

# The (mis)behavior of behavior analysis storage estimates

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**Abstract.** This paper investigates the dependence of estimates of reservoir storage capacity derived using behavior analysis on the length of inflow sequence used for overyear reservoir simulation. It has generally been assumed that simulation using behavior analysis, which incorporates a given probability of failure, will always give steady state estimates of the storage capacity (apart from the effects of the initial reservoir condition). The results reported here show that it may take sequence lengths as much as 1000 years or more for the mean of the distribution of storage capacity estimates to approach a stationary value. For some cases with high draft and high inflow variability, we show that a significant swing of the mean storage estimates from an initial downward bias into an upward bias occurs before their convergence to a stationary level. However, the median storage estimates always showed downward bias which sometimes decreased very slowly with increasing sequence length. We provide explanations for these observations and discuss some of the implications on the choice of inflow sequence length in determining reservoir storage capacities using behavior analysis.

## Introduction

Behavior analysis simulates storage fluctuations in a reservoir subject to a given draft (which can be constant or can vary in time) and a given sequence of inflows. When applied to a semi-infinite reservoir, it can be used to determine directly the reservoir storage capacity necessary to supply, without failure (i.e., with a reliability of 100%), the given draft over the entire period defined by the inflow sequence used. Variants of this approach have been used, for example, by *Rippl* [1883], *Hazen* [1914], and *Thomas and Burden* [1963], whose “sequent peak algorithm” has become widely used in the United States [*Vogel and Stedinger*, 1988].

When applied to a finite reservoir of a particular storage capacity  $S$  (a common practice outside the United States), behavior analysis allows a direct computation of the frequency of failures in the delivery of a given draft. Alternatively, by iterating this direct computation for the same inflow sequence and different values of the storage capacity  $S$ , an estimate can be obtained (by successive approximations) of a storage capacity required for supplying a given draft with a prescribed rate of failure (i.e., for a reliability given by some preset percentage less than 100%). This iterative approach can also be used to obtain an estimate of a draft that could be supplied by a reservoir of a given capacity with a prescribed reliability.

In the current study, we are concerned with the estimation of the reservoir storage capacity for given values of constant draft and reliability for overyear systems. The rate of failure (the complement of system reliability) can be defined in a number

of ways (*Kritskiy and Menkel* [1952], as cited and discussed by *Klemeš et al.* [1981]). The definition which is used here is the occurrence based, or annual, probability of failure. This is the proportion of the years in the entire inflow sequence analyzed in which the required draft is not supplied. The storage estimate is derived by the iterative procedure described above, altering the assumed reservoir capacity until the minimum capacity is found which results in the simulated system achieving the required reliability. This minimum capacity will be referred to as the “behavior analysis storage estimate” [see *McMahon and Mein*, 1978].

For behavior analysis the assumption has typically been that apart from the influence of the initial reservoir condition, the analysis yields an unbiased storage estimate for a steady state system reliability [see, e.g., *Klemeš et al.*, 1981]. The major objective of this paper is to report some unusual results of behavior analysis simulations using synthetically generated inflow sequences. Of particular interest is the finding of an unexpected swing in some cases of the mean storage estimates from an initial downward bias into an upward bias before their convergence to a stationary level with very long sequences.

## Concatenated Behavior Analysis

Numerical behavior analysis involves using a mass balance equation of inflows to and outflows from a finite conceptual storage:

$$X_{t+1} = X_t + q_t - D_t \quad (1a)$$

$$X_{t+1} > S, X_{t+1} = S \quad (1b)$$

$$X_{t+1} < 0.0, X_{t+1} = 0.0 \quad (1c)$$

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where  $X_{t+1}$  is the reservoir content at the beginning of the  $t + 1$ th time period,  $q_t$  is the inflow during the  $t$ th period,  $D_t$  is the draft in the  $t$ th period, and  $S$  is the capacity of the storage. For the purposes of this study, all losses will be neglected to simplify the analysis, hence terms dealing with net evaporation and other losses have not been included. These assumptions would certainly not be applicable where a real reservoir design is considered, but they would not be critical in affecting the general results in this study.

Equation (1) is applied to inflow sequences year by year, thereby updating the reservoir level for overyear simulation. The initial reservoir condition ( $X_0$ ) is typically assumed to be full [see *McMahon and Mein*, 1978], although any initial condition ranging from empty to full could be used. The annual probability of failure for the system is then estimated by dividing the number of years in which the required draft could not be supplied by the number of years in the total sequence.

