THE RETURN PERIOD OF A RESERVOIR SYSTEM FAILURE

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ABSTRACT

It has become standard practice in the design of hydraulic structures to employ the average return period of a flood as the design event. An analogous index is introduced for the design of a water supply system: the average return period of a reservoir system failure, defined as the expected number of years until the occurrence of the first reservoir system failure. Other indices of the reliability of a water supply system are introduced. The use of these reliability indices is discussed in the context of a water supply reservoir design application.

1. INTRODUCTION

The storage-yield relation is the traditional tool used by water resource engineers to determine the required capacity of a storage reservoir to maintain a prespecified reservoir release. Although stochastic streamflow models have been available for a few decades, the most common approach in practice has been to base estimates of the required capacity of a storage reservoir upon application of the sequent peak algorithm (Thomas and Burden, 1963) or Rippl's mass curve (Rippl, 1883) to the historical streamflow record. While this approach is still advocated in recent textbooks on the subject (Clark et al., 1977; Steel and McGhee, 1979), it ignores the reliability associated with the resulting reservoir design capacity. Other textbooks (Linsley and Franzini, 1979; Linsley et al., 1982) discuss the application of stochastic streamflow models in conjunction with the sequent peak algorithm to generate the cumulative distribution function (cdf) of required reservoir storage, S, corresponding to a fixed planning period of length N years.

The cdf of S describes the relationship between the required storage capacity to meet a stated yield and the probability of failure-free reservoir operation, p, over an N-year planning period. Thus p is a measure of the reliability with which a reservoir of size S will provide failure-free operation over an N-year planning period. Duckstein et al. (1986) define p in a more general context where p is simply one performance index (PI^4) among a set of ten possible indices. Similarly Plate and Duckstein (1986) term p the project reliability for hydraulic design applications.

Engineers are often asked to convert statements of reliability over an N-year planning period to equivalent statements of annual reliability, R_a, or vice versa. Relationships between the annual reliability and the reliability over an N-year planning period associated with the design of flood control structures were developed by Thomas (1948) and further analyzed by Gumbel (1955) and Yen (1970) for independent events. These relations are in widespread use as evidenced by their inclusion in many textbooks on hydrology (Chow, pg. 8-34, 1964; Haan, pp. 70-75, 1977; and Linsley et al., pp. 349-350, 1982). This study develops analogous relationships between the annual reservoir reliability, R_a, and the probability of failure-free reservoir operations, p, over an N-year planning period.

Although use of stochastic streamflow models in conjunction with the sequent peak algorithm to estimate the cdf of S has been advocated by many authors since Fiering (1963), none of these investigators have evaluated which quantile, S_p, to choose in a design application. Similarly, when the sequent peak algorithm is applied to a single n-year historic streamflow record one is not sure which value of p, the probability of n=N year failure-free reservoir operation, to assign to the resulting estimate of the required storage capacity S. Here the length of the available historic streamflow sequence, n, is distinguished from the planning period or economic life of the proposed structure, N.

This study introduces a new index, T, the average return period of a reservoir system failure, analogous to the average return period of a flood event. Duckstein et al. (1986) also define the average return period, T, in a more general context where again T is simply one performance index (PI^5) among a set of ten possible indices. The relationships between T, p and N developed here are instrumental in choosing an appropriate quantile S_p for design purposes. Other indices are developed and their utility in choosing an appropriate design storage capacity is also considered.

2. RELATIONSHIP BETWEEN THE ANNUAL RELIABILITY OF A RESERVOIR SYSTEM AND THE PROBABILITY OF FAILURE-FREE OPERATION OVER AN N-YEAR PLANNING PERIOD

In this section, storage reservoir behavior is modeled, following Stedinger et al. (1983), concentrating upon both failure and regular (non-failure) years. In a given year a storage reservoir may be in either one of two states: (1) failure or (2) regular operation. Here a "failure" year is considered one in which the stated yield could not be met and a "regular" year is one in which the stated yield is provided or exceeded. While this analysis includes both the year in which the first reservoir system failure occurs as well as the duration of the failure, the actual magnitudes of the failures are ignored. For a more complete discussion of reliability measures associated with the frequency of failure years, failure durations and magnitudes of failures see Klemes (1979), Klemes et al. (1981) and Hashimoto et al. (1982).

