliency and vulnerability, as well as the S-R-Y relationship, so that, for example, one could illustrate the frequency, magnitude, and duration of reservoir system failure sequences. Otherwise, one is often lost in the myriad of computer output from more complex reservoir system simulation exercises.

The goal of this study is to develop a set of simple expressions which both enhance our understanding of the behavior of water supply systems and provide an explanation of over-year reservoir system behavior. A related study by Vogel(1987) uses a two-state Markov model of reservoir storage states to derive and validate relationships among N-year no-failure reliability, p, and steady-state reliability of failure, q, for reservoir systems dominated by within-year behavior. However, that study does not connect reliability and resiliency indices to other system parameters such as storage capacity, yield, or streamflow statistics as is done here, nor does it deal with over-year reservoir systems. Hashimoto et al. (1982) describes the use and importance of reliability, vulnerability, and resiliency indices for exposing the consequence of reservoir system failures.

DEFINITION OF SOME WATER SUPPLY SYSTEM PERFORMANCE INDICES

System Reliability

Two schools of thought exist regarding the reliability of water supply systems. In the U.S., system reliability is usually defined as the probability of no-failure reservoir operations, p, over an N-year planning period. This is the interpretation of reliability which results when one applies the sequent peak algorithm in conjunction with a stochastic streamflow model (see Vogel, 1987 for a discussion). Alternatively, in Australia and elsewhere, system reliability is usually defined in terms of the steady-state probability of a system failure q, where a failure is defined as the inability of the system to deliver the desired yield or demand. We show later on how both of these reliability definitions may be related mathematically.

System Resilience

Hazen (1914), followed by Sudler (1927), Hurst (1951), and others introduced one of the most useful indices of reservoir system performance, which we term the resilience index

$$m = \frac{(1 - \alpha)\mu}{\sigma} = \frac{(1 - \alpha)}{C_v} \tag{1}$$

where α is the annual system demand or yield as a fraction of the mean annual inflow, μ, σ is the standard deviation of the annual inflows and C_v is the coefficient of variation of the annual streamflows. Perrens and Howell (1972) termed m the standardized inflow. After its use by Hurst (1951), the nondimensional index m has subsequently found use in both analytic investigations in "water storage theory" (Pegram et. al., 1980; Buchberger and Maidment, 1989) and in Monte-Carlo investigations of the storage-reliability-yield relationship (Perrens and Howell, 1972; Vogel and Stedinger, 1987). Vogel and Stedinger (1987) suggest that as long as $0 \leq m \leq 1$, the system is dominated by over-year behavior, whereas if $m > 1$, the system is dominated by within-year behavior. Based on these findings, the scope of this paper is limited to those cases where m is between 0 and 1. We show later on that m is related to the probability that a storage reservoir will recover from a failure, hence it is an ideal measure of reservoir system resiliency. That is, reservoirs with values of m near 0 are less likely to recover from a failure than reservoirs with values of m near unity. Systems with low resiliency (m near zero) are characterized by having either large values of C_v, or α, or both. Similarly, reservoirs with values of m near or above unity are more likely to refill once empty. Therefore, such systems are more likely to exhibit within-year rather than over-year behavior.

Since resilient reservoir systems (large resiliency index m) tend to have either small demand levels α or small coefficients of variation, one expects that regions with low streamflow variability will contain more resilient reservoir systems than regions with high streamflow variability, for a fixed demand level. Similarly, demands levels generally increase over time, thus one expects a general reduction in the overall resiliency of reservoir systems over time.

GENERAL STORAGE-RELIABILITY-YIELD RELATIONSHIPS

When one attempts to develop the S-R-Y relationship for an actual reservoir system, stochastic streamflow models are often employed in combination with a reservoir simulation model developed for the system in question. For reservoir systems dominated by over-year storage requirements, a variety of generalized analytical S-R-Y relationships are available for providing a preliminary estimate of the S-R-Y relationship. Klemes (1987), Vogel and Stedinger (1987), Votruba and Broza(1989), Phatarfod(1989), and Buchberger and Maidment (1989) provide recent reviews of the literature relating to the development of analytical S-R-Y relationships.