In a study using simulation results based on the generation of monthly synthetic data sequences and behavior analysis, *McMahon et al.* [1972] found that the mean value of storage estimates increased with increasing sequence length up to a maximum steady state level at 100 to 200 years. The explanation given was that the first failure in the sequence was dependent on the initially full storage condition used, whereas subsequent failures could occur without the reservoir refilling to capacity. *Moran* [1959] pointed out that reliability estimates for a given draft and reservoir capacity based on behavior analysis would be biased because of the simulation not starting in a stationary state. *Moran* felt that these effects would be mostly negated by ignoring the first 10 to 20 years in, for example, a 1000 year simulation, basing the estimate on the remaining years in the sequence. For systems with high drafts and highly variable inflows, *McMahon and Mein* [1978] recommended a sequence length of at least 100 years would be required to overcome the effects of the assumed initial conditions.

In order to remove the influence of initial conditions on behavior analysis storage estimates, the inflow sequence used is often concatenated with itself, the resulting concatenated sequence being routed through the reservoir simulation beginning with the reservoir full (i.e.,  $X_0$  for the first cycle equal to the reservoir capacity). In this way, two complete cycles of the inflow sequence are consecutively routed through the system, with the reliability being estimated only for the second cycle. This practice has been used from the beginning of this century in Europe [see *Klemeš et al.*, 1981] and was recommended independently in Australia by *Moran* [1959] and by *Thomas and Burden* [1963] for sequent peak analysis in the United States. The estimates produced by this method in this study will be referred to as the concatenated behavior estimates.

The commonly held belief (following *Moran* [1959]) appears to be that the averages of any of the three parameters in the storage-reliability-yield (S-R-Y) relationship obtained from behavior analysis of even short concatenated input sequences are good approximations of their stationary values. The analysis reported here will show that averages of storage estimates can exhibit substantial biases.

## Method of Analysis

### Generation of Inflow Sequences

The inflow sequences for analysis were generated using a simple lag one annual autoregressive model [Fiering, 1961]

$$q_{t+1} = \mu + \rho_1(q_t - \mu) + z_t\sigma\sqrt{(1 - \rho_1^2)} \quad (2)$$

with  $q_{t+1}$  as the annual inflow in year  $t + 1$ ,  $\mu$  as the mean of the annual inflows,  $z_t$  as a normal random variate with mean zero and standard deviation one,  $\sigma$  as the standard deviation of annual inflows, and  $\rho_1$  as the autocorrelation coefficient of annual inflows. Sequences were generated for four values of the coefficient of variation of annual inflows ( $C_v$ ): 0.2, 0.5, 0.7, and 1.0, each with a mean annual inflow of 100 units. In most of the cases reported the value of  $\rho_1$  was zero.

For the case of  $C_v = 0.2$ , (2) was used directly to calculate the annual inflows, the assumption being that these inflows have a normal distribution. For the higher  $C_v$  values it was assumed that the distributions of annual inflows were described by a three parameter lognormal distribution (LN3), with coefficients of skew ( $\gamma$ ) assumed as given by (3).

$$\gamma = 2C_v \quad (3)$$

By enforcing (3) it should be noted that the resulting LN3 distribution will very closely resemble a Gamma distribution. The values of  $C_v$  and  $\gamma$  chosen are considered to be representative of a wide variety of streams in Australia as well as Europe and North America (considering the global data examined by *McMahon et al.* [1992]). Generation of the inflows with a LN3 distribution was carried out using a modified version of (2) in association with the moment transformation equations proposed by *Matalas* [1967], the log-transformed inflow  $y_t$  being generated and then exponentiated to produce synthetic inflows in the real domain,

$$y_t = \ln(q_t - \tau) \quad (4)$$

where  $\tau$  is the lower bound for a LN3 distribution, and  $y_t$  is the log-transformed annual inflow at time  $t$ .

### Experimental Design

For each set of inflow parameters, 15 different sequence lengths were generated (varying from 10 to 5000 years). For each sequence length ( $n$ ),  $k$  replicates were produced such that

$$kn = 5,000,000 \quad (5)$$

Thus, for example, 500,000 replicates of 10-year sequences and 1000 replicates of 5000-year sequences were generated. The generated sequences were used for a concatenated behavior analysis (starting with the reservoir full) with a reliability of 90% and several drafts (see Table 1).