Let the row vector $\underset{\sim}{X}_y = (x_{1y}, x_{2y})$ specify the probability that a reservoir system is in either: (1) the failure state or (2) the regular (non-failure) state in year y. Also assume as did Stedinger et al. (1983), that $\underset{\sim}{X}_y$, y = 1, ..., N forms a Markov chain with probability transition matrix

$$A = \begin{bmatrix} 1 - r & r \\ f & 1 - f \end{bmatrix} \quad (1)$$

where f is the probability that a failure year follows a regular year and r is the probability that a regular year follows a failure year. The two-state Markov model becomes

$$\underline{X}_{y+1} = \underline{X}_y A \quad (2)$$

Figure 1 depicts the two-state Markov model. The Markov assumption introduces memory into the process although the transition of the reservoir system in any given year y is only influenced by the reservoir system state in the preceding year y-1. More complex models are possible and have been considered by Klemes (1967, 1969). The simple model introduced here provides an approximation to the behavior of reservoir storage state transitions, and, most importantly, this model formulation leads to the simple and useful reliability indices developed in the following sections.

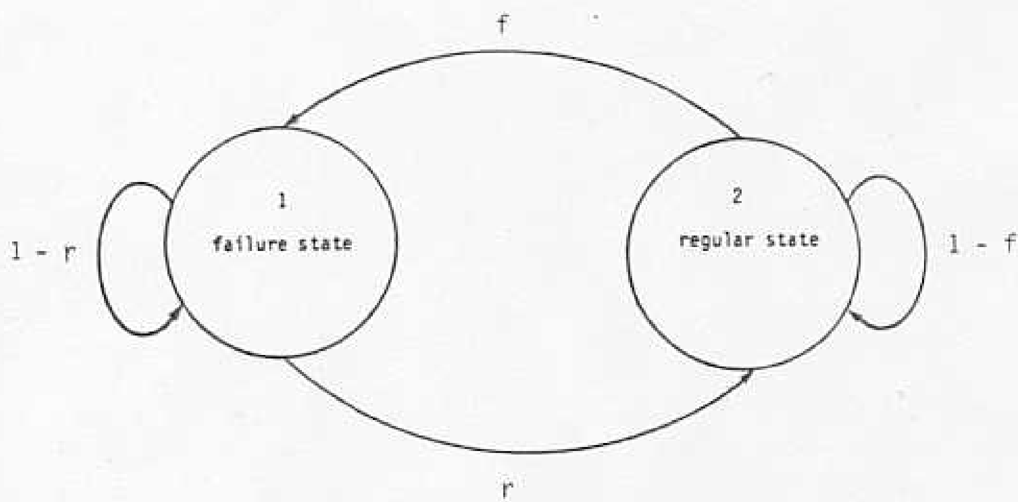

Figure 1 - The Two-State Markov Model of Reservoir System States

As y increases, $\underline{X}_y$ reaches a steady-state, and the solution to (2) becomes

$$\lim_{y \to \infty} \underline{X}_y = \left(\frac{f}{r+f} , \frac{r}{r+f} \right) \tag{3}$$

for $0 \leq |rf| \leq 1$. A derivation of this result may be found in Jackson (1975). Thus in the long run, the probabilities that the reservoir system will be in the failure or regular state are f/(r+f) and r/(r+f) respectively, regardless of the initial state of the reservoir system. Hence r/(r+f) is the steady-state probability of regular operation or the annual reliability R_a.

When the sequent peak algorithm is employed to determine the smallest storage reservoir design capacity required to assure regular or failure-free operation over an N-year planning period with probability p, then p becomes a steady-state probability. This is due to the fact that the sequent peak algorithm, as advocated by Thomas and Burden (1963), wraps the streamflow record around which generates the steady-state solution to the problem posed.