Storage-Reliability-Yield Relationship for Normal Inflows

Buchberger and Maidment (1989) show that for independent normal inflows the relationship between the steady-state probability of failure q, the storage ratio K, and the resilience index m is given by

$$q = \theta_1(m, K) + \theta_2(m, K) \tag{2}$$

where Θ_1 and Θ_2 are functions of m and K too lengthy to report here. The storage ratio K is the ratio of the reservoir capacity S to the standard deviation of the inflows σ.

Vogel (1985) developed analytical approximations to the relationship among probability of no-failure operations over an N-year planning period, p, resilience index, m, storage ratio, K, and the lag-one serial correlation of annual flows, ρ, for AR(1) normal inflows. His approximation

$$K = f(m, p, \rho) \tag{3}$$

takes the form of a set of regression equations too lengthy to report here. Similarly Pegram (1980) reports relations among K,m, and q in tabular form.

Storage-Reliability-Yield Relationships for Lognormal Inflows

Vogel and Stedinger (1987) developed approximate multivariate relationships for lognormal inflows of the form

$$K = g(m, \rho, C_v, p, N) \tag{4}$$

where

K = standardized storage ratio = S/σ
m = resiliency index = $(1-\alpha)\mu/\sigma$
C_v = coefficient of variation of inflows = σ/μ
ρ = lag-one correlation of inflows
p = probability of no-failure reservoir operations over an N-year period
N = planning period

with the function g in (4) is based on a set of regression equations too complex to reproduce here. Similarly Pegram (1980) provides tabular results of the S-R-Y relationship for correlated and uncorrelated lognormal inflows for selected cases.

APPLICABILITY OF GENERAL STORAGE-RELIABILITY-YIELD RELATIONSHIPS

Most general analytical S-R-Y relationships are inadequate for design purposes because they cannot be general and at the same time account for complexities such as the seasonal nature of evaporation, precipitation, streamflow, and operating rules. Phatarfod (1989) recommends using Monte-Carlo simulation methods for handling specific reservoir design problems and using general analytical S-R-Y relationships for obtaining qualitative results and for obtaining insight into the mathematics of reservoir operations. Monte-Carlo simulations of reservoir systems using monthly or even daily time steps are so detailed that it is easy to miss general, yet important, features of the reservoir operations. For example, significant attention in the literature has been devoted to the development and application of monthly stochastic streamflow models for use in reservoir operations studies, yet few studies have evaluated the general relationships among reservoir system reliability, resilience, and vulnerability. Similarly few studies have addressed which definition of reliability to use and more importantly what level of reliability is suitable for the proper design and/or operation of a reservoir system.

Many investigators dispense with general over-year S-R-Y relationships immediately since they are thought to be too simplistic to capture the overall complexity of real water supply systems. For example, Vogel and Hellstrom (1988) showed that for the Quabbin Reservoir system which provides the water supply for much of eastern Massachusetts, an annual simulation of the system was almost indistinguishable from a monthly simulation of the system. This is expected since the quoted firm yield of 300 mgd for this system corresponds to α = 0.915 and C_v =

0.34, hence m = 0.25. As long as m remains in the range $0 \leq m \leq 1$, the system will be dominated by over-year behavior and seasonal variability of operations and hydrologic processes becomes moot in terms of the overall reservoir system behavior.

A TWO-STATE MARKOV MODEL OF RESERVOIR SYSTEM STATES

Storage-reliability-yield equations are useful for describing certain aspects of reservoir system behavior, yet such relationships are unable to describe the system resilience and vulnerability in terms of the duration and magnitude of reservoir system failures. For this purpose we consider a two-state Markov model.

A two-state Markov model allows us to relate system storage, reliability, and yield to the frequency, magnitude, and duration of reservoir system failures. In addition, the two-state Markov model allows us to relate steady-state reliability 1-q, to the N-year no-failure system reliability p. Another advantage of the two-state Markov model is its simplicity and therefore its ease of manipulation. Others have successfully exploited a two-state Markov model for representing sequences of reservoir surplus and failures (see Klemes, 1967; Jackson, 1975; Hirsch, 1979; Stedinger et.al, 1983; and Vogel, 1987). However, those studies have not provided a direct link between the two-state Markov model and a simple reservoir system model.