## Results

Effects of inflow sequence length on storage estimates will manifest themselves as differences in the distributions of these estimates derived from sequences of different lengths. Figure 1 shows a representation (in terms of quantiles and means) of the distributions of concatenated behavior storage estimates over the range of sequence lengths investigated for the case where draft was 90 units and  $C_v = 0.7$  for independent inflows. For this case the distribution of storage estimates is substantially influenced by the sequence length analyzed. Although the quantile estimates and the mean converge to stationary levels after approximately 1000 years, there is a definite swing from an initial downward bias to an upward bias of the

**Table 1.** Combinations of Streamflow Characteristics and Reservoir Drafts Investigated

Streamflow Characteristics				Annual Draft, units	$m$
Streamflow Distribution	$C_v$	$\gamma$	$\rho_1$		
Normal	0.2	0.0	0.0	90	0.5
				85	0.75
				80	1.0
				90	0.2
				85	0.3
	0.5	1.0	0.0	80	0.4
				70	0.6
				90	0.14
				80	0.29
				70	0.43
LN3	0.7	1.4	0.0	50	0.71
				90	0.14
				90	0.14
				90	0.14
				90	0.1
	1.0	2.0	0.0	80	0.2
				70	0.3
				50	0.5

Mean annual flow is 100 units. Reliability is 90%.

Hazen's [1914] nondimensional parameter  $m$  is defined in equation 6.

upper quantile and mean storage estimates before this convergence is achieved.

This swing in the bias of the mean storage estimates was not expected and, to our knowledge, has not been previously reported. The swing in bias is more pronounced for the higher quantile storage estimates, corresponding to an initial increase in the degree of (positive) skew before a significant reduction in skew occurs as convergence progresses. This behavior of the storage estimate distributions is well captured by considering the behavior of the means and medians of the storage estimates as shown in Figure 1. The swing into upward bias is shown as a "hump" in the plot of mean storage estimate versus inflow sequence length used in concatenated behavior analysis.

The hump was not exhibited at all for median storage estimates.

### Behavior of Mean and Median Storage Estimates

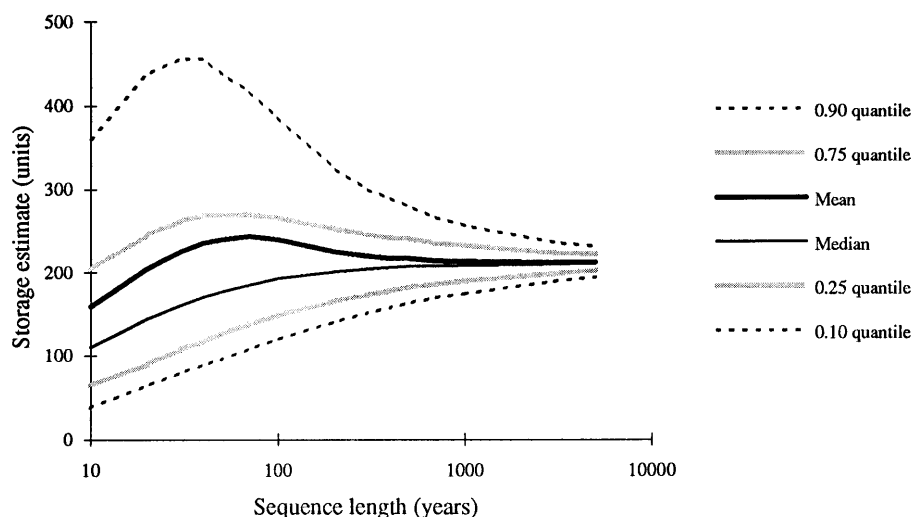
Figure 2 shows plots of the mean and median concatenated behavior storage estimates against sequence length for the four cases of  $C_v$  examined. It can be seen that convergence of both mean and median storage estimates to stationary levels was generally complete for sequence lengths of 1000 years. The swing into upward bias of the mean storage estimates did not occur for all cases investigated, steady convergence without appearance of the hump being associated generally with lower levels of draft and lower values of  $C_v$ . For a given draft, increasing  $C_v$  resulted in accentuation of the swing in bias as indicated by a more pronounced hump in mean storage estimates and occurrence of humps at longer sequence lengths. The swing was also accentuated by increased draft for a given value of  $C_v$ .