Now suppose we choose a reservoir capacity equal to S_p (the p^{th} quantile of the distribution of S) using the sequent peak algorithm. Then the steady-state probability of regular (failure-free) operation over an N-year planning period, p, equals the probability of normal operation the first year r/(r+f) times the probability that subsequent years remain free of failures:

$$p = \frac{r}{r+f} (1-f)^{(N-1)} \tag{4}$$

The critical parameters of this model which must be estimated are r and f since p and N are usually taken as fixed values. Stedinger et al. (1983) and Vogel (1985a) investigated equation (4) and concluded that knowledge of p and N are not sufficient to determine r, f or R_a unless one knows the values of r and f which is unlikely to be the case in practice. Vogel (1985a) showed that the average return period of a reservoir system failure does not depend upon the value of r in practice.

In this study (4) is simplified considerably by conditioning the analysis upon the occurrence of regular or non-failure reservoir operations during the first year. Now the steady-state probability of regular operation over an N-year planning period, p, conditioned upon regular reservoir operations in the first year equals the probability that all years, subsequent to the first year, remain free of reservoir system failures:

$$p = (1-f)^{(N-1)} \tag{5}$$

Now due to this innovation p no longer depends upon r and (5) may be solved directly for f without resorting to a numerical algorithm as is required in the solution of (4).

$$f = 1 - p^{(1/(N-1))} \tag{6}$$

Furthermore f is completely specified by our knowledge of p and N and does not require assumptions regarding the value of r.

3. THE AVERAGE RETURN PERIOD OF A RESERVOIR SYSTEM FAILURE

In association with flood studies dealing with sequences of peak annual streamflows, Gumbel (1941) and Thomas (1948) defined the return period as the interval between flood events, where a flood event is defined as an annual peak flow above some threshold. Alternatively the return period may be thought of as the number of years until the occurrence of the first flood event. Since Thomas (1948), the meaning of the return period has changed. For example, Haan (1977, pg. 3) defines the return period as the average elapsed time between occurrences of a flood event. Thus, the return period was initially defined as the random time to an event, yet its meaning has changed to become the expected value of that random variable. This study distinguishes between these two definitions by using the terms return period and <u>average</u> return period.

Drawing an analogy to the average return period of a flood, the average return period of a reservoir system failure may be defined as the expectation of the return period, which is the number of years before the occurrence of the first reservoir system failure.

Let Z be the year in which the first reservoir system failure occurs. Then the steady-state probability of the first failure occurring in the Z^{th} year, conditioned upon regular reservoir operation in the first year, equals the probability of Z-2 years of regular operation followed by a failure year. The probability mass function (pmf) of the time to the first failure is

$$P[Z = z] = \begin{cases} 0 & \text{if } z = 1 \\ f(1-f)^{z-2} & \text{if } z \geq 2 \end{cases} \tag{7}$$

Then the average return period of a reservoir system failure is

$$T = \mu_z = \frac{1 + f}{f} \tag{8}$$

which may be combined with (6) to obtain

$$T = \frac{2 - p^{(1/(N-1))}}{1 - p^{(1/(N-1))}} \tag{9}$$

Similarly one obtains from (7) the variance of Z simply as

$$\sigma_z^2 = \frac{1 - f}{f^2} \tag{10}$$

Equations (7) through (10) provide a measure of the likelihood of future reservoir system failures. In particular, equation (9) is a simple relationship between T, p and N which provides a useful tool for determining which quantile, S_p, to choose in a design application. Equation (9) is the counterpart to the well known relationship between the average return period of a flood discharge, the planning period N, and the non-exceedence probability of that flood discharge documented in most textbooks on hydrology (see for example Chow 1964, Figure 9-61 and Linsley et al., 1982, Table 11-7). The simplicity of equations (7) through (10) results in large part from conditioning the entire analysis upon regular (or non-failure) reservoir system operation during the first year. If an unconditional approach is employed as was the case in the development of (4), equations (7) through (10) become much more complex because the expressions for P[Z = z], T and σ^2 include both r and f in addition to p and N, as is shown in Vogel (1985a). The equations developed here are recommended over those in Vogel (1985a) since the conditional approach makes physical sense and the resulting expressions are much simpler.