Klemes (1969) employed a multi-state Markov chain model in an effort to describe the complex structure of sequences of reservoir surplus and failures that arise from reasonable assumptions regarding the character of inflow and demand processes. Since a primary objective of this study is to derive relatively simple expressions to aid in the understanding of reservoir system behavior, the m-state Markov chain model formulation employed by Moran (1954), Klemes (1969), and others must be simplified considerably at the potential expense of misrepresenting the complexity of reservoir surplus and failure sequences. Vogel (1987) documents that a two-state Markov model can accurately represent within-year reservoir systems and we extend those results here to a wide class of over-year reservoir systems.

Klemes (1977) showed that the number of discrete storage states required to assess the reliability of a storage reservoir with a desired level of accuracy is usually well above two states. It is usually infeasible for an over-year reservoir system to pass from full to empty in one year hence most investigators have employed more than two states to model reservoir state transitions. However, if one defines one state as the failure state and another as the nonfailure state, we show that such a two-state Markov model of reservoir state transitions provides an adequate description of the frequency and magnitude of reservoir system failure durations for systems with m > 0.2. Figure 1 illustrates the two-states in the Markov model.

Model Development

Let the row vector $\mathbf{Y}_t = (y_{1t}, y_{2t})$ specify the probability that a reservoir system is either in : (1) the failure state; or (2) the regular (nonfailure) state in year t. A

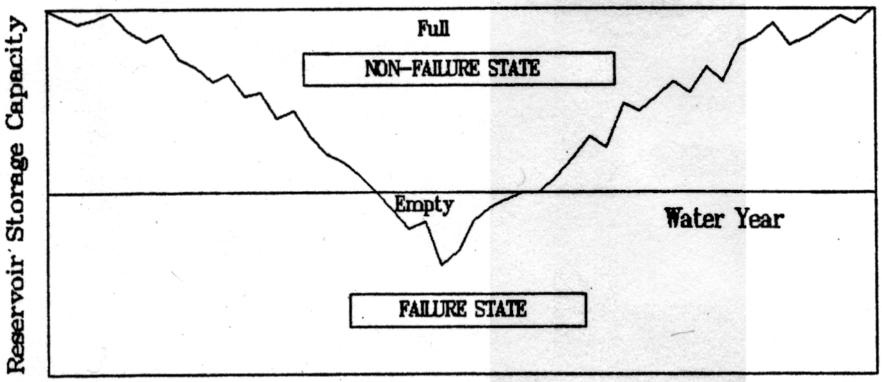

Figure 1. Reservoir system states for the two-state Markov model.

failure state occurs when the water in storage plus the inflow during year t are less than the contracted demand $\alpha\mu$. We assume that the states associated with $\mathbf{Y}_t$, t = 1,...,N form a Markov chain with the transition probability matrix

$$\mathbf{A} = \begin{bmatrix} 1-r & r \\ f & 1-f \end{bmatrix} \tag{5}$$

where f = the probability that a failure year follows a regular year; and r= the probability that a regular year follows a failure year. Now the probability distribution of reservoir system states follows

$$\mathbf{Y}_{t+1} = \mathbf{Y}_t \mathbf{A} \tag{6}$$

As t increases, $\mathbf{Y}_t$ reaches a steady-state, and the solution to (6) becomes

$$\lim_{t\to\infty} \mathbf{Y}_t = \left[\frac{f}{r+f}, \frac{r}{r+f} \right] \tag{7}$$

Jackson (1975) provides the derivation of this result. Thus, the steady-state probability that the reservoir will be in the failure or regular states are f/(r+f) and r/(r+f), respectively, regardless of the initial state of the reservoir system. The steady-state system reliability, 1-q, can be related to the two-state Markov model using $1\text{-}q = r/(r+f)$ or

$$q = 1 - \frac{r}{r+f} = \frac{f}{r+f} \tag{8}$$

Equation (8) provides the link between the two-state Markov model and S-R-Y relationships based upon a steady-steady probability of failure.

To fully specify the two-state Markov model, we require estimates of r and f in (8). Estimation of the transition probability r is accomplished by first recalling its definition as the probability that the reservoir system transfers from the failure (empty) state to the normal (nonempty) state. The failure state is defined as the condition when the water storage plus the inflow for that period Q_t, is less than the demand $\alpha\mu$. Once a failure has occurred, r becomes the conditional probability

$$r = P\{Q_{t+1} \geq \alpha\mu \mid Q_t < \alpha\mu\} \tag{9}$$

As long as the inflows are independent ($\rho = 0$), the conditional probability statement in (9) becomes

$$r = P\{Q \geq \alpha\mu\} \tag{10}$$

which reduces to $r = \Phi(m)$ for independent normal inflows. Similarly for independent lognormal inflows (10) reduces to $r = 1 - \Phi((\ln(\alpha\mu)-\mu_y)/\sigma_y)$ where $y = \ln(Q)$. Either index, r or m, may be considered representative of the resilience of a reservoir system.