With the combination of draft and  $C_v$  characterized using the nondimensional parameter  $m$  first used by Hazen [1914]

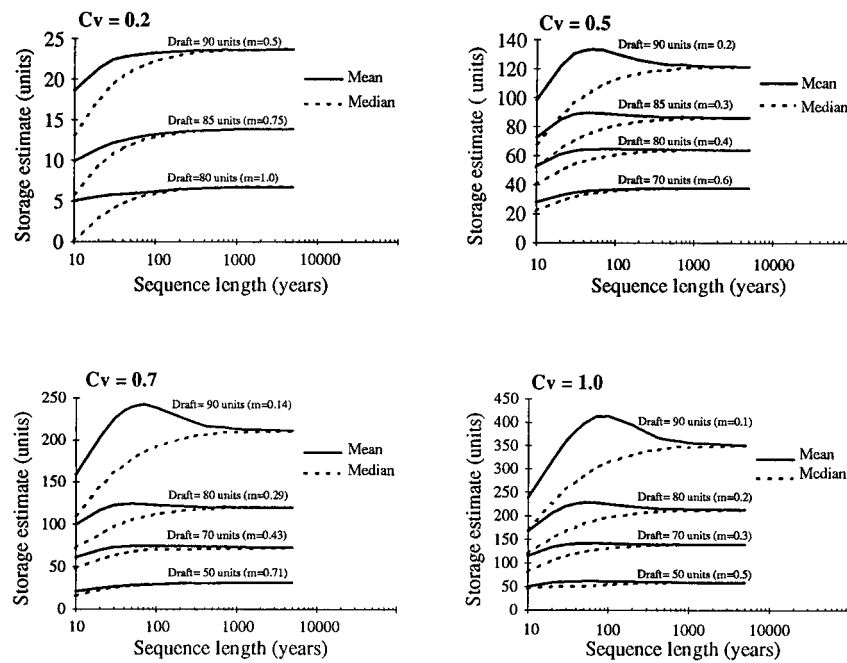
$$m = (1 - D)/C_v \quad (6)$$

where  $D$  is the annual draft as a proportion of mean annual inflow, it was found that the swing into upward bias of concatenated behavior mean storage estimates was apparent only for values of  $m$  below about 0.5. Potentially, an estimate of  $m$  could be used as a check of  $C_v$  and  $D$  values for particular design applications as an indicator of the likelihood of the swing in bias occurring if concatenated behavior analysis is used to derive storage estimates.

Although both mean and median storage estimates showed initial downward bias, the plots of median storage estimates showed no humps at all, the values consistently rising asymptotically to the long-term value as sequence length increased. For all cases examined the mean-to-median ratio was initially greater than 1 (indicating positively skewed storage estimate distributions), this ratio approaching unity for nearly all cases (except where  $m = 0.1$ ) as the long-term stationary values were achieved. Where the humps in the mean storage estimate



**Figure 1.** The 0.10, 0.25, 0.75, and 0.90 quantiles, mean and median storage estimates for concatenated behavior analysis where draft = 90 units,  $C_v = 0.7$ ,  $m = 0.14$ , and  $\rho_1 = 0.0$  plotted against sequence length;  $kn = 5,000,000$  and reliability = 90%.



**Figure 2.** Plots of the concatenated behavior analysis mean and median storage estimates (over  $k$  replicates) against length of streamflow sequence ( $n$ ) for normally distributed annual flows ( $C_v = 0.2$ ) and log normally distributed annual flows ( $C_v = 0.5, 0.7$ , and  $1.0$ ), all with  $\rho_1 = 0.0$ , for various drafts.  $kn = 5,000,000$  and reliability = 90%.

plots occurred, the mean-to-median ratios showed marked initial increases as sequence length increased before finally converging to 1. This is indicative of an increase in the positive skew of the storage estimate distributions for these cases before a decrease in skew commences (coinciding with the peak of the hump) as convergence progresses.