4. OTHER INDICES OF RESERVOIR SYSTEM PERFORMANCE

The average return period of a reservoir system failure, T, is simply the average number of years prior to the first reservoir system failure. Perhaps a more reasonable statistic would be to report the "likely recurrence interval" which is defined here to be that interval of time over which reservoir system failures are likely to occur, say 90% of the time.

The q^{th} percentile of the distribution of the year in which the first reservoir system failure occurs, Z_q, may be obtained by choosing the largest value of Z_q such that

$$\sum_{z=2}^{Z_q} f(1 - f)^{z-2} \leq q \tag{11}$$

which may be solved for Z_q as follows

$$Z_q = \frac{\ln(1-q)}{\ln(1-f)} + 1 \tag{12}$$

where f is uniquely determined from (6) given values of p and N. Again equation (12) yields a very simple expression for a percentile of the distribution of return periods due in large part to having conditioned the entire analysis upon regular (or non-failure) reservoir system operation during the first year. Without resorting to this conditional analysis, the resulting expression for Z_q depends upon both r and f in addition to p and N and must be solved using a numerical algorithm as shown in Vogel (1985a).

5. SUMMARY

In the design of hydraulic structures it has become standard practice to employ the average return period of a flood discharge as the design event. This study developed an analogous index for the design of a water supply system: the average return period of a reservoir system failure. The resulting expressions are simplified dramatically by conditioning the entire analysis upon regular (or non-failure) reservoir operations during the first year. Percentiles of the distribution of the return period of reservoir system failures or simply the average return period of a reservoir system failure are readily estimated from the simple expressions developed here. These expressions are of particular value for the following reasons:

(1) The return period concept is a widely accepted index of reliability in the field of water resources engineering.

(2) The reliability indices developed here are simple to understand and easy to apply.

(3) Use of these indices provides a measure of the likelihood of future reservoir system failures which until now was unavailable in such a simple form. For an example of the use of the reliability indices developed here see Vogel (1985a, 1985b).

In a recent national assessment of our nation's water resources, the Water Resources Council (1978) concluded that 17 of the nation's 21 water resource regions have or will have a serious problem of inadequate surface-water supply by the year 2000. As increasingly marginal surface-water supply sites are pressed into service, target yields at both existing and proposed sites can only increase. In many instances, increased demands are being met by more efficient management and utilization of existing reservoir systems rather than by construction of new facilities (for an example of this recent phenomenon see Sheer and Flynn, 1983). Whether new facilities are envisaged or the existing reservoir system is to be operated more efficiently, the storage-reliability-yield relationship is a fundamental ingredient. The use of stochastic streamflow models in conjunction with the sequent peak algorithm may be used to develop the storage-reliability-yield relationship. The reliability indices developed here may then be employed to develop explicit statements regarding the likelihood of future reservoir system failures.

Past and recent research has identified weaknesses and potential problems with the traditional techniques for estimating the storage-reliability-yield relationship. Fiering (1967) documents important shortcomings associated with the strict use of the historic streamflow record. Stochastic streamflow models were developed to circumvent the shortcomings of the use of the historical required storage alone. Recent research indicates that stochastic streamflow models can be used to significantly improve the precision of estimates of the storage-reliability-yield relationship in comparison to the traditional approach of employing the historical streamflow record alone (Vogel, 1985a; Vogel and Stedinger, 1986) As is to be expected, the precision of estimates of the storage-reliability-yield relationship depends primarily upon the length of the available historic streamflow record. The precision of these estimates may be improved by employing streamflow record augmentation and/or extension procedures (Vogel and Stedinger, 1985). However, even for relatively long records, Vogel (1985a) and Vogel and Stedinger (1986) document substantial sampling variability associated with estimates of S_p. Given the short streamflow records available in most practical situations it has become evident that one should incorporate streamflow model parameter uncertainty into reservoir design and operations studies to obtain an honest account of the true likelihood of reservoir system failures (see Stedinger and Taylor, 1982 and Stedinger et al. 1985).

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