Once r is determined, f is found by rearranging (8) to obtain

$$f = r\left[\frac{q}{1-q}\right] \tag{11}$$

Note that systems with r near unity (m large) correspond to within-year systems. Hence one may consider using the index r to distinguish between systems dominated by over-year (r small) behavior from systems dominated by within-year (r large) behavior.

The Duration of a Reservoir System Failure

Vogel(1987) shows that the probability mass function for the length of a reservoir system failure for a two-state Markov model is given by

$$P\{L = \lambda\} = r\,(1 - r)^{\lambda-1}\;; \qquad \text{for } \lambda \geq 1 \tag{12}$$

where L is the length of a failure sequence. Since L is geometrically distributed it has mean, $E[L] = 1/r$, variance $Var[L] = (1-r)/r^2$, and coefficient of variation $C_v[L] = (1-r)^{1/2}$.

A UNIFIED VIEW OF RESERVOIR SYSTEM RELIABILITY

In general there are two approaches to the determination of the yield or storage capacity of a reservoir system. One approach used in the U.S. is to determine the no-failure yield (often called the firm yield) which can be met over a particular planning period with a specified reliability. An approach used elsewhere is to determine the yield which can be delivered with a specified steady-state reliability 1-q. Unfortunately, these two approaches are often seen as unrelated and disconnected. Both of these schools of thought can be linked using a two-state Markov model, leading to completely consistent estimates of the reliability of reservoir systems regardless of which school of thought one happens to follow.

When the sequent peak algorithm (see Loucks et.al., 1981) is used to determine the smallest reservoir system design capacity S, required to assure regular or failure-free operation over an N-year planning period with probability p, then p is a steady-state probability over that planning period. This is because the sequent peak algorithm wraps the streamflow record itself, thus generating the steady-state solution to the problem posed. If we employ the two-state Markov model, the steady-state probability of regular (failure-free) operation over an N-year period, p, is simply the steady-state probability of normal operations in the first year 1-q, times the probability that subsequent years remain free of failures:

$$p = (1 - q)(1 - f)^{N-1} \tag{13}$$

Equation (13) relates the index of reliability commonly used in the U.S. (the probability of failure-free operation over an n-year period p) to the index of reliability commonly used elsewhere (the steady-state system reliability 1-q). Hence one can employ the two-state Markov model to compare S-R-Y relationships developed using completely different interpretations of system reliability.

MONTE-CARLO EXPERIMENTS

All of the experiments follow the same general procedure. First 100 million independent normal inflows with $\mu=1$, $\sigma=0.2$ and $\rho=0.0$ were generated. Similarly 100 million independent lognormal inflows were generated with skewness $\gamma=0.25$, 0.5, and 1.0.

Assuming a full reservoir capacity equal to S at the beginning of each 100 million year simulation, the experiment proceeds by determining the amount of water in storage in each of the 100 million years. If the reservoir contents plus the inflow in a given period is less than the required demand $\alpha\mu$, a failure is documented. If the reservoir contents plus the inflow in a given period is greater than the storage capacity S, the excess or surplus is spilled or lost from the system.

Validation of Two-State Markov Model for Describing Failure Durations

Figure 2 compares the theoretical and simulated mean E[L] and coefficient of variation $C_v[L]$ of system failure durations as a function of the resilience index m and failure probability q for independent normal inflows. Overall, very good agreement is obtained between the theoretical and simulated mean and coefficient of variation of failure durations. The agreement is less satisfactory for resilience indices, m, less than or equal to 0.2. This is due to the fact that systems with low resilience index tend not to refill once empty, since they have by definition high demand and low resiliency. Therefore, more than two-states are required to capture the behavior of these systems. For cases where $m > 0.2$ and $0.005 < q < 0.1$, we conclude that the two-state Markov model provides an accurate description of the distribution of system failure durations. The results for lognormal inflows were just as accurate as the results for normal inflows, however they are not reported here due to space limitations.