#### Impact of Reliability and Serial Correlation on Reservoir Storage Estimates

Some of the experiments were repeated setting the required reliability at 95%, the results showing the same behavior as that already described. Another assumption used in the above simulations was that the annual inflows were serially independent. It has been well known for many years that, in general, as the degree of serial correlation increases, so does the required storage for a given reliability and draft [e.g., see *Perrens and Howell, 1972; Srikanthan, 1985*]. This would be expected because of the serial correlation causing critical deficit periods to become more extreme. To investigate the effects of serial correlation in the context of the current study, a comparison was made between  $\rho_1$  values of 0.0, 0.15, 0.3, and 0.5 for the case where  $C_v = 0.7$  and draft = 90 units (Figure 3). Not only did the mean and median storage estimates increase with greater serial correlation, but the hump occurred later (as did the convergence of the mean-median ratio to 1) and was more pronounced (as was the initial increase in mean-median ratio).

#### Effect of Initial Conditions

A number of experiments were run to explore the influence of the initial reservoir condition. For these, simulation was initially performed without concatenating the inflow sequences, the single cycles analyzed being started with the reservoir level at various fixed proportions of capacity. Typical results (for draft = 90 units and  $C_v = 0.7$ ) are shown in

Figure 4a. It can be seen that for this case the influence of the initial condition is effectively nullified for inflow sequences longer than about 500 years. Interestingly, the swing into upward bias of the storage estimates observed for concatenated behavior analysis also occurred in this case where the initial storage level was fixed, even for an initial reservoir level fixed at 0.1 times capacity.

The influence of initial conditions for single-cycle behavior analysis is not surprising, considering that extreme initial conditions in such cases will introduce a significant initial bias that will only be overcome for sequence lengths long enough such that the initial error disappears when its effects are averaged over the entire sequence. Experiments were also run starting concatenated behavior analysis with initial conditions set at various fixed proportions of capacity. The results (for draft = 90 units and  $C_v = 0.7$ ) shown in Figure 4b indicate that initial conditions still influence concatenated behavior analysis storage estimates up to relatively long sequence lengths (about 300 years in this case). However, this influence disappears for shorter inflow sequence lengths compared to single-cycle behavior analysis (300 years compared to 500 years for the simulations shown in Figure 4).

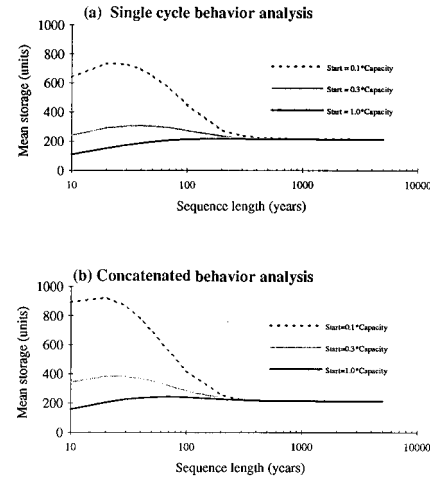
#### Discussion

It follows from the stochastic theory of storage [*Moran, 1959*] that the reliability of a reservoir operating on a single realization of the inflow process (i.e., one inflow sequence) can theoretically attain steady state only as the sequence length approaches infinity. For synthetic inflow series simulating annual inflows, Moran suggested that a sequence length of 1000 years should be sufficient to obtain an estimate of the steady state reliability with an accuracy needed for practical purposes. Moran further noted that it would not be worth analyzing

sequences longer than about 1000 years because of the inherent uncertainty associated with usually short historic records. *Stedinger et al.* [1985] also discuss the effects of parameter uncertainty in this context, including the incorporation of such parameter uncertainty into stochastic techniques.

Since the service life of real reservoirs is closer to only about 100 years and their operating procedures are likely to change even within such a relatively short period, *Klemeš* [1967, 1969] proposed to evaluate the distributions of reliability within short periods of operation. To this end he presented two techniques, one applicable only for annual inflows [*Klemeš*, 1967] and the other using monthly inflows [*Klemeš*, 1969]. Application of both techniques to the Teplice Dam project in Czechoslovakia showed a clear dependence of the reliability distributions on the length of the period of reservoir operation considered. In this regard the findings reported in this paper are consistent with *Klemeš*' results.

However, there is an important difference: *Klemeš*' [1967, 1969] results show no bias other than that due to the initial storage state, the convergence of the mean reliability to the steady state being uniform. When *Klemeš* used a steady state storage distribution as the initial condition, the mean reliability was always equal to its steady state value regardless of the length of the period considered. Since both of *Klemeš*' techniques are based on probabilistic analyses of inflow distributions (a combinatorial analysis of "wet" and "dry" reservoir states in the first paper and the Gould probability matrix technique in the second paper) rather than on behavior analysis of individual realizations of the inflow process, it seems plausible that the swing in bias of the mean storage estimates identified



**Figure 4.** Comparison of mean storage estimates for various starting conditions for (a) single cycle (nonconcatenated) and (b) concatenated behavior analysis with  $C_v = 0.7$ , draft = 90 units, and  $m = 0.14$ ;  $kn = 5,000,000$  and reliability = 90%,  $\rho_1 = 0.0$ .

in our analysis is a phenomenon exhibited only by storage behavior corresponding to individual short inflow sequences.

In particular, we believe that the behaviors of the biases in mean and median storage estimates are caused by the interaction of three phenomena. The first is related to the limited range of the storage estimate distribution for small inflow sequence lengths, the second is a frequency bias for small sequence lengths, and the third is related to the highly variable and skewed nature of the distribution of storage estimates.

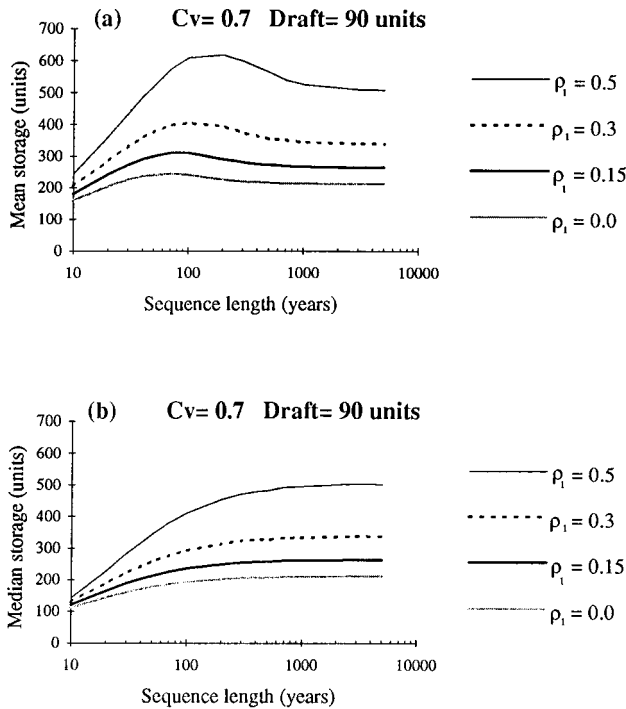
#### Limited Range of Storage Estimate Distribution for Short Inflow Sequence Lengths

For short inflow sequence lengths ( $n$ ), storage estimates ( $S(n)$ ) equal to zero may result because of the limited hydrologic experience. That limited experience also puts a bound on the largest possible value of  $S(n)$ . Even if every inflow in a short sequence were zero, the total demand over  $n$  periods is only  $nD$ . Thus, in practice the concatenated behavior analysis algorithm (starting full) will produce an upper bound for  $S(n)$  of  $2nD$ , which increases as sequence length increases. This can have a significant effect, particularly in causing downward bias in the upper tails of the distribution of  $S(n)$  for short-sequence lengths, as suggested by Figure 1.

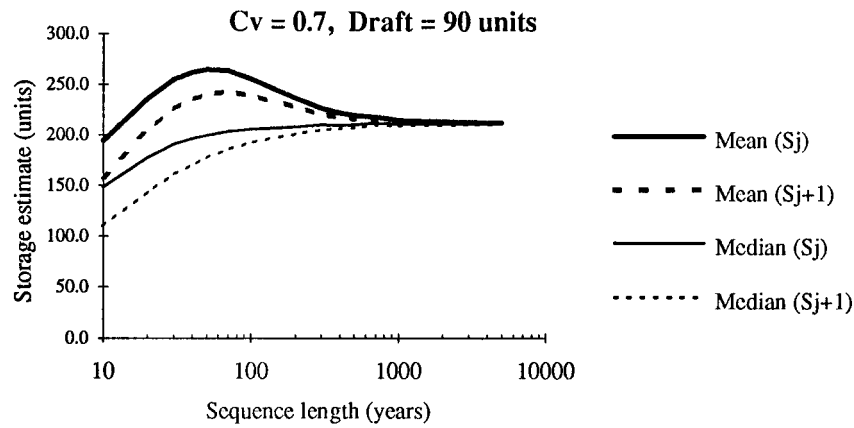
Similarly, if the sequence length ( $n$ ) is shorter than the length of critical drought sequences, then reservoir behavior in the second  $n$  years of a concatenated sequence (i.e., during the second or test cycle of concatenated behavior analysis) will not exhibit a critical range of drawdown behavior. Thus the distribution of  $S(n)$  will be stochastically too small, where concatenated behavior analysis is started with the reservoir full. This effect combined with the limited range of  $S(n)$  for small  $n$  imposes a downward bias on estimates of  $S(n)$  for small sequence lengths.