Validation of General, Analytic Storage-Reliability-Yield Relationships and the Two-State Markov Model For Independent Normal Inflows

Two analytical S-R-Y models were evaluated for use when streamflows are normal and independent. Buchberger and Maidment (1989, equation 27) provide analytic expressions for the relationship between the storage ratio S/σ , resiliency index m, and the steady state probability of failure q, for systems fed by independent normal inflows.

Vogel (1985) provides approximate expressions describing the relationship between S/σ, m, and the probability of no-failure operations over N-years, p. To allow for comparison with Buchberger and Maidment (1989), Vogel (1985) was combined with the two-state Markov model to convert reliability p to reliability 1-q using equation (13).

Both Buchberger and Maidment (1989) and Vogel (1985) are compared in Figure 3 along with Monte-Carlo simulation results and tabulated results from Pegram (1980). We conclude that the two-state Markov model successfully converts no-failure reliability p over an $N=50$ year planning period to steady-state reliability 1-q. Once again, the only exception is for values of m less than or equal to 0.2.

Validation of General, Analytic Storage-Reliability-Yield Relationships and the Two-State Markov Model for Independent Lognormal Inflows

Vogel and Stedinger (1987) provide analytic expressions for the relationship

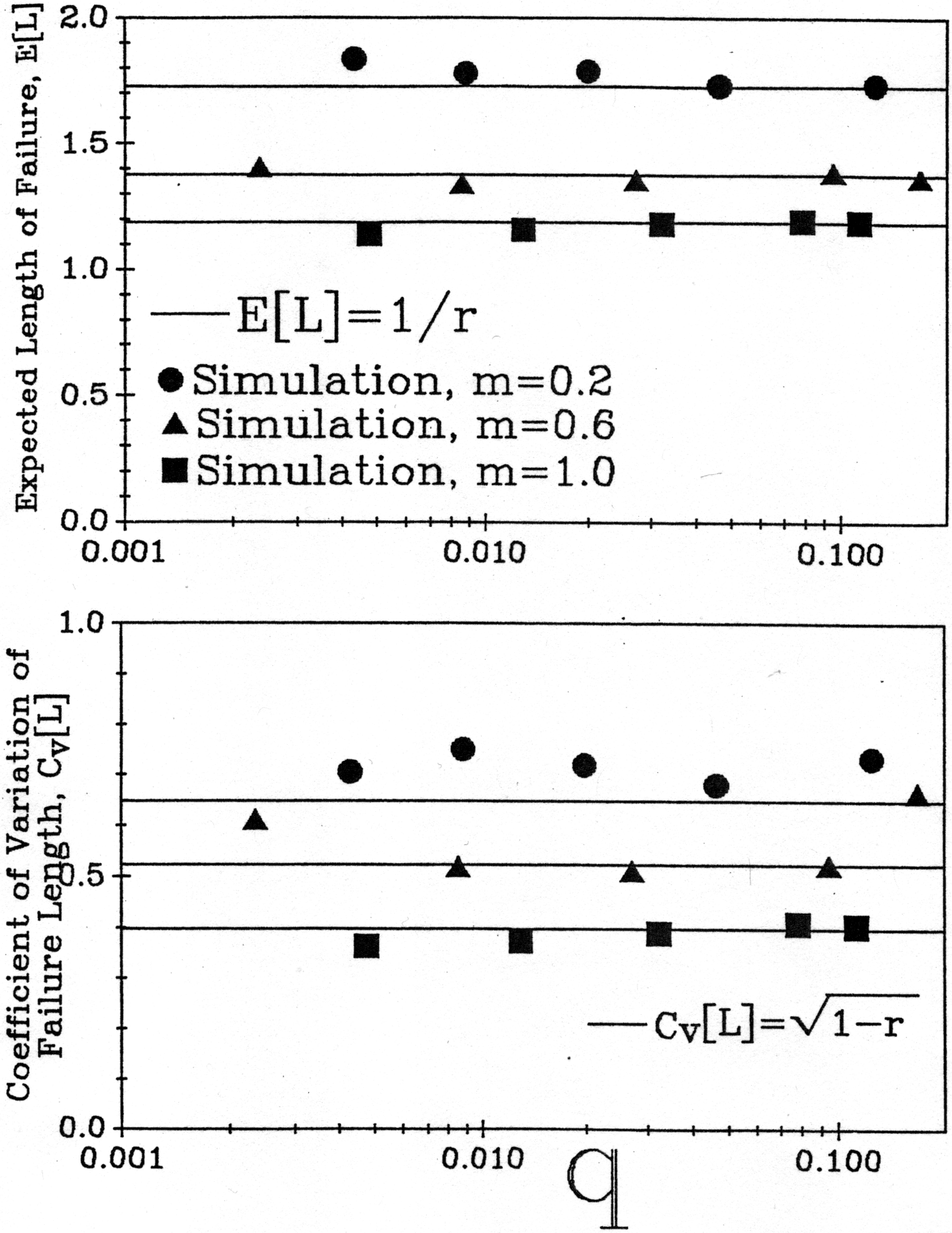

Figure 2. Comparison of theoretical and simulated average failure duration and coefficient of variation of failure duration as a function of failure probability q and resilience index m, for independent normal inflows.

between standardized storage S/σ, planning period, N, skewness of the inflows, γ, serial correlation of the inflows, ρ, and the resiliency index, m for AR(1) lognormal inflows. Vogel and Stedinger (1987) was compared with exact results from Pegram (1980) and Monte-Carlo simulations. We conclude that the two-state Markov model

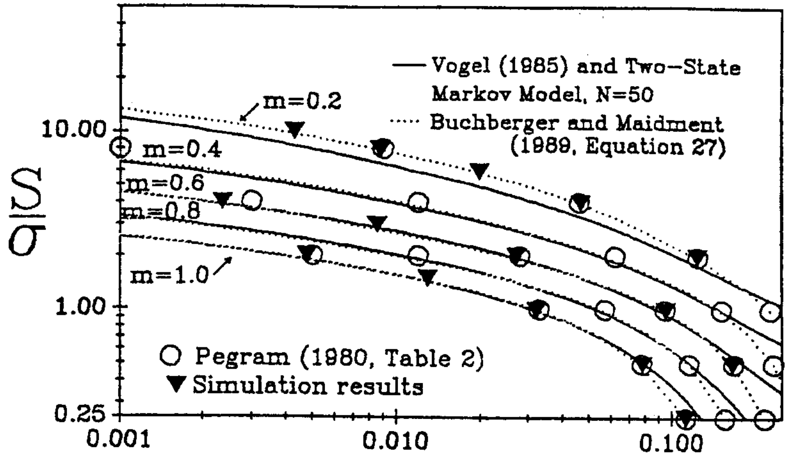

Figure 3. Comparison of the storage ratio, S/σ as a function of failure probability q, and resilience index m, for independent normal inflows.

accurately converts reliability p to reliability 1-q. Once again, the only exception is for values of m less than or equal to 0.2.

CONCLUSION

This study has shown that a two-state Markov model provides a satisfactory approximation to the mean and coefficient of variation of reservoir failure durations for systems dominated by over-year behavior and fed by independent normal and lognormal inflows. Vogel (1987) found that a two-state Markov model can also accurately represent reservoir surplus and failure sequences for systems dominated by within-year behavior.

In the U.S., reservoir design and operation studies usually focus upon the critical drought in each inflow sequence, hence reliability is normally quoted in terms of the probability of failure-free reservoir operations over an N-year period. Another approach used elsewhere defines the storage-yield relationship in terms of the steady-state probability of a reservoir system failure. The two-state Markov model enabled us to explain the relationship between N-year failure free reliability p and steady-state reliability 1-q for over-year reservoir systems providing a unified view of system reliability.

Most importantly, this study demonstrates that a two-state Markov model can adequately represent the structure of failure sequences to the extent that it can be used to convert reliability statements from one school of thought to another, providing a unified view of reservoir system reliability and resilience.

This study has also reviewed simple analytic Storage-Reliability-Yield (S-R-Y) relations (Buchberger and Maidment, 1989; Vogel ,1985; Vogel and Stedinger, 1987) which describe the approximate behavior of over-year reservoir systems fed by independent normal and lognormal inflows. Monte-Carlo experiments confirm the ability of the two-state Markov model combined with the cited S-R-Y relations to explain the reliability, resiliency, and vulnerability of over-year water supply systems. A sequel to this study will document the variety of applications one may envision using the procedures described for explaining the general behavior of over-year water supply systems.

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