#### Frequency Bias for Small Inflow Sequence Length

The algorithm described above for estimating the concatenated behavior analysis storage estimates uses (for finite  $n$ ) a failure frequency ( $f(n)$ ) that is greater than the target long run failure frequency ( $f$ ). Mathematically,



**Figure 3.** Comparison of (a) mean and (b) median concatenated behavior analysis storage estimates between simulation runs for  $\rho_1 = 0.0, 0.15, 0.3$ , and  $0.5$ , where  $C_v = 0.7$ , draft = 90 units, and  $m = 0.14$ ;  $kn = 5,000,000$  and reliability = 90%.



**Figure 5.** Comparison of mean and median storage estimates for  $S_{j+1}$  and  $S_j$ , for concatenated behavior analysis with  $C_v = 0.7$ , draft = 90 units, and  $m = 0.14$ ;  $kn = 5,000,000$  and reliability = 90%,  $\rho_1 = 0.0$ .

$$f(n) > f \quad (7)$$

Where  $n$  is a multiple of 10 (as assumed for the simulations performed),  $nf$  equals some integer  $j$ . Considering the computation of  $S(n)$ , it will be found that for a particular sequence, for any  $S > S_1$ , no failures occur. For  $S < S_1$ , at least one failure occurs. In particular, for  $S = S_j$ ,  $(j - 1)$  failures occur with strictly positive shortages (ignoring ties) and one zero shortage failure (storage volume zero) occurs. However,  $S$  can be further reduced until a volume  $S_{j+1}$  is reached at which  $j$  positive failures occur (ignoring ties) plus one zero shortage failure. For any  $S(n)$  with

$$S_j \geq S(n) \geq S_{j+1} \quad (8)$$

only  $j$  failures occur. All of the storage estimates within this range have a frequency of failure  $j/n = f$  over  $n$  years. For small  $n$  this interval can be wide, but as  $n$  approaches infinity it would tend toward zero width. The algorithm used to calculate the concatenated storage estimates described above always estimates  $S(n)$  as  $S_{j+1}$ . Thus, for small  $n$  it introduces a further source of downward bias relative to  $S(n \rightarrow \infty)$ . Use of  $S(n) = S_j$  would introduce a source of upward bias as an estimate of  $S(n \rightarrow \infty)$ .

To evaluate the influence of the frequency bias, a further simulation was performed such that  $S_j$  estimates (for the case where  $C_v = 0.7$  and draft = 90 units) could be compared with  $S_{j+1}$  estimates. The means and medians of both these estimates are plotted in Figure 5. As expected, the  $S_j$  and the  $S_{j+1}$  estimates converge for large  $n$ , with the  $S_{j+1}$  estimates smaller than the  $S_j$  estimates. However, the  $S_j$  estimates are still downward biased relative to the long-term values. This indicates that the downward bias due to the influences described earlier (given the full starting condition) is stronger than the upward frequency bias for the  $S_j$  estimates. For the  $S_{j+1}$  estimates the downward frequency bias compounds the downward bias due to the effects described above, although its influence is less significant.

#### **$S(n)$ Distribution and Mean Storage Estimates**

As discussed earlier, the distribution of  $S(n)$  is bounded below and has an upper bound ( $2nD$ ) which increases as  $n$  increases. For modest  $n$  and systems with high draft and high inflow variability (small  $m$ ) the distribution of  $S(n)$  can have a very large coefficient of variation,  $C_v(S(n))$ , and large positive

skew coefficient. In particular, the skew phenomenon is illustrated by the results of *Vogel and Stedinger* [1987]. Thus, for small  $n$ , as  $n$  increases, the upper bound on  $S(n)$  increases, resulting in increases in the median storage estimates as well as in  $C_v(S(n))$ . An increasing  $C_v(S(n))$  results in an increasing skew in the  $S(n)$  distribution. In some cases, with modest  $n$  and small  $m$  (below about 0.5 according to the simulation results reported in this paper), this increasing skew overcomes the downward bias from the other two phenomena discussed earlier, resulting in the swing of the mean storage estimates into upward bias.

However, as  $n$  continues to increase, the distribution of  $S(n)$  becomes more stable with a mean approaching  $S(n \rightarrow \infty)$  and a decreasing  $C_v(S(n))$ , resulting from a more complete pattern of drought events (upon which  $S(n)$  depends) being included in the much longer sequences. As a result, the upward bias is reduced and the mean storage estimates converge to the stationary level from above, although slowly, where more severe critical drawdown periods occur (as was observed in our simulations for smaller  $m$  values, as in Figure 2, and larger inflow  $\rho_1$  values, as in Figure 3). This explains the appearance of the humps in mean and upper quantile storage estimates observed for some of the cases examined.

#### **Conclusions**

In summary the overyear storage estimates derived using concatenated behavior analysis are significantly influenced by the length of annual inflow sequence analyzed. Storage estimates approached a stationary level by about 1000 years or more for all cases examined. For combinations of high draft and high inflow  $C_v$  (associated with  $m$  values below about 0.5) a significant swing of the mean storage estimates from an initial downward bias into an upward bias was observed before their convergence to the stationary level. This behavior was accentuated by increased draft and  $C_v$ , as well as by increasing levels of serial correlation in the inflow sequences, and was also observed in some cases where nonconcatenated behavior analysis was performed with a fixed initial storage condition (for example, 0.1 and 0.3 times capacity as in Figure 4a).

The analysis reported in this paper illustrates that the problem of computing estimates of the storage volume needed to supply a specified demand with a given annual reliability is much more complicated than initially realized, particularly for

systems with high levels of draft. The S-R-Y relationship estimated using behavior analysis appears to be in a transient state for annual inflow sequence lengths up to about 1000 years before a stationary relationship is achieved, and this must be understood where any variant of behavior analysis (concatenated or not) is used. This is important given that behavior analysis has been recommended by *McMahon and Mein* [1986] as a storage design technique and is widely used outside the United States. Users of behavior analysis should also have an appreciation of the potential for the mean and upper quantile storage estimates to swing into an upward bias before converging to a stationary level for cases where *Hazen's* [1914] nondimensional standardized inflow parameter,  $m$ , is less than about 0.5. Finally, beyond the bias issue there is the additional question (although not addressed in this paper) of how many streamflow replicates are required to obtain a mean or quantile estimator with the desired precision.

Planners should realize that a new reservoir will not have the long run reliability in its early years of operation. For example, in practice a reservoir initially needs to start filling, while its demands may not have fully developed thus giving it more opportunity to fill. Long run S-R-Y relationships are only benchmarks for planning and should not be used in an attempt to describe operation in the early years of a project. Further, it should be realized that in future decades the mix of demands on releases and storage levels will alter. S-R-Y relationships provide rough estimates of reliability that allow different project proposals for one set of water demands to be compared, as well as some general comparison of the reliability of systems developed by different water uses to meet different demands in different regions of the world. It seems reasonable to ask how big a reservoir would need to be to provide the target water deliveries with a reasonable reliability.

Nevertheless, the results reported in this paper raise the question as to how behavior analysis should be used to estimate storage capacity. The answer depends on which aspect of reservoir operation is important for the analysis at hand. Where planners are interested in long run S-R-Y relationships for comparing alternative projects and systems, the bias in mean and median storage capacity estimates should be negligible if replicate inflow sequences are at least 500 to 1000 years in length. However, where planners are interested in reservoir operation over short planning horizons, the length of the planning horizon of interest will dictate the length of inflow sequences which should be used in conjunction with the appropriate initial reservoir volume. In such instances, planners should be aware of the issues raised in this paper which can have significant impacts on the simulations and resulting storage capacity estimates.

This study has been concerned only with overyear S-R-Y relationships for fixed draft and reliability. There is also a need to examine the case of a fixed storage capacity with varying draft and reliability, as is the case in many practical instances involving reservoir planning. For such cases, because of expected lower levels of sensitivity of draft and reliability compared to capacity, it may be that reliability and draft for a given storage capacity are much less influenced by inflow sequence length than is storage capacity for fixed reliability and draft.